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FINITE ELEMENT MODEL VALIDATION AND PREDICTIONS USING DYNAMIC REDUCTION TECHNIQUES

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Abstract. *Finite element (FE) model updating and validation techniques are formulated as single and multi-objective optimization problems. A multi-objective optimization framework results in multiple Pareto optimal models that are consistent with the measured data and the residuals used to measure the discrepancies between the measured and the FE model predicted characteristics. The uncertainty in the Pareto optimal models can then be propagated to predict the uncertainty in the response predictions. Gradient-based optimization algorithms, such as the Normal Boundary Intersection algorithm, are used to compute the Pareto optimal solutions. These iterative algorithms require repeated solutions of the FE model for various values of the model parameters, as well as repeated computation of the gradients of the response characteristics involved in the residuals. For FE models with very high number of degrees of freedom, of the order of millions, repeated solutions of the FE models can be computationally very demanding. Component mode synthesis (CMS) methods are integrated into the updating method in order to reduce the computational effort required for performing the single- and multi-objective optimization problems. Exploiting certain schemes often encountered in FE model parameterization