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AN EFFICIENT META-MODELING APPROACH FOR NATURAL FREQUENCY APPROXIMATION: THE *Q*-METHOD

A. Gallina[1], **L. Pichler**[1,2], **T. Uhl**[1], **L. A. Bergman**[2]

[1]Department of Robotics and Mechatronics, AGH University of Science and Technology
al. Mickiewicza 30, 30059, Krakow, Poland
e-mail: {agallina,tuhl}@agh.edu.pl

[2] Department of Aerospace Engineering, University of Illinois at Urbana-Champaign
104 S. Wright Street, Urbana, IL 61801, USA
e-mail: Lukas.Pichler@ymail.com , lbergman@illinois.edu

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Abstract. *The variability and uncertainty of structural parameters have a large impact on the modal properties and need to be considered for an adequate assessment of the dynamic response. Recently, so-called modal meta-models were used to approximate the natural frequencies of structures modeled with finite elements when structural modifications are present. The present paper proposes a viable meta-modeling approach which, based on the approximation of the coefficients of the characteristic polynomial, succeeds in accurately predicting the natural frequencies of the system in presence of veering assuming stiffness variability. The meta-model is based on* $m^p + p$ *finite element analyses, where* m *denotes the number of analyzed modes and* p *the number of parameters. The number of required analyses is independent of the model or of the complexity of the evolution of the modal parameters. Furthermore, the obtained modal meta-model is used in combination with a procedure for efficient detection of veering and crossing phenomena without resorting to time-consuming mode tracking procedures.*

1 INTRODUCTION

The evolution of the modal parameters (e.g., eigenfrequencies) as a function of changing structural parameters provides important information required for the response assessment of a structure. In many cases, the studied modal parameters show fairly smooth evolutions which suggest the utilization of direct approximations of these modal quantities as functions of the varying structural parameters, requiring only few finite element analyses. This fact proves to be especially advantageous in the case of optimization or uncertainty analysis, where many analyses with different parameter combinations need to be carried out. The utilization of approximations based on the results of several finite element runs, which are carried out according to a planned design, provides a means for reducing the computational costs. The dependence on a large number of full finite element simulations can thus be eliminated by employing fast surrogate or meta-models [1]. Relatively simple approximations based on linear regression, polynomial interpolation, kriging, radial basis functions, etc. are generally used for meta-modeling. In structural dynamics, so-called modal meta-models are introduced and refer to the case when these techniques are used to approximate modal quantities, i.e., eigenfrequencies, eigenvectors, mode shapes, modal mass, etc. Examples are the approximation of the modal parameters of a structure using linear regression with polynomial functions [2], the approximation of the eigenvalues of civil structures using polynomial functions [3] or the kriging approximation of the modal mass and stiffness as utilized in [4]. In recent investigations concerning reliability and uncertainty analysis, the linear combination of the modal parameters of a reference system is used to represent the varying modal parameters [5].

Despite the versatility and simplicity to use direct modal meta-modeling - direct refers to *direct approximation* of the response quantities of interest - recent investigations [2] have pointed out its limitations which arise when the dynamic behavior of undamped structures exhibits mode degeneration. In the presence of degenerate phenomena such as mode crossing, mode veering [6] or mode coalescence [7], the evolution of the modal parameters generally strongly varies from a smooth evolution. Consequently, the direct approximation of the modal properties gets more involved under these conditions, because only a considerably increasing number of design points can improve the quality of the approximation.

The issue of inaccurate or erroneous approximations arising from the utilization of direct meta-modeling was discussed in [8] and indirect modal meta-modeling is proposed. The idea of the indirect approach is that quantities involved in the structural eigenvalue problem are approximated before proceeding with the solution of the eigenvalue problem (see Fig. 1). As an example, in the simplest case the structural matrices (i.e., the mass and stiffness matrices) can be approximated; such approaches based on the interpolation of the structural matrices evaluated at some supporting points are used for the eigenfrequency and eigenvector approximation and are introduced in, e.g. [8, 9].

