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DAMAGE DETECTION IN PLATE USING GRADIENT SEARCH SENSITIVITY METHOD OF FE MODEL UPDATING

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Abstract. *Damage Detection in the structure is an active research area since decades. Number of the research studies has been done to meet this objective, particularly, the identification of the damage size and location. The finite element (FE) updating methods have also been used in the literature for the damage size and location identification. However, most of the studies are generally related to the beam type structures with the crack/damage. This concept has now been extended to the plate type structure with damage. Here again, the modal properties (natural frequencies and mode shapes) of the damaged plate have been used to estimate the damage size assuming the location of the damage was known. In the present study, the damage in the plate has been simulated in the FE model of the plate by the reduction in the plate thickness in the small portion of the plate and then the thickness of the plate has been chosen as the updating parameter. The Gradient Search based Sensitivity Model Updating method has then been applied to calculate the plate thickness for the damaged area from the FE model of the healthy plate (without damage). The proposed method has been examined on a simple simulated example of a steel plate. The paper presents the proposed method and the result from a simulated example.*

1 INTRODUCTION

Accurate condition assessment of engineering structures has become increasingly important. Current damage detection methods are either visual or localised experimental methods such as acoustic or ultrasonic methods, magnetic or thermal field methods, etc. [1]. All of these experimental techniques require that the vicinity of the damage is known a priori and that the portion of the structure being inspected is readily accessible. Furthermore, these tests are cumbersome and expensive. The need for quantitative global damage detection methods that can be applied to complex structures, has led to the development of methods that examine changes in vibration. Doebling et al. [2] give an extensive overview of vibration-based detection methods. The vibration characteristics of a structure are the function of its damping, stiffness and mass which in turn are a function of the condition of the structure. Among the various dynamic parameters which are used for damage detection, modal frequency and mode shape are the ones most commonly used. Salawu [3] presents a review on the use of modal frequency changes for damage diagnostics. It was later on confirmed that the modal frequencies are insensitive to the local condition of the structure and as such has limitations for damage isolation. In order to isolate the damage, mode shapes are necessary, which increase the data handling as well as increased instrumentation for accurate measurement. In order to overcome these shortcomings, several different parameters were studied, like the mode shape curvature [4-5], dynamically measured flexibility [6-7], damping [8-9] etc. All the monitored parameters have their set of advantages and disadvantages and the search of an elusive method which can overcome all the problems faced during damage detection is still on.

In recent years, sensitivity-based Finite Element (FE) model updating has been successfully used for damage assessment. In general, FE model updating aims at adjusting the dynamics of the mathematical model so that it becomes close to the experimental model. Many methods have been proposed for the updating of the mathematical model [10]. Some of the methods are direct while others are iterative. The direct methods compute a closed-form direct solution for the global stiffness and/or mass matrices using the structural equations of motion and the orthogonality equations. The changes made in the global matrix do not often make any physical sense. The iterative methods update the physical model variables like the Young's modulus, density, thickness, etc. by minimising the differences between the numerical and experimental vibration data. This gradient based sensitivity approach is an iterative process making use of updating variables to alter the stiffness matrix so as to achieve matching dynamics of the mathematical and the experimental models.

The present paper deal with determination of the extent of damage in plates using the gradient search sensitivity based approach when the location of damage is assumed to be known. The FE model of the plate was developed in the commercial FE code ABAQUS. The computed modal parameters have then been integrated with the computational code developed in the MatLab to calculate the sensitivity matrix and the updating parameter (plate thickness) by minimising the error in the penalty function by the iterative process till the problem converges to the required solution. The proposed method has been examined on a simple simulated example of a steel plate. The paper presents the proposed method, results from a simulated example and possible extension of proposed method to identify damage location as well as its size to meet the requirement of Level 3 in Damage Detection.

2 THEOROTICAL BACKGROUND

The error function at *n*th iteration is given by [10]

$$\boldsymbol{\varepsilon}_n = \boldsymbol{\delta}\mathbf{z}_n - \mathbf{S}_n \boldsymbol{\delta\theta}_n \tag{1}$$

where, $\boldsymbol{\delta}\mathbf{z}_n$ is the vectors of the eigen values error between the calculated experimental natural frequencies and corresponding computed natural frequencies from the FE model assuming only modes are used for this purpose.

S_n is the sensitivity matrix which is the first derivative of the eigenvalues with respect to the updating parameters

$\delta\theta_n$ is the change in the updating parameter at the n^{th} iteration.

