COMPDYN 2011
III ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis, V. Plevris (eds.)
Corfu, Greece, 25–28 May 2011

BENDING CYCLIC LOADING ON PRESSURIZED ELBOWS – FINITE ELEMENT ANALYSES

Jan Ferino[1], Elisabetta Mecozzi[2], Antonio Lucci[2],Giuseppe Demofonti [2]

[1] Centro Sviluppo Materiali S.p.A
Pula, Italy
j.ferino@c-s-m.it

[2] Centro Sviluppo Materiali S.p.A
Roma, Italy
e.mecozzi@c-s-m.it, a.lucci@c-s-m.it, g.demofonti@c-s-m.it

Keywords: Instructions, ECCOMAS Thematic Conference, Structural Dynamics, Earthquake Engineering, Proceedings.

Abstract. *Industrial piping can be subjected to severe cyclic loading associated to temporary ground deformations induced by seismic activity. One of the reasons of piping failures under earthquakes can be identified in the accumulation of plastic strains along with biaxial ratcheting of elbows subjected to in-plane cyclic bending. This paper deals with the estimation, using finite element analysis, of the strain evolution in a representative OD 8" X52 grade piping section including an elbow subjected to cyclic in-plane bending and internal pressure.*

Lemaitre-Chaboche nonlinear kinematic hardening model for plasticity has been employed in FEA to represent the ratcheting phenomenon more efficiently with respect with standard kinematic hardening rule.

Numerical results provide an estimation of the plastic strain evolution at critical locations, which may result in crack formation or material damage, and valuable information to address a full future scale testing program that will be undertaken within the INDUSE project

1 INRODUCTION

CSM is involved in the European RFCS research program INDUSE [1] which is aimed at developing design guidelines and recommendations, for the seismic analysis and design of industrial equipment steel structures, such as liquid storage tanks, pressure vessels and piping. In this context, CSM is in charge to perform experimental and numerical activities on piping elbows subjected to cyclic in plane bending loading, involving large imposed displacements in the presence of internal pressure.
This paper deals with the estimation, using finite element analysis, of the strain evolution in a representative 8” piping section including an elbow subjected to cyclic in-plane bending and internal pressure. Numerical results provide an estimation of the plastic strain evolution in critical locations, which may result in crack formation or material damage, and valuable information to address the future full-scale testing program.

The behavior of pipelines under large imposed displacements typical of earthquake induced permanent ground deformations (PGD) and other seismic phenomena such as liquefaction and subsidence etc. is recognized to be one of the most important issues in pipeline research. In fact pipelines are frequently required to cross areas where human activity is very intense. Piping components such as bends and elbows connections are recognized to be critical as the deformation tend to concentrate in such locations due to their lower stiffness. It is not surprising that an important experimental and numerical activity has been undertaken in Japan as a consequence of the intense seismic activity. [2] studied the deformation and cracking behavior of induction bends X80 grade subjected to closing bending moment and internal pressure by means of experimental and numerical analysis. Closing and opening bending experiments with internal pressure were conducted by [3] while [4] studied closing bending mode by means of FEA. Ultimate bending capacity of pressurized elbows for the in plane and out of plane bending load has been extensively studied by [5]and [6] by comparing experimental results with analytical formulations and numerical FE predictions.
Several failures have also been recorded for not buried industrial subjected to repetitive alternating loads induced by temporary ground deformations (TGD). In such conditions once again piping connections are subjected to severe deformations as the presence of internal pressure causes biaxial ratcheting. Under such loading conditions trough wall cracking is reported to be one of the main causes of failure as a consequence of the accumulation of plastic strains in the hoop direction at flank position. Recently [7] studied the cyclic behavior of elbows without pressure while [8] investigated also the biaxial ratcheting effect caused internal pressure.

2 FE ANALYSIS

2.1 Geometry overview

The geometry of the piping section analyzed is shown in Figure 1 and its relevant characteristics are reported in Table 1: Relevant characteristics. The elbow element is connected to two straight pipes 1100 mm long and characterized by the same thickness. The length is enough to ensure that no stress interference in the elbow area deriving from the applied loads at the ends occurs. The bending load on pipe elbow has been obtained imposing an alternating displacement along the line
connecting the two pipe ends

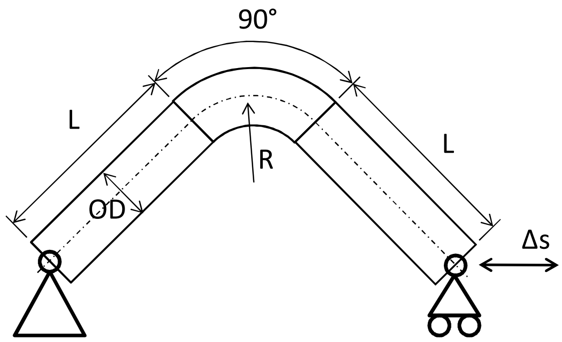

Figure 1: Overview of the studied geometry.

Outside Diameter (D) (mm)	Wall Thickness (T) (mm)	D/t	R/D	Elbow angle (°)	Straight pipe length (L) (mm)
219.1	8.2	26.7	1.5	90°	1100

Table 1: Relevant characteristics

To investigate the effect of the internal pressure during the cyclic deformation process two levels have been considered: zero pressure and a pressure associated with the 50% of SMYS design factor according to the following relation:

$$p = \frac{k \cdot SMYS \cdot 2t}{D} \quad (1)$$

with:

D	:	pipe outer diameter
t	:	Pipe wall thickness
$SMYS$	:	Specified Minimum Yield Strength
k	:	design factor

Internal pressure has been applied first and then maintained constant during the whole cyclic bending. Pressure have also been applied to the pipe ends to account for end cap effect axial induced load. A summary of the load cases analyzed have been is reported in the following Table 2

Load type	Load sequence	Displacement amplitude mm	Internal pressure MPa
Monotonic	opening mode	200mm	0 13.25 (u.f. = 0.5)
	closing mode	200mm	0 13.25 (u.f. = 0.5)
Cyclic	1st cycle opening	+/- 100 mm	0 13.25 (u.f. = 0.5)
	1st cycle closing		0 13.25 (u.f. = 0.5)
	1st cycle closing	+/- 50 mm	0 13.25 (u.f. = 0.5)

Table 2: Summary of analyzed load cases

2.2 Finite element model Description

As shown in Figure 1a representative section of a piping containing an elbow and two sleeve pipes has been modeled by using the commercial software MSC.Marc. To allow for a better estimation of the effect of the accumulated plastic strains on the wall thickness during cycling solid elements have been employed.

Exploiting the planes of symmetry of the geometry and loading conditions the modeling was limited to one fourth of the whole geometry, applying appropriate boundary conditions, thus allowing for more detailed representation of the wall thickness. The mesh adopted and a detail of the welded connection between elbow and pipes are shown Figure 2.