When the evolution of the eigenfrequencies is of interest, the approximation of the coefficients of a reduced characteristic polynomial can be used to analyze the influence of structural parameter changes to the eigenfrequencies. Such an approach was first introduced in [10] and a clear theoretical description of the approach in the presence of stiffness variations was presented in [11]. The proposed procedure yields very accurate predictions of the natural frequencies. It can be shown that the coefficients of the reduced characteristic polynomial demonstrate smooth behavior when linear changes are introduced for the elements of the *stiffness matrix*. Independent of the model and the complexity of the modal behavior, the procedure always requires $m^p + p$ finite element runs, where m and p are the number of analyzed modes and input parameters, respectively. Differently from a direct modal meta-modeling approach, in the indirect

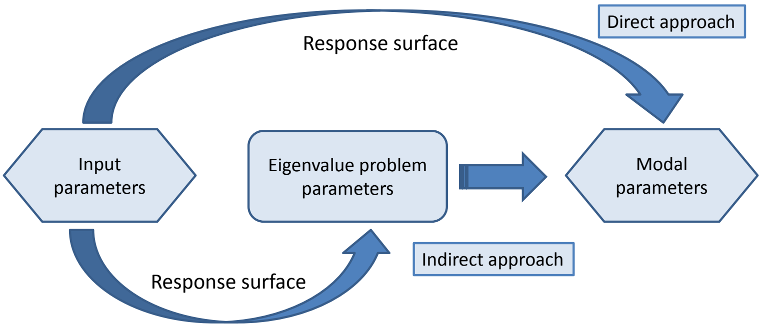


Figure 1: Modal meta-modeling: direct and indirect approaches.

approach based on the approximation of the chareacteristic polynomial, no mode tracking procedure is required to account for mode switching. This means that the method does not provide any information about the physical evolution of the modes. To obtain nontheless a continuous (physical) evolution of the eigenfrequencies a step-wise procedure is applied. The procedure uses the fast predictions of the modal meta-model to identify and quantify crossing or veering phenomena. A major advantage of this approach is that the presence of the veering regions can be determined. This further allows the design of a system which is not prone to unexpected dynamic behavior associated with mode degeneration.

In this paper, first the procedure as described in more detail in [11] is presented in Section 2 and is then combined with an algorithm for eigenfrequency loci reconstruction. This stepwise procedure (Section 3, see also [10]) allows us to detect mode degeneration within the analyzed parameter space. Finally, the effectiveness of the proposed approach is demonstrated in Section 4 for a system affected by crossing and veering.

2 THE *q*-METHOD

The q-method is a meta-modeling strategy for eigenfrequency approximation of the a structure subjected to stiffness variability. The procedure consists of directly approximating the coefficients of the system's characteristic polynomial. In the following it is shown that $m^p + p$ finite element runs provide a sufficient base for achieving a correct description of the characteristic polynomial coefficients and consequently of the eigenfrequencies evoltion, with m the number of analyzed modes and p input parameters. Moreover, a viable approach for the approximation of the coefficients is proposed.

The eigenvalue problem for a system at its nominal configuration can be written as

$$(\mathbf{K}_0 - \lambda_0 \mathbf{M}_0)\, \phi_0 = 0\,, \tag{1}$$

with $\mathbf{K}_0$, $\mathbf{M}_0$, λ_0 and ϕ_0 denoting the stiffness matrix, mass matrix, eigenvalue and eigenvector of the nominal system, respectively. A possible approximation of the modal parameters at a perturbed system configuration is obtained by applying modal reduction [12]. Modal reduction implies the definition of a eigenvector modal basis $\mathbf{\Phi}_0$ that allows one to approximate the perturbed eigevector ϕ^* as a linear combination of the assumed modal basis

$$\phi^* = \mathbf{\Phi}_0 \boldsymbol{\alpha}\,. \tag{2}$$

Under this assumption and for constant mass matrix, it is possible to write a reduced eigenvalue problem of the perturbed system as

$$\left(\mathbf{\Phi}_0^T \mathbf{K} \mathbf{\Phi}_0 - \lambda^* \mathbf{\Phi}_0^T \mathbf{M_0} \mathbf{\Phi}_0\right) \boldsymbol{\alpha} = 0\,, \tag{3}$$