The size of the sensitivity matrix depends on the number of updating parameters and number of modes used for this purpose. The penalty function (*J*) at the n^{th} iteration is given by

$$J(\boldsymbol{\delta\theta}) = \boldsymbol{\varepsilon}_n^T \mathbf{W} \boldsymbol{\varepsilon}_n \tag{2}$$

where ***W*** is the positive diagonal weighing matrix. The vector of the updating parameters can be calculated by minimizing *J* with respect to $\delta\theta$ which involves differentiating of *J* with respect to each element of $\delta\theta$ and setting the result equal to 0. The solution obtained at each step is a weighted least square solution. Finally this leads to the following equation for the vector of updating parameters at each iteration.

$$\boldsymbol{\theta}_{n+1} = \boldsymbol{\theta}_n + (\mathbf{S}_n^T \mathbf{W} \mathbf{S}_n)^{-1} \mathbf{S}_n^T \mathbf{W} (\boldsymbol{\delta}\mathbf{z}_n) \tag{3}$$

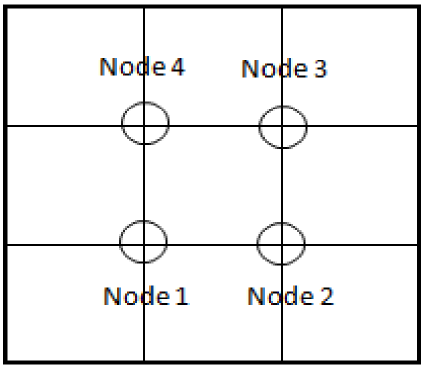
The iterative process will be carried out till the targeted accuracy is reached. The target accuracy determines the computational effort needed to be put in and the target should be optimal to ensure efficient computation. The elements of the sensitivity matrix can be computed as [10-12].

$$\mathbf{S}_{i,j} = \frac{\partial \lambda_{ci}}{\partial \theta_j} = \boldsymbol{\varphi}_i^T \left[\frac{\partial \mathbf{K}}{\partial \theta_j} - \lambda_{ci} \frac{\partial \mathbf{M}}{\partial \theta_j} \right] \boldsymbol{\varphi}_i \tag{4}$$

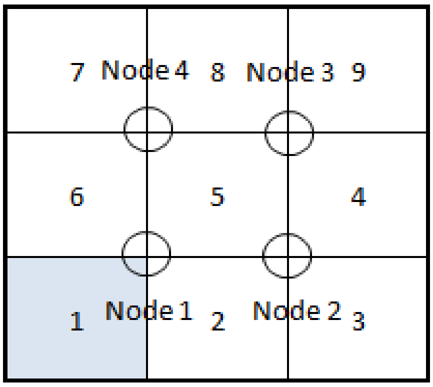
where, $\mathbf{S}_{i,j}$ is the element of the sensitivity matrix for the computed *i*th eigenvalue (λ_{ci}) w.r.to the *j*th updating parameter (θ_j). The system matrix, **K, M** are the stiffness and mass matrices respectively and $\boldsymbol{\varphi}_i$ is the normalized eigenvectors for the *i*th mode.

3 EXAMPLE

A test plate of 1.2 × 1.2 m and thickness of 1mm has been considered. The material of the plate was assumed to be steel with Young's Modulus 210GPa, Poisson's ratio 0.33 and density 7800kg/m^3. For the simulated example, the plate is divided into 9 elements as shown in Figure 1(a) and all the 4 edges of the plate were assumed to be fixed supports. It has also been assumed that the Nodes 1 to 4 are measured location for vibration during modal testing using tri-axial accelerometers. Hence the measured degree of freedoms (DoFs) at each measurement node is 3 (2 in plane and 1 out of plane). The thickness of the element 1 is assumed to be 0.8mm (20% less from the original thickness of 1mm) which represents the damage in the plate.



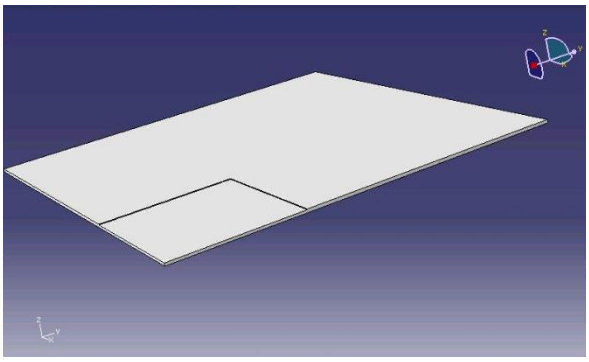
(a) Plate with 4 nodes for vibration measurement