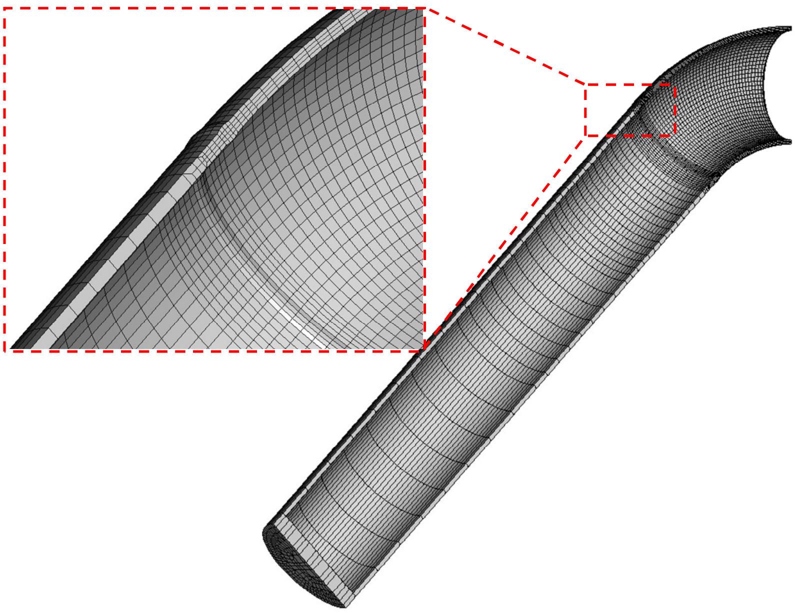

Figure 2: Overview of the mesh employed

The element type used is the three dimensional arbitrary distorted brick which is based in a full integration formulation containing 8 integration points.
To take into account the stresses and strains intensification of, the presence of the welds between the elbow and the straight pipes has been modeled adopting a 1mm increase of thickness in the weld cap. No specific material properties have been assigned to the weld.
In order to reduce calculation resources different element sizes have been used for the mesh the model along length. .
A total amount of 13609 nodes and 9674 elements have been used. More in detail the following subdivision has been implemented:

- 48 elements along elbow axial direction, with thickening close to the weld area;
- 35 elements along straight pipe axial direction, with a refining close to the weld area and a coarsening proceeding towards pipe ends;
- In the circumferential direction a number of 40 elements has been used.
- 4 elements have been used through the wall thickness for the whole elbow length,
- a gradual transition from 4 to 1 element have been adopted trough the wall thickness of straight pipe connected to the elbow;
-

The accuracy of the adopted mesh in describing the elastic plastic behavior of the piping and the concentration of stresses and strains at critical location has been verified trough sensitivity analyses.
To properly account for high deformations that are expected to occur during cyclic process, the following options have been activated for the simulation:

- large displacements;
- large strain additive strain decomposition;
- constant dilatation (incompressible material in plastic range).;

2.3 Material model employed in FEA

The currently available FE software are able to predict with good accuracy the behavior of the material in elastic-plastic regime under monotonic loading, even for a multi-axial stress state. On the other hand, cyclic behavior involves much more complex phenomena which interacts together during loading: ratcheting, hardening or softening, mean stress relaxation etc.
The non linear kinematic hardening, or Lemaitre-Chaboche rule [9] has been adopted to describe of the cyclic material properties.
This model eliminates the typical shortcomings of the standard kinematic hardening rule to describe the cyclic phenomena. Bauschinger effect is better represented as the transition from elastic to plastic regime at strain reversal is smoother, ratcheting, mean strain relaxation, cyclic hardening and softening can also be modeled.
More in detail, this model combines a non linear kinematic rule and an isotropic hardening rule so that Von Mises yield surface undergoes to the following evolution law:

$$F = J_2(\sigma_{ij} - \alpha_{ij}) - R - k = 0, \tag{2}$$

Where:

$$J_2(\sigma_{ij} - \alpha_{ij}) = \sqrt{\frac{3}{3}\left(\sigma'_{ij} - \alpha'_{ij}\right) \cdot \left(\sigma'_{ij} - \alpha'_{ij}\right)} \tag{3}$$

$\sigma_{ij'}$ and $\alpha_{ij'}$ are respectively the deviatoric components of the stress tensor and the back-stress tensor, the latter tracking the position of the centre of yield surface. R is a scalar variable which accounts for the isotropic expansion or contraction of the surface during cycling loading and is associated with the accumulated plastic strain ε_e^p and k is the initial elastic limit of the material. The evolution law for α_{ij} is:

$$d\alpha_{ij} = 2/3 \cdot C \cdot d\varepsilon_{ij}^{p} - \gamma \cdot \alpha_{ij} \cdot d\varepsilon_{e}^{p} \quad (4)$$

The previous equation (which was initially proposed by Armstrong-Fredrick) is composed by a linear term which is identical to the Prager rule and expresses a proportionality between σ_{ij} and ε_{ij}^{p} to which a nonlinear term function of the accumulated plastic strain is superimposed was initially proposed by Armstrong-Fredrick. The evolution law for the R variable is:

$$dR = b \cdot (Q - R) \cdot d\varepsilon_{e}^{p} \quad (5)$$

Current version of MSC Marc® only provides the basic Lemaitre-Chaboche rule, while more advanced versions of this model, which adopt a superposition of multiple hardening rules each with a own set of parameters exist. Following values of characteristics parameters, obtained from literature [10] have been adopted.

- k = 250 MPa;
- C = 75000 MPa;
- γ = 400.

Within the INDUSE research program base material characterization including true stress true strain monotonic curves as well as cyclic hardening and ratcheting experiments are planned allowing for Lemaitre Chaboche model parameters to be determined.

3 FE ANALYSES RESULTS

3.1 Monotonic behavior

The monotonic behavior of the piping section has been initially analyzed in order to study its basic response to simple bending in opening and closing mode. An imposed displacement of 200 mm has been applied on pipe ends and internal pressure effect have been also considered. The response in terms of applied bending moment at symmetry section of the elbow versus curvature calculated for the whole elbow length is reported in Figure 3, where the elbow curvature is initially shifted to zero for easier reading of results, it is noticeable that the effect of the internal pressure clearly influence the results.

In closing mode after the peak load is reached, there is a slow decrease of the bending moment indicating that buckling takes place. For low D/t values as in the case under study no wrinkling of the compressed side takes place, instead buckling is driven by ovalization which leads to a reduction in the section moment of inertia. Initially the material work hardening counteracts this reduction until a compensation of two competing effect is reached thus resulting in a maximum of the bending moment.

On the contrary in opening mode the moment of inertia is increased due to the orientation along the bending plane of the maximum diameter elbow. Thus work-hardening and ovalization sum their effect. In this case a collapse for high tensile strains is expected.

In closing mode an increase of internal pressure can be considered as beneficial as it increases both the peak moment and the corresponding curvature. In this case the internal pressure counteracts the ovalization of the pipe cross section which is developed along the line connecting the theoretical neutral axis on both flanks of the elbow, and is responsible of the decrease of the moment of inertia of the pipes.