with $\mathbf{K}$ and λ^* representing the stiffness matrix and the approximated eigenvalue of the perturbed system. The reduced eigenvalue problem yields very accurate modal parameters approximation as long as the analyzed eigenvectors of the perturbed system remains within the same eigenvector subspace spanned by the truncated set of nominal modes $\mathbf{\Phi}_0$. This is the case for mode veering where the eigenvectors, even though undergoing large variations within the veering region, stay and rigidly rotate within the same eigenvector subspace. Therefore, modal reduction is effective in the presence of mode veering provided that the assumed modal basis includes all pairs of eigenvectors involved in the veering phenomenon. Under this condition and using a truncated modal basis of m eigenvectors, the m modes of the system with perturbed stiffness will be studied in the following. Mass-normalizing the eigenvectors of $\mathbf{\Phi}_0$ Eq. (3) becomes

$$\left(\mathbf{K}^* - \lambda^* \mathbf{I}\right) \boldsymbol{\alpha} = 0\,, \tag{4}$$

with

$$\mathbf{K}^* \;=\; \mathbf{\Phi}_0^T \mathbf{K} \mathbf{\Phi}_0\,. \tag{5}$$

Each element k^*_{uv} of $\mathbf{K}^*$ can be written as a sum of all terms of the perturbed stiffness matrix

$$k^*_{uv} = \sum_{i=1}^{n} \phi_{0ui} \sum_{j=1}^{n} \phi_{0vj} k_{ij}\,, \tag{6}$$

with n the number of degrees of freedom of the model and ϕ_{0ui} meaning the u-th degree of freedom of the i-th mode of the modal basis. Eq. (6) shows that the k^*_{uv} are linear functions of k_{ij}. When the elements k_{ij} undergo linear changes with respect to an assumed input parameter, the elements k^*_{uv} also represent a linear behavior with respect to the same input parameter.

The characteristic polynomial associated with the reduced eigenvalue problem of Eq. (4) is given by

$$\lambda^{*m} + q_{m-1}\lambda^{*m-1} + ... + q_1\lambda^* + q_0 = 0\,. \tag{7}$$

The q coefficients (thus the name q-method) of the Eq. (10) can be expressed in terms of the elements k^*_{uv} through the relations

$$\begin{aligned}
q_{m-1} =& (-1)^1 \sum_{r=1}^{m} |\mathbf{K}^*_{r,r}| = (-1)^1 \sum_{r=1}^{m} k^*_{rr} \\
q_{m-2} =& (-1)^2 \sum_{r=1}^{m-1} \sum_{s=r+1}^{m} |\mathbf{K}^*_{\{r,s\},\{r,s\}}| \\
q_{m-3} =& (-1)^3 \sum_{r=1}^{m-2} \sum_{s=r+1}^{m-1} \sum_{t=s+1}^{m} |\mathbf{K}^*_{\{r,s,t\},\{r,s,t\}}| \\
& \dots \\
q_0 =& (-1)^m |\mathbf{K}^*|\,,
\end{aligned} \tag{8}$$

with $|\mathbf{K}^*_{\{r,s,t\},\{r,s,t\}}|$ denoting the determinant of a principal minor of $\mathbf{K}^*$

$$|\mathbf{K}^*_{\{r,s,t\},\{r,s,t\}}| = \begin{vmatrix} k^*_{rr} & k^*_{rs} & k^*_{rt} \\ k^*_{sr} & k^*_{ss} & k^*_{st} \\ k^*_{tr} & k^*_{ts} & k^*_{tt} \end{vmatrix}. \tag{9}$$

According to Eq. (6) and 8, it is seen that q_{m-1} is a linear function of the factors k_{ij}, q_{m-2} a second order polynomial with interactions, q_{m-3} a third order polynomial with second order interactions, and so forth. The highest order function is due to q_0 which is a m-th order polynomial with $m-1$ order interactions. Any variation of the structural parameters which is linear with k_{ij} is related to the q coefficients by the same kind of relationship. Thus, when a number p of structural parameters are analyzed simultaneously, a correct approximation of the m coefficients q can be obtained through a set of m-th order polynomial functions with $(m-1)$th order interactions. Polynomial regression can efficiently interpolate these functions using $m^p + p$ observations.