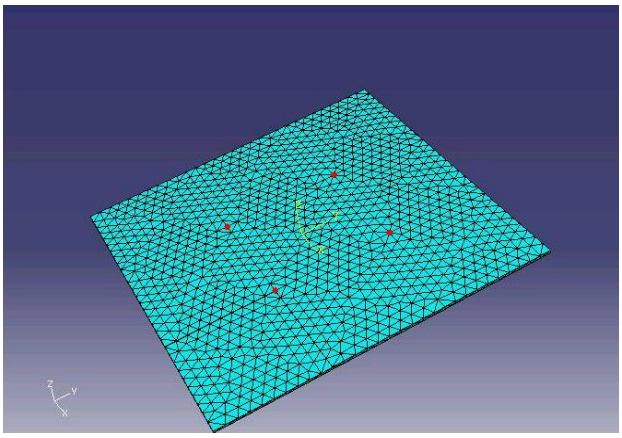
(b) Damage in Element 1 asuumed

Figure 1 Simulated example of a damage plate

An FE model of the simulated example of a damage plate shown in Figure 1 has also been constructed in the FE code ABAQUS which is shown in Figure 2. The FE model has then been used to calculate the natural frequencies and the mode shapes related to 12 DoFs at 4 measurement locations shown in Figure 1. The calculated natural frequencies are assumed as the experimental natural frequencies (Target data) which are listed in Table 1.



(a) Plate geometry in FE model



(b) FE mesh

Figure 2 An FE model of the plate shown in Figure 1

3.1. Damage Detection

Now it has been assumed that the damage location is known but the extent of the damage is unknown. Hence the flexural rigidity of the plate $\left(D = \frac{Et^3}{12(1-\nu^2)}\right)$ has been chosen as the updating parameter. It is because the flexural rigidity is nearly proportional to the plate stiffness matrix, **K**. A computation program has also been written the MatLab code to calculate the sensitivity matrix using Equation (4) based on the calculated natural frequencies and the modeshapes (eigenvalues) at the measured 12 DoFs and then iterative process as per Equation (3). Initial guess of the damage is assumed to be 0.7mm to start the iteration. The first 6 modes are used for this iterative process. The natural frequencies for initial guess are also listed in Table 1. At the end of each iteration, the change in the updating parameter, *dD,* is converted to the change in the plate thickness, *dt*, as

$$dt = \sqrt[3]{\frac{12dD(1-\nu^2)}{E}} \qquad (5)$$

The calculated new thickness of the plate is then used into the FE model to estimate natural frequencies and mode shapes for the next iteration. The process is continued still convergence. Table 1 list the updated thickness and natural frequencies with iteration and the error at each mode with iterations in Table 2 and graphically in Figure 3 which confirms the success of the iterative technique proposed.

Table 1 Updating parameter and the natural frequencies with iterations

Iteration		1	2	3	4	5
Extent	0.8	0.7	0.73	0.75	0.77	0.8
Mode 1	280.411	281.343	281.618	280.703	280.755	280.523
Mode 2	589.161	590.495	591.287	589.201	589.186	589.159
Mode 3	597.970	601.914	603.244	599.809	598.957	597.796
Mode 4	846.398	851.671	854.073	848.879	848.099	847.038
Mode 5	1030.060	1034.770	1035.840	1032.180	1030.870	1030.670
Mode 6	1053.130	1060.240	1061.600	1057.020	1054.480	1053.880

Table 2 Error at each mode with iterations

Iteration	1	2	3	4	5
Extent	0.7	0.73	0.75	0.77	0.8
Mode 1	-0.332	-0.430	-0.104	-0.123	-0.040
Mode 2	-0.226	-0.361	-0.007	-0.004	0.000
Mode 3	-0.660	-0.882	-0.308	-0.165	0.029
Mode 4	-0.623	-0.907	-0.293	-0.201	-0.076
Mode 5	-0.457	-0.561	-0.206	-0.079	-0.059
Mode 6	-0.675	-0.804	-0.369	-0.128	-0.071

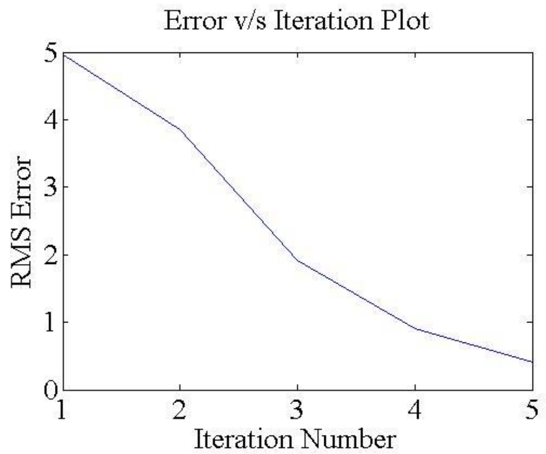


Figure 3 Iterative process showing convergence

4 CONCLUSION

The model updating method has been proposed to find the extent of damage in a plate type structure. The proposed method has been validated through a simple simulated example but it needs further validation through the experimental examples to enhance the confidence level in the proposed method. Currently just one updating parameter has been used but the proposed method can further be extended such that it can identify both location and size of damage in the plate.

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