In opening mode the behavior is different: at increasing bending the moment arm reduces so that always more axial tension force is needed to bend the elbow. Such high value of axial force influences the results and decreases the effect of the internal pressure. In fact passing from zero pressure to 13.2 MPa there is only a slight change in peak moment and corresponding curvature. Furthermore the pressure in this case is not beneficial as it contrast the elongation on elbow cross section along the line connecting the extrados and the intrados. In presence of pressure for the opening mode the failure mode is the necking of the intrados pipe wall as shown in Figure 6.

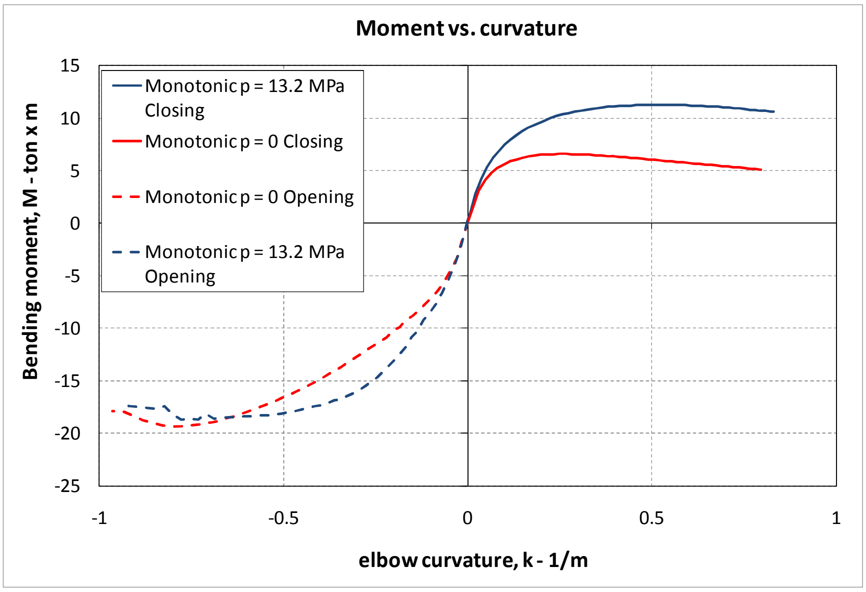

Figure 3: Monotonic bending response in opening and closing mode with and without internal pressure

Opening and closing mode ovalization shape of the elbow cross section are reported in Figure 4 together with tee original un-deformed section. In case of opening the effect of the internal pressure is clearly visible and has the effect to maintain the section more circular. In closing mode, for pressure case, the ovality at peak moment is greater because pressure allows for a larger curvature to be reached.
Errore. L'origine riferimento non è stata trovata. and **Errore. L'origine riferimento non è stata trovata.** show the critical areas, in terms of concentration of strains, of the elbow at peak moment condition with and without pressure, both in closing and opening mode. In case of closing mode the critical position is the flank of the elbow where high strains in circumferential direction are observed. In opening mode if no pressure is applied high tensile strains in axial direction are observed in the intrados of the elbow in the half length section as a result of the combined effect of bending and tensile axial load. With the internal pressure the position of the critical section moves along intrados slightly away from half-length section causing tearing of the pipe wall due to high tensile strains in axial direction.

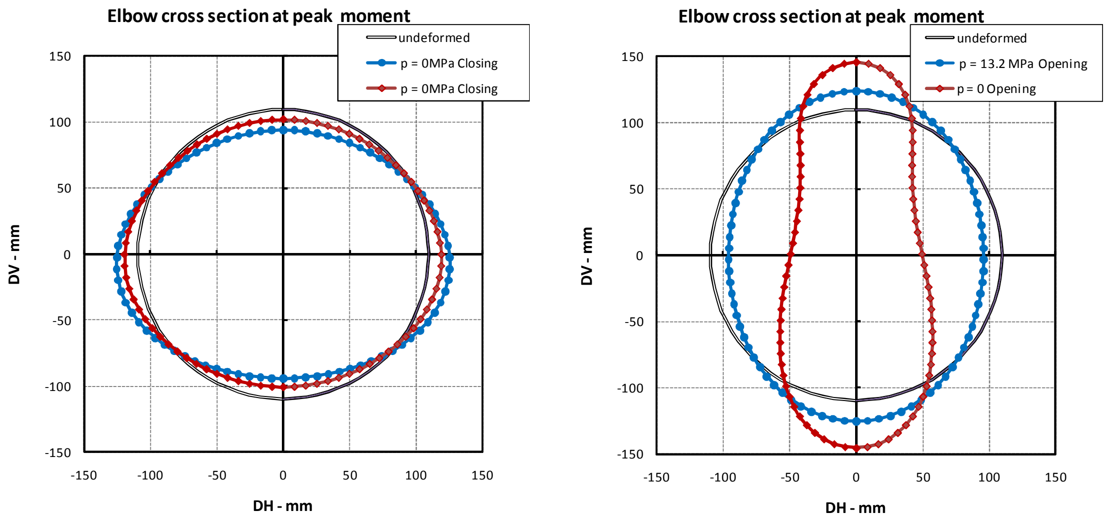

Figure 4: Cross section shape at half length of the elbow at peak bending moment in closing mode, left and in opening modem right

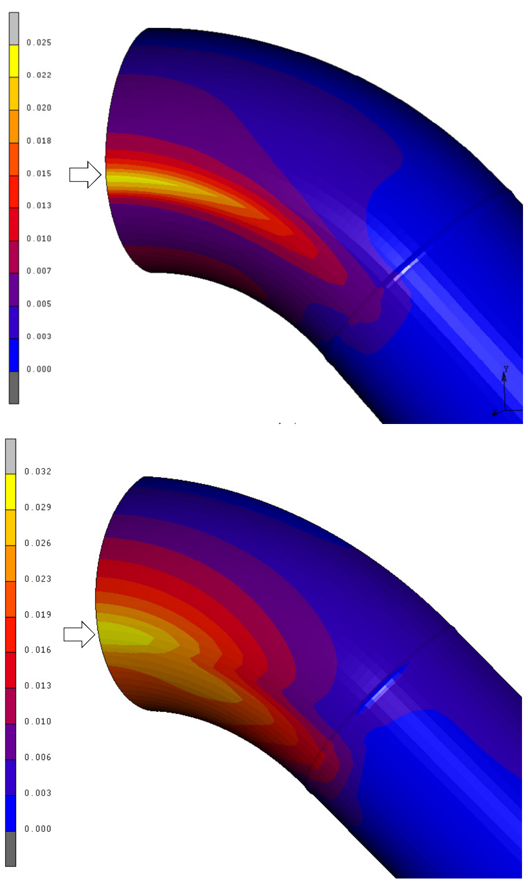

Figure 5: Accumulated equivalent plastic strains at peak closing moment at zero pressure, up and at 13.2MPa down