The approximation of the q coefficients by using Eq. (8) is not handy as it involves operations with the stiffness matrix which is usually very large. A more viable approach exploits the exiting relation between the coefficients of a polynomial equation and its roots, that is between the characteristic polynomial coefficients and the eigenfrequencies associated to the reduced eigenvalue problem Eq. (4). Indeed, rewriting Eq. (10) as

$$(\lambda^* - \lambda^*{}_1)(\lambda^* - \lambda^*{}_2)...(\lambda^* - \lambda^*{}_m) = 0\,, \tag{10}$$

and solving the multiplication by binomials, it is possible to express the q in terms of the roots of λ^*. This allows one to calculate the q coefficient at the $m^p + p$ design sites directly from the results of FE analyses. Then, the q coefficients are approximated within the design space via polynomial regression and the eigenvalues at untried points are calculated by solving the approximated mth-order characteristic polynomial.

The big advantage of the proposed methodology with respect to a direct modal meta-modeling consists of being insensitive to the complexity of the modal behavior. However, it is important to bear in mind that the method is effective as long as all analyzed modes move within the subspace spanned by the same modal basis within the whole domain of interest. As mentioned above, this means also that in the presence of veering phenomena, the assumed modal basis has to accommodate aways both modes involved in the veering. The detection of coupled modes resulting from veering phenomena can be accomplished by the modal assurance criterion (MAC) matrix histogram.

From the calculation of the MAC values between every possible pair of eigenvectors and design points it is possible to assemble the MAC matrix histogram. In this matrix, each element (i.e., each histogram) shows the frequency of occurrence of a certain MAC value (see Fig. 2). The *y*-axis shows the number of occurrences and the *x*-axis labels the range of the MAC value form 0 to 1 which for convenience is subdivided into 10 parts. A dispersion on the diagonal of this matrix clearly indicates mode veering. Fig. 2 shows that the modes *j* and *k* are coupled with each other but decoupled from mode *i*.

3 MODE VEERING DETECTION

The q-method approximates the coefficients of the characteristic polynomial and can be used as an efficient tool to investigate mode degeneration phenomena. This is accomplished in the

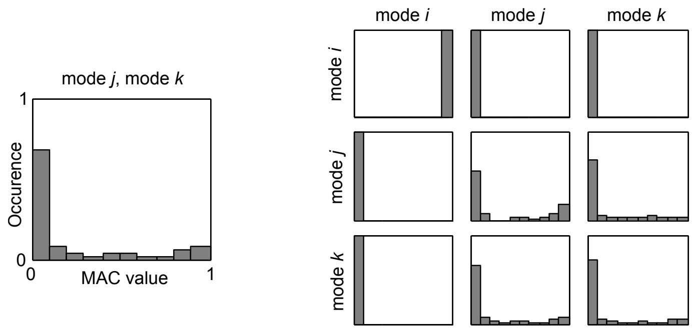


Figure 2: MAC matrix histogram. The modes *j* and *k* are coupled with each other but decoupled from *i*.

following way. The meta-model of the q-coefficients is used to evaluate the eigenfrequencies by the solution of Eq. (10). Due to the fact that no eigenvectors are calculated, no information about the presence of veering or crossing can be extracted from MAC analysis. However, the capability of the modal meta-model to correctly predicting the eigenfrequencies can be utilized for developing an efficient iterative procedure that detects possible mode degeneration within the parameter space. The step-wise algorithm is based on tangential approximations and at each step it is checked whether a change in the mode order is occurring within the range of Δ. In the sample graph of Fig. 3 this would be the case for the step ② − ③. The eigenfrequen-

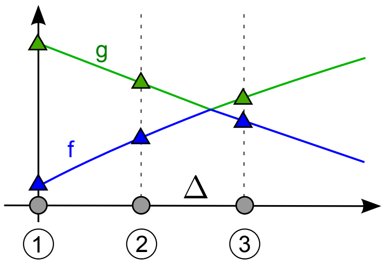


Figure 3: Step-wise algorithm.