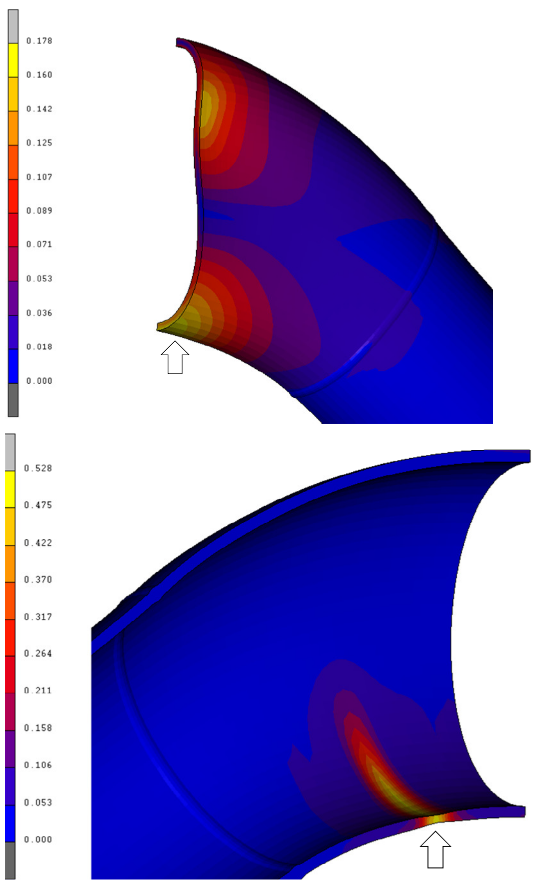

Figure 6; Accumulated equivalent plastic strains at peak opening moment at zero pressure, up and at 13.2MPa down

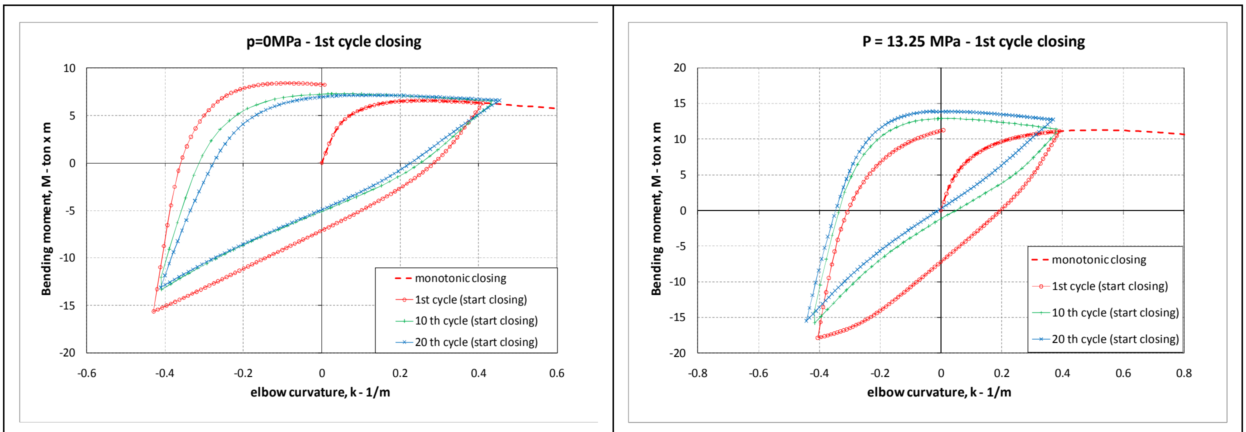

Figure 9: Cyclic response with zero pressure for cycling starting in closing mode: without pressure, left and for 13.2 MPa right.

3.2 Cyclic behavior

Cyclic behavior has been investigated for different values of internal pressure and imposed displacements of +/-100mm at one pipe end, this correspond to impose a curvature of +/-0.4m^{-1} a variation of +/-12% of the curvature at elbow yielding.
In **Errore. L'origine riferimento non è stata trovata.** the shape of the elbow cross section is reported for the zero pressure case, before cycling and after 20 cycles both for cycling starting in opening or in closing mode. The same result is reported by
, for the internal pressure case. In both cases shape differences totally have totally disappeared after 20 cycles.
The shape differs from the basic ones observed in opening and closing modes for monotonic loading, this is particularly true in the case of zero pressure case, where a downward shift of the maximum width amplitude accompanied by a high value of distortion in such locations can be reported. In general greater influence from closing mode is observed, in fact the ovality is always more pronounced along the horizontal DH diameter. This occurrence is present independently of the pressure and can be explained recalling the monotonic behavior for closing and opening bending moment reported in Figure 3: for the particular loading configuration considered, the resistance of the elbow in closing mode is much less if compared with opening mode both in terms of maximum moment than curvature. In other words a 100mm alternating displacement causes the curvature to be far beyond the curvature at peak closing moment but still below the critical curvature for opening moment.
As previously calculated, internal pressure of 13.2 MPa has the effect to introduce a constant hoop stress level of the 50% of SMYS, that in combination with stresses beyond yielding induced in longitudinal direction by bending process results in biaxial ratcheting.
Ratcheting effect is clearly visible in Figure 8 where it is possible to observe that the elbow underwent a marked cycle by cycle increase of perimeter. Furthermore hoop stress reduced the general distortion of the cross section and markedly increase the hoop deformations and consequently the perimeter.
Cyclic moment *vs* curvature diagrams for bending process starting in closing are reported in
, for 1st ,10th and 20th cycle both with zero pressure and for 13MPa of pressure. Very little difference are observed in stress strain distribution and in the moment curvature response if the load cycle starts in opening or closing mode. After a few cycles all differences disappear. and such diagrams are not reported for the opening starting mode.

If no pressure is applied the moment-curvature cycles are stable in position, while with pressure the bending moment is seen to shift in the closing moment part of the diagram. This can be explained by the increase of the moment of inertia of the pipe section caused by ratcheting. From Figure 8 in fact no decrease on DV is observed while DH increase.
To study the effect of the displacement amplitude on the cyclic bending process, two additional simulations where carried out. For the cyclic loading starting in closing mode, a displacement of +/-50mm was imposed at one pipe end, corresponding to a curvature of +/-0.2m^{-1} a variation of +/-6% of the curvature at elbow yielding.
The simulation was carried out up to 40 cycles twice the number of +/- 100mm load sequence. Deformed shapes observed at mid length are compared in Figure 10 and Figure 11 with the ones for the larger displacement case. It is possible to notice that independently of the pressure, raising the number of cycles and reducing the amplitude caused an increase of the DV diameter and a reduction on DH with respect to the lower cycles case analyzed. Furthermore in case of pressure DH is larger than un-deformed diameter.

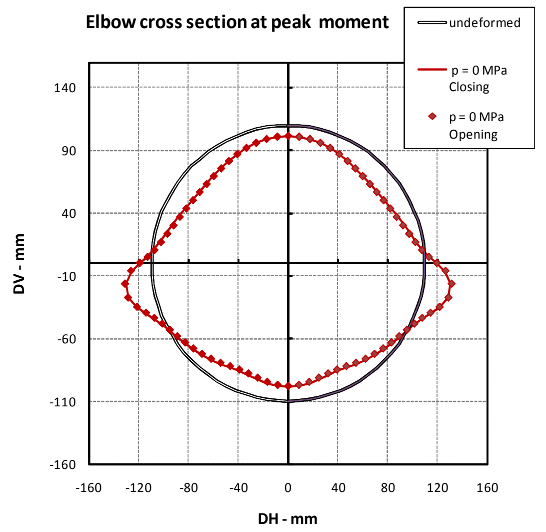

Figure 7: Cross section shape at half length of the elbow after 20 cycles for and imposed displacement of 100mm and zero internal pressure in case of 1st cycle in closing and opening mode

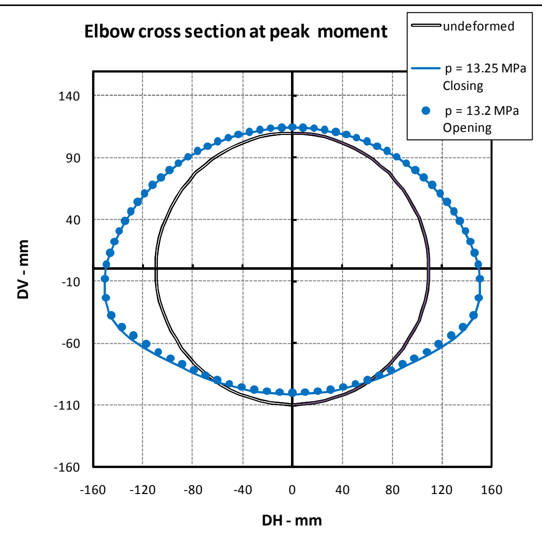

Figure 8: Cross section shape at half length of the elbow after 20 cycles for and imposed displacement of 100mm and 13.2 MPa internal pressure in case of 1st cycle in closing and opening mode

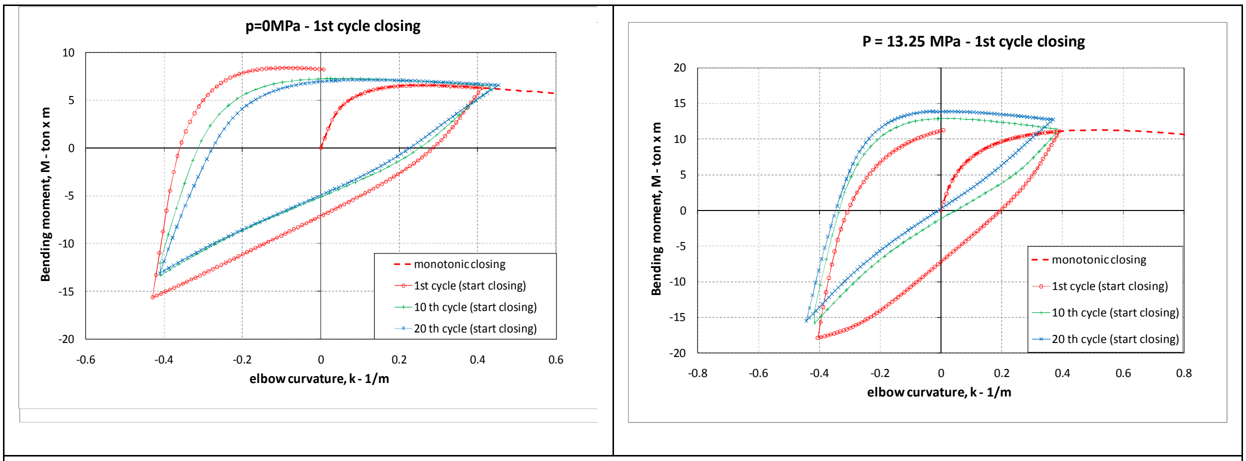

Figure 9: Cyclic response with zero pressure for cycling starting in closing mode: without pressure, left and for 13.2 MPa right.

Cyclic moment *vs* curvature diagrams for bending process starting with closing are reported in , for 1st ,10th and 20th cycles, with zero pressure and in the presence of 13.2MPa of pressure. While in absence of internal pressure the range of bending moment and curvature are stable internal pressure induces a progressive decrease of the average curvature and an increase of the bending moment at peak closing. This last effect can be explained by the increase of moment of inertia associated by DH growth due to pressure. In case of pressure cycles are also seen to translate in the opening part of the diagram indicating that a global straightening has taken place.

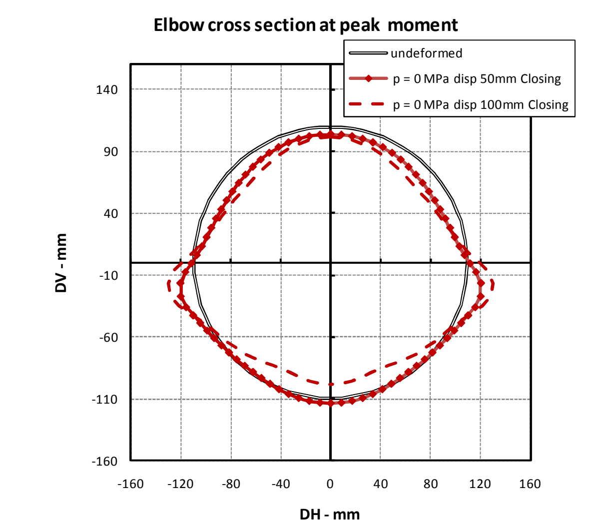

Figure 10: Cross section shape at half length of the elbow after cycling for imposed displacement of 100mm and 50mm at zero internal pressure, 1st cycle in closing mode

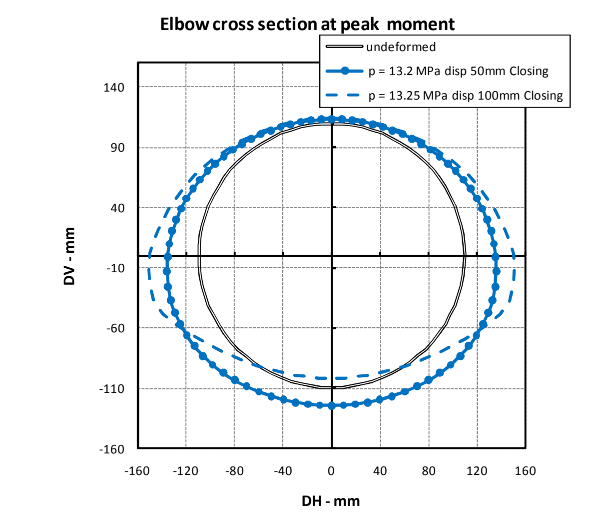

Figure 11: Cross section shape at half length of the elbow after cycling for imposed displacement of 100mm and 50mm at 13.2;Pa internal pressure, 1st cycle in closing mode