cies obtained from the q-method are denoted as f (blue: $\overline{acb}$) and g (green $\overline{dce}$) as shown in Fig. 3 and 4; the eigenfrequencies given after the construction of the eigenfrequency locus are denoted as F ($\overline{ace}$) and G ($\overline{dcb}$). The correct representation of the evolution of the modes is accomplished in the following way. First, initial values are calculated using the meta-model; they are denoted with the superscript $^{(0)}$, i.e., $F^{(0)} = f^{(0)}$ and $G^{(0)} = g^{(0)}$. Then, the new values of the eigenfrequencies at position $^{(1)}$ are calculated using finite tangential approximations

$$F^{(1)} = \begin{cases} f^{(1)}, & \text{if } \|F^{(0)} + \frac{\partial f^{(0)}}{\partial q}\Delta - f^{(1)}\| < \|F^{(0)} + \frac{\partial f^{(0)}}{\partial q}\Delta - g^{(1)}\| \\ g^{(1)}, & \text{if } \|F^{(0)} + \frac{\partial f^{(0)}}{\partial q}\Delta - f^{(1)}\| > \|F^{(0)} + \frac{\partial f^{(0)}}{\partial q}\Delta - g^{(1)}\| \end{cases} \tag{11}$$

and

$$G^{(1)} = \begin{cases} g^{(1)}, & \text{if } \|G^{(0)} + \frac{\partial g^{(0)}}{\partial q}\Delta - g^{(1)}\| < \|G^{(0)} + \frac{\partial g^{(0)}}{\partial q}\Delta - f^{(1)}\| \\ f^{(1)}, & \text{if } \|G^{(0)} + \frac{\partial g^{(0)}}{\partial q}\Delta - g^{(1)}\| > \|G^{(0)} + \frac{\partial g^{(0)}}{\partial q}\Delta - f^{(1)}\| \end{cases} \tag{12}$$

In Eq. (11) $\frac{\partial f^{(0)}}{\partial q}$ denotes the derivative at $f^{(0)}$ and in Eq. (12) $\frac{\partial g^{(0)}}{\partial q}$ denotes the derivative at $g^{(0)}$ which are calculated numerically using a small increment ∂q; $\partial q \ll \Delta$. If the second lines of Eq. (11) and (12) appear to be correct, this is an indication of a crossing or veering occurrence within the interval Δ as shown in Fig. 4 for ② − ③.

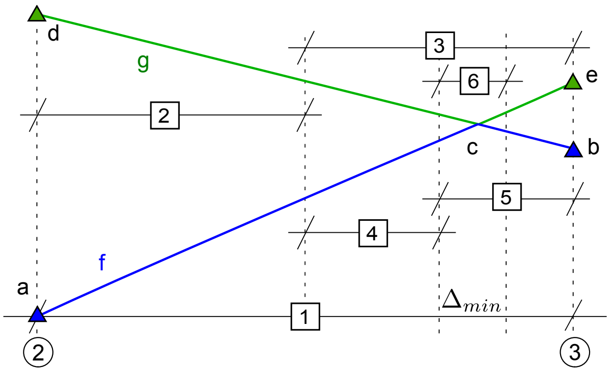


Figure 4: Bisection algorithm.

At every subsequent step, the interval is halved, until either the initial crossing is revealed to be a veering, or an established Δ_{min} is reached. In the former case, the smallest Δ_{out} is stored, and the veering point P_v is determined. Then the domain Ω_v of mode degeneration is assumed to be within the interval $[P_v - I_v, P_v + I_v]$ with $I_v = \Delta_{out}$ providing a metric for the veering intensity. In the latter case, this means that either the modes are crossing, or the modes are sharply veering within the current Δ_{min}. In both cases, the crossing/veering point P_v is determined and the domain Ω_v within which mode degeneration occurs $[P_v - I_v, P_v + I_v]$ with $I_v = \Delta_{min}$ calculated. In case of abrupt veering or crossing, a so-called veering/crossing line can be drawn by connecting the points P_v; otherwise the evaluation of the region Ω_v is more meaningful. In general, the procedure will never distinguish between mode switching and mode veering with veering region smaller than Δ_{min}, however, by reducing the value of Δ_{min} it is possible to recognize sharper and sharper veering phenomena.