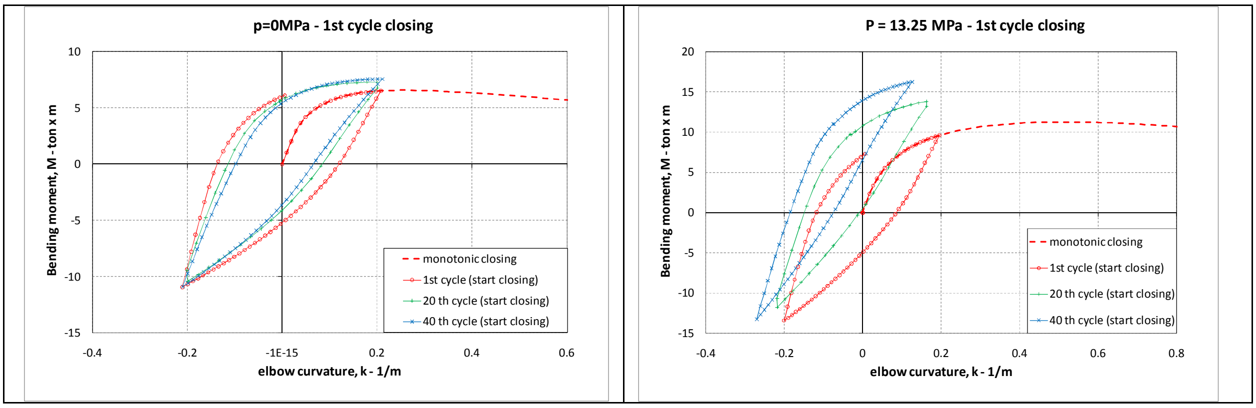

Figure 12: Cyclic response for +/50mm displacement amplitude with zero pressure for cycling starting in closing mode: without pressure, left and for 13.2 MPa right.

From Figure 13 and Figure 14 it is possible to observe the deformed shape of the elbow at the end of cycling process starting in closing mode. If no pressure is applied both load sequence lead to similar results; a sharp corner at flank is visible and high tensile hoop plastic strain is present at the external surface, while in the inner surface is compressive. When internal pressure is applied results differ for the two sequences considered: when larger cyclic curvature is imposed local buckling occurs producing an outward bulge. It is interesting to note that such behavior was not observed in monotonic compression even for larger curvatures and is to attribute to the combined effect of the steady internal pressure and accumulation of plastic strain in longitudinal direction.

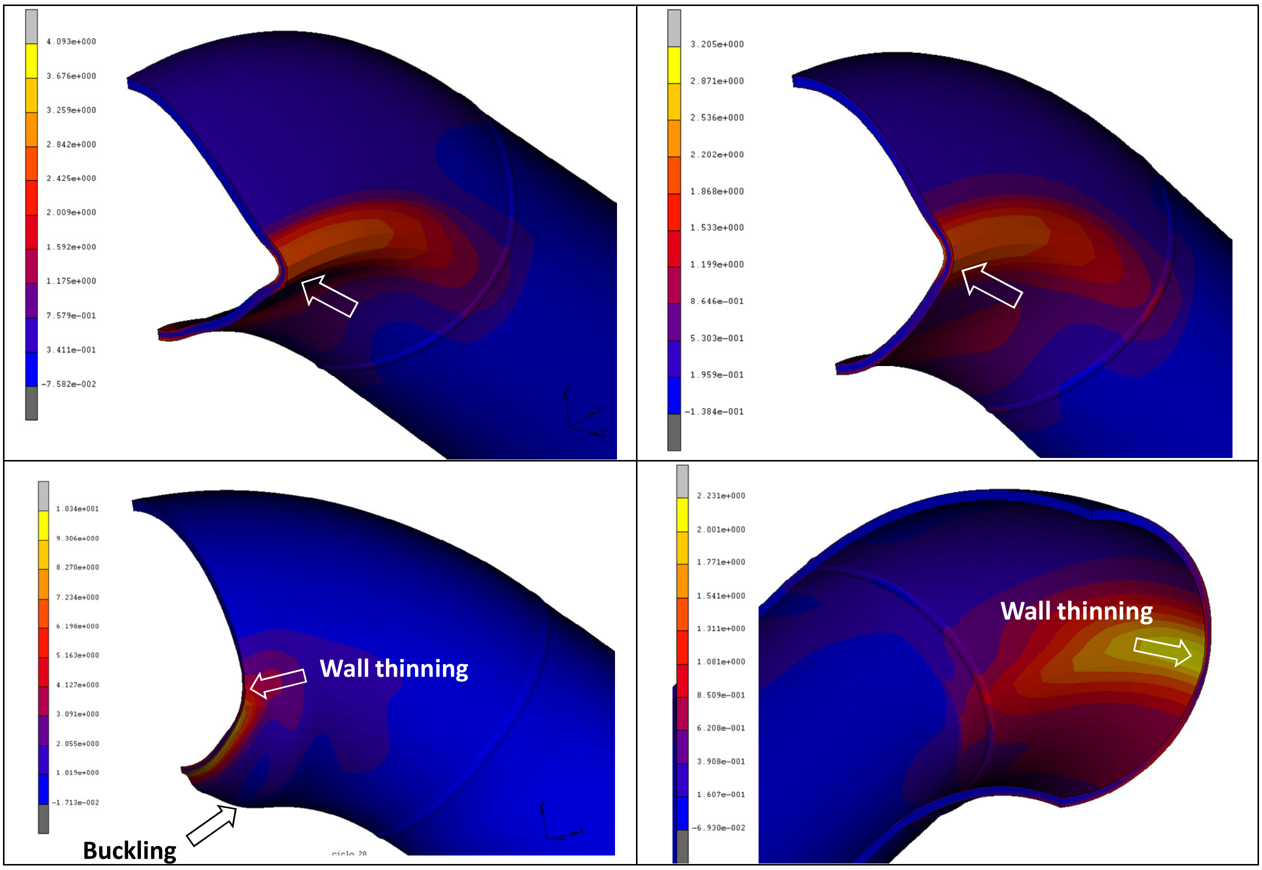

Figure 13: Accumulated equivalent plastic strains after 20 cycles at zero pressure, up and at 13.2MPa down.	Figure 14; Accumulated equivalent plastic strains after 39 cycles at zero pressure, up and at 13.2MPa down

Local strains in such critical locations have been traced along cycles in Figure 15 and Figure 16. Without internal pressure flank position is subjected to tensile hoop strain at the external surface and compressive at internal surface. Alternate tensile strains on pipe surface are to be considered critical because can induce surface cracking that can propagate through the pipe wall. For 20 cycles at larger displacement/curvature sequence tensile and compressive plastic strains have a larger range and tend to stabilize approximately at 15th cycle, while for smaller imposed curvature the range is narrower and an uniform growth is observed. It can be concluded that external surface is critical for the zero pressure case as high tensile plastic strain localize. The reduction of the imposed alternating curvature and the increase in cycle number leads to a slower and continuous increase of the mean values of the hoop strains.

As shown in Figure 16, if pressure is applied in both the internal and external pipe wall sides tensile hoop strains are detectable. For the larger displacements sequence hoop alternating range is very high in the internal surface of the pipe, so that this location has to be considered critical for crack penetration. No stabilization is observed and on both internal and external sides. When alternating displacement amplitude is reduced the rate of hoop strain growth reduces and never reaches the high values of the larger displacements sequence, even for 40 cycles. Hoop strains both in the internal and the external pipe wall are similar in value and trend being internal side slightly more critical.

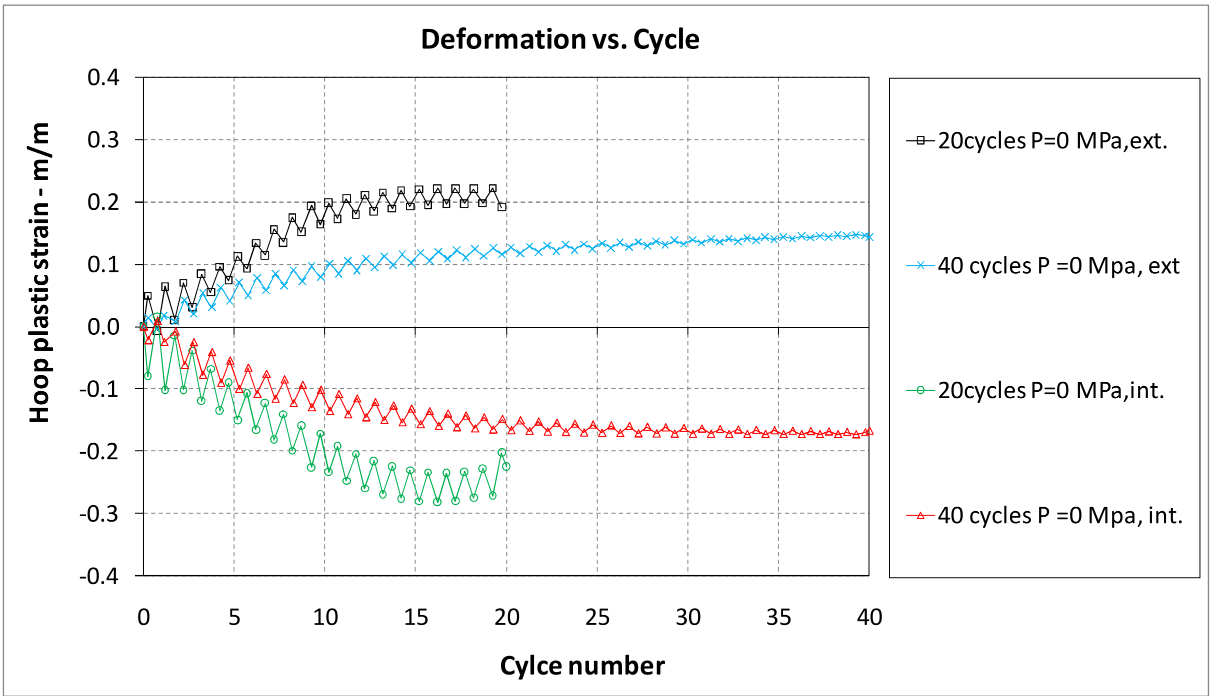

Figure 15:Hoop plastic strains at most critical locations during cycling without pressure

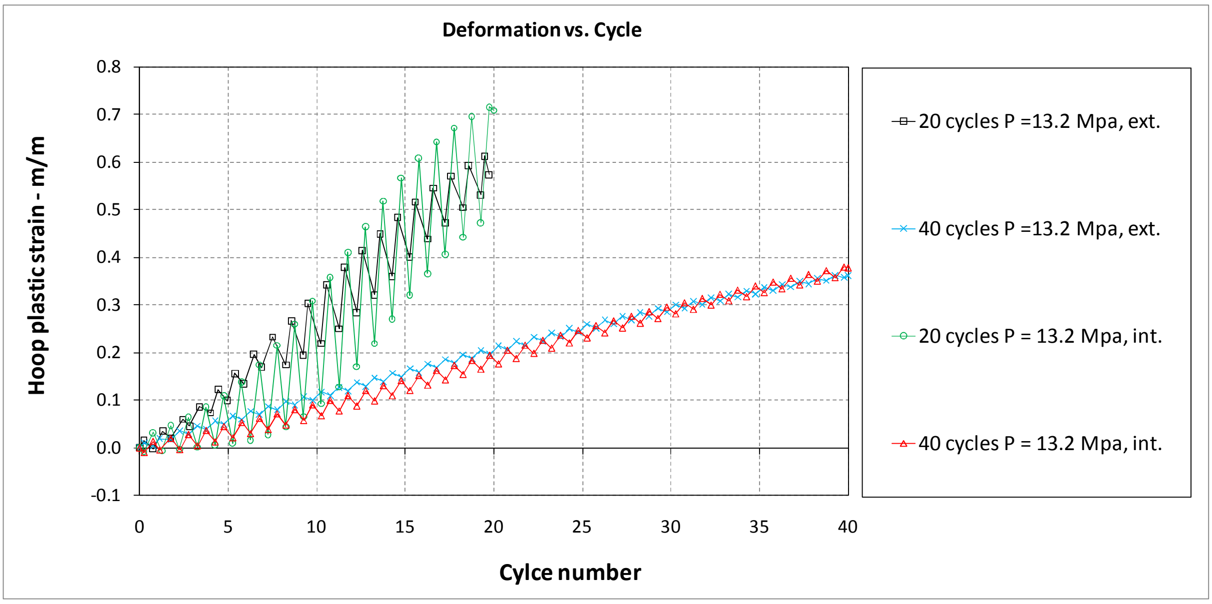

Figure 16: Hoop plastic strains at most critical locations during cycling with internal pressure

Figure 17 shows a comparison of all cyclic sequences starting in closing mode for pressurized and un-pressurized case based on the wall thickness reduction observed at mid length of the elbow.

If no pressure is applied wall thickness reduction is present in a region between the flank and the intrados. Increase of cycle number and reduction of imposed load amplitude results in a lower accumulation of strains at such locations.

The presence of the internal pressure induces high thickness reduction especially for the area between flank and intrados. Higher thickness reduction accompanied by a more irregular distribution is observed for the larger curvature sequence .In general the extrados does not seem to experience thickness significant variation as in the flank to intrados region. On the other hand, due to production process, wall thickness at the intrados pipe is slightly higher if compared with other parts of the pipe and can be considered less prone to cracking.