4 NUMERICAL EXAMPLE

4.1 Model description

An assessment of the presented procedure and its capability to predict a physically meaningful evolution of the eigenvalue loci in the presence of veering is completed via a numerical example. A two-bay frame, modeled with finite elements provides a test case therefore; despite the simplicity of the structure, the model shows complex behavior of its modal parameters.
The frame model consists of three vertical plates which are connected through a horizontal plate. Two horizontal bars, linking the vertical plates, produce an additional weak coupling between the vertical plates. The frame is unconstrained. The underlying FE model consists of 320 shell elements and 423 nodes leading to 2538 degrees of freedom. Fig. 5 shows the FE model and Table 1 lists its properties at the nominal configuration.

The eigenfrequency loci evolution is approximated when the Young's moduli E_1 and E_2 of the plates 1 and 2 are changed. Both Young's moduli vary $\pm 30\%$ with respect to their nominal values, resulting in a range of $0.7 \cdot \mathrm{E}_{1,2} - 1.3 \cdot \mathrm{E}_{1,2}$.

Element	Young's modulus [Pa]	mass density ρ [kg/m^3]	ν	thickness[m] a,b	L[m]	H[m]
plates $1-4$	2.1×10^{11}	7860	0.3	0.002	0.2	1
bars	2.1×10^{11}	7860	0.3	$0.002, 0.0002$	0.5	-

Table 1: Nominal configuration of the frame.

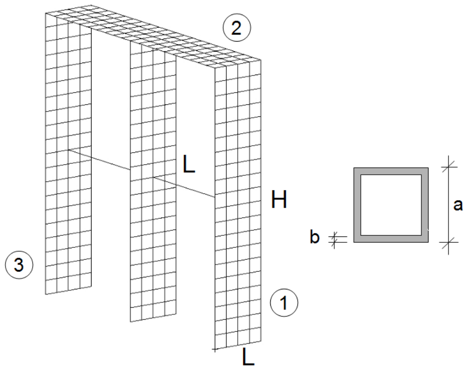


Figure 5: M-shaped frame structure.

4.2 Response surface approximation

First the MAC matrix histograms (Fig. 6) was calculated from a small number of FE runs for detecting coupled modes. The MAC matrix histogram shows a clear coupling for the modes 2 to 4. Thus, these 4 modes are selected for the further analysis.

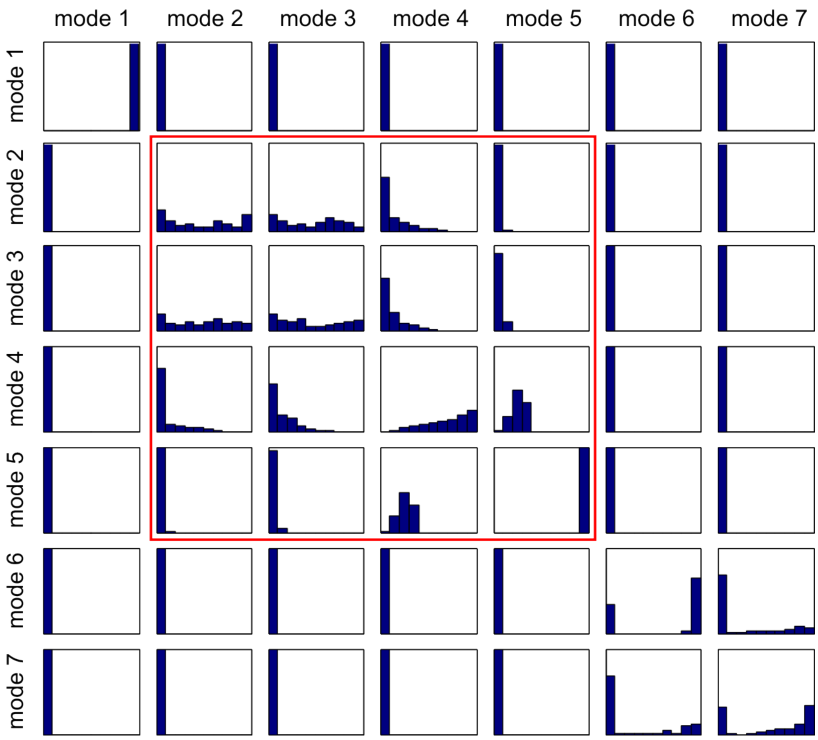
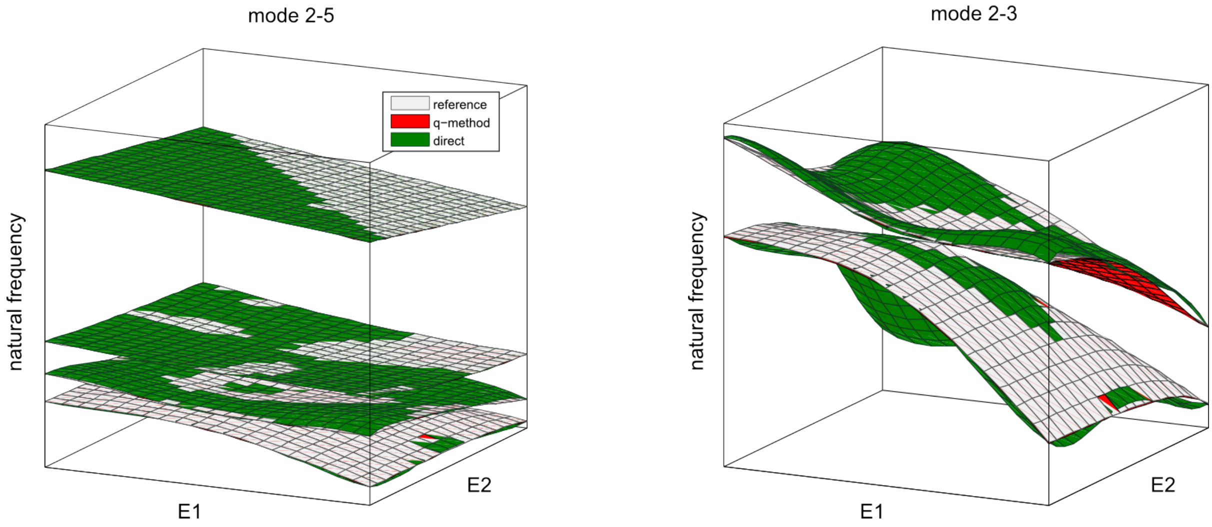


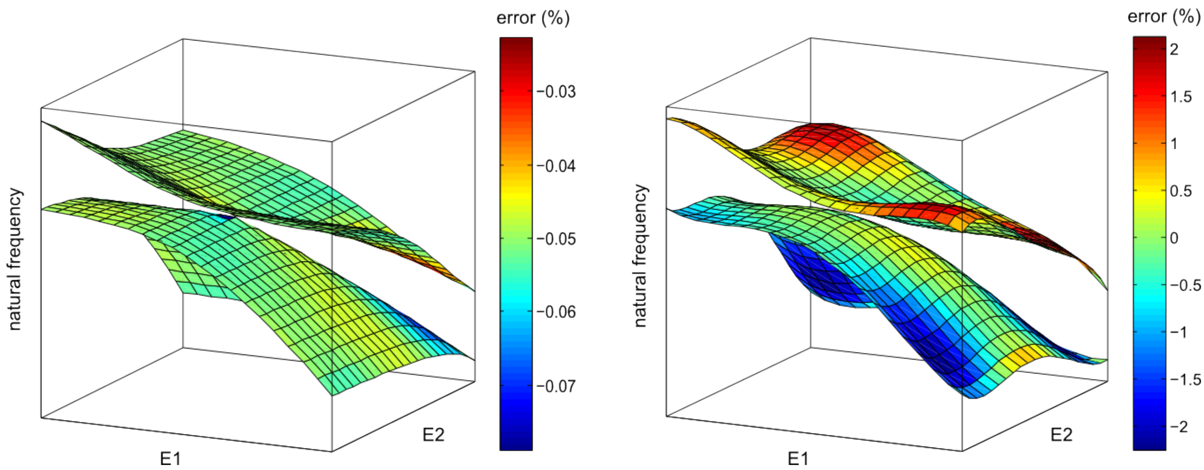
Figure 6: MAC histogram.

According to the law proposed in Section 2, $4^2 + 2 = 18$ numerical runs should suffice to accurately describe the natural frequency evolution of the analyzed modes. Thus, a 18-sample Latin hypercube sampling was carried out and 4th order polynomials (with interactions) fitted to the observations at the 18 design points via polynomial regression. The results from the q-method has been compared with the results from a direct meta-modeling approach and with the exact numerical solution calculated on a 21×21 regular grid. In particular, both meta-modeling techniques evidence high accuracy for modes 4 and 5, those not affected by degeneration (Fig. 7(a)). However, the q-method performs much better than the direct approach for modes 2 and 3, that coalesce within the parameter space as presented in Fig. 7(b). In order to assess the quality of meta-models prediction, the relative error with respect to the reference solution has been calculated at modes 2 and 3. The q-method reveals to be much more accurate than the direct method, as shown in the plots of Fig. 8.



(a) (b)

Figure 7: Comparison of the eigenfrequency variation: exact solution vs. q-method and direct approximation.



(a) q-method (b) direct method

Figure 8: Measured per cent error of the approximated responses with respect to the reference solution.

4.3 Detection of mode coalescence

The step-wise procedure was applied to a 4×20 grid which was imposed on the input parameter space. The procedure succeeded in detecting the region of mode degeneration, as presented in Section 3. The solid lines in Fig. 9 show the veering region, and the extension of the domain is given by the veering intensity $I_v = \Delta_{out}$. Close to the occurrence of mode coalescence, the veering intensity factor assumes a small value which also suggests the occurrence of abrupt veering or crossing.

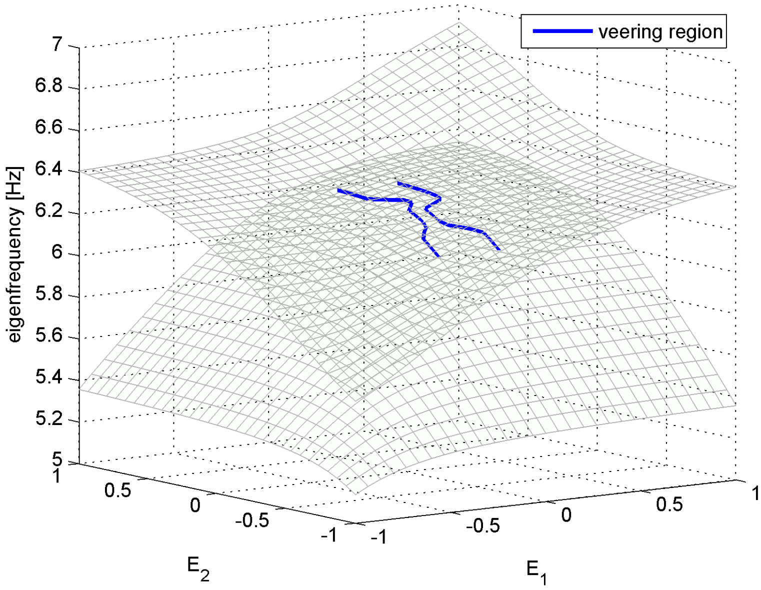


Figure 9: Spacial occurrence of coalescence.

5 CONCLUSIONS

The utilization of indirect meta-modeling for the investigation of the eigenfrequency loci evolution has been discussed in this work. A sufficient number of design sites for constructing accurate meta-models of the natural frequencies has been assessed independently of the presence of mode degeneration phenomena within the analyzed parameter space. Combined with a step-wise mode tracking procedure based on the natural frequency analysis, the modal meta-model is able to provide meaningful information about the physical mode evolution. This is an advantage with respect to direct modal meta-modeling that has difficulty in dealing with systems affected by mode veering, crossing and coalescence. A numerical example discussed the application of the methodology on a structure affected by mode coalescence. The combination of the surrogate model with the step-wise procedure provides a tool to improve the design through the identification of regions where the system dynamic behavior undergoes large modal parameter variations. In addition, it has been shown that for the examined structure, the proposed indirect meta-model is clearly outperforming a direct approximation. At present the procedure is able to only deal with systems subjected to stiffness variability. Future works will concern with the extension of the procedure to also account for mass modifications.

6 ACKNOWLEDGMENTS

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