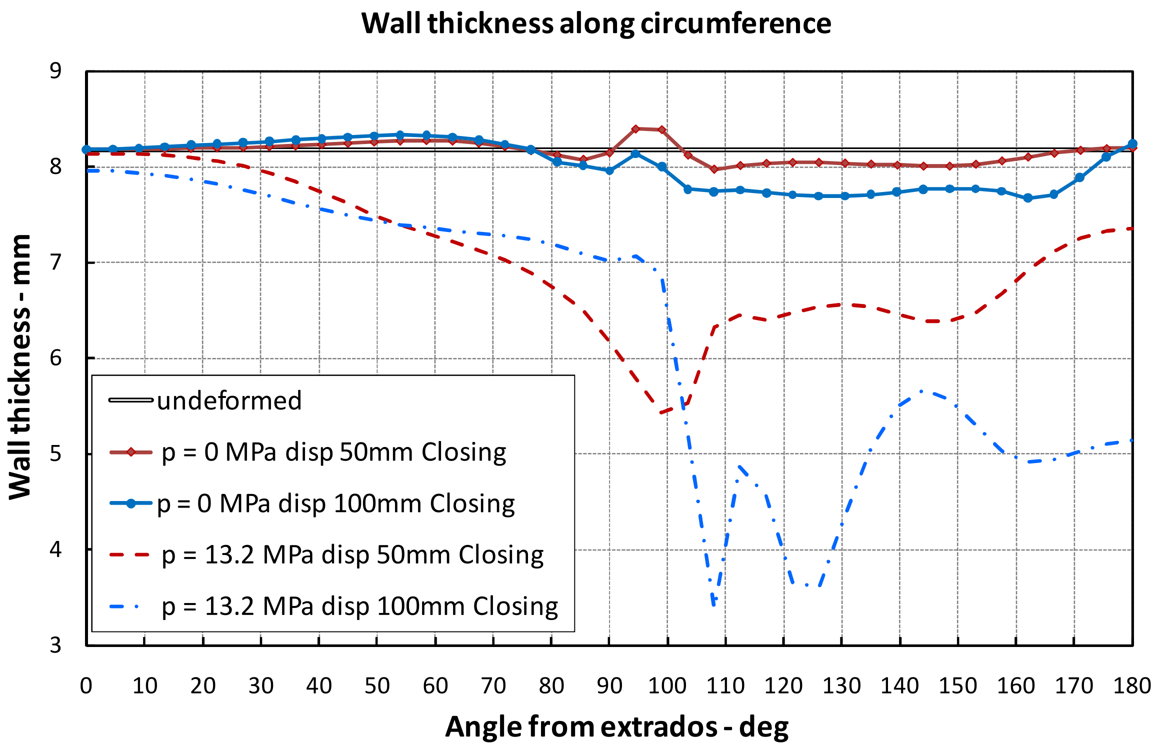

Figure 17: Wall thickness distribution along circumference

4 CONCLUSIONS

In this paper in plane bending of a representative 8" OD x 8.2mmWT grade API5L X52 industrial piping section containing a 90° elbow has been studied in this paper by means of finite element analysis, both in monotonic and cyclic conditions. The nonlinear kinematic hardening model (Lemaitre Chaboche) has been employed to take into account for biaxial ratcheting phenomenon.

Monotonic bending simulations show that opening mode show higher resistance capacity in terms of moment and curvature reached before failure, with respect to closing mode.

Cyclical load sequences starting in opening and closing mode have been investigated and after a few cycles the differences between the two load types, totally disappear.

Results show that, for the configuration presented for a symmetric alternating bending load sequence, closing bending deformation mode dominates the cyclic behavior. The most critical position is the elbow flank, both in presence and absence of the internal pressure.

The effect of the internal pressure have been investigated and results show that for monotonic case it can be considered beneficial in closing bending, while slightly negative in opening bending.

If cyclic loading is applied internal pressure induces biaxial ratcheting, causing the growth of large hoop strains that can lead to crack penetration risk. Large wall thinning is also observed.

For the particular range of imposed curvature/displacement considered the effect of reducing the cycles amplitude to one half while increasing up to twice the number of cycles, leads to a lower final accumulation of plastic strains in critical sections.

The performed analysis gave valuable information concerning stress-strain and loads to be addressed to a full scale testing activity that will be conducted within the INDUSE research program.

AKNOWLEDGEMENTS

This work was carried out with a financial grant from the Research Fund for Coal and Steel of the European Commission, within INDUSE project: "STRUCTURAL SAFETY OF INDUSTRIAL STEEL TANKS, PRESSURE VESSELS AND PIPING SYSTEMS UNDER SEISMIC LOADING", Grant No. RFSR-CT-2009-00022

REFERENCES

[1] INDUSE-Structural Safety of Industrial Steel Tanks, Pressure Vessels and Piping Systems Under Seismic Loading, http://www.mie.uth.gr/induse;

[2] R. Muraoka, N.Ishikawa, S.Endo, M.Yoshikawa, N.Suzuki, J.Kondo, M.Takagishi, *Deformation and Ductile Cracking Behavior of X80 Grade Induction Bends*, 4th International Pipeline Conference, Calgary, Alberta, Canada September 29–October 3, 2002.

[3] C. Miki, T. Kobayashi, N. Oguchi, T. Uchida, A. Suganuma, A. Katoh, Deformation and fracture properties of steel pipe bend with internal pressure subjected to in-plane bending. *12th World Conference on Earthquake Engineering, Auckland, New Zealand, 30 January - 4 February*2000.

[4] M. A. Shalabya, M. Y. A. Younanb, *Nonlinear analysis and plastic deformation of pipe elbows subjected to in-plane bending*, International Journal of Pressure Vessels and Piping, 75,603-611,1998.

[5] S. A. Karamanos, D. Tsouvalas, A. M. Gresnigt, Ultimate Bending Capacity and Buckling of Pressurized 90 deg Steel Elbows, *J. Pressure Vessel Technology*. 128, 348-356, 2006

[6] S. A. Karamanos, E. Giakoumatos, A. M. Gresnigt, Nonlinear Response And Failure Of Steel Elbows Under In-Plane Bending And Pressure, *J. Pressure Vessel Technol*. Volume 125, 393-402, 2003

[7] K. Takahashi, S. Tsunoi, T. Hara, T. Ueno, A. Mikami, H. Takada, K. Ando,M. Shiratori, Experimental study of low-cycle fatigue of pipe elbows with local wall thinning and life estimation using finite element analysis. *International Journal of Pressure Vessels and Piping* 87 211-219, 2010.

[8] X. Chen, B. Gao, G. Chen, Ratcheting Study of Pressurized Elbows Subjected to Reversed In-Plane Bending. *Journal of Pressure Vessel Technology,*128, 525-532, 2006.

[9] J. Lemaitre, J.L Chaboche, *Mechanics of Solid Materials,* Cambridge University Press, ISBN 0-521-47758, 1994.

[10] V. Semiga, A. Dinovitza, S.Tiku, R. Batisse, M. Zarea, T. Zimmermann, *Full-scale Experimental Validation on Mechanical Damage Assessment Models,* 17th Joint Technical Meeting on Pipeline Research, Milan, Italy, May 11/15 2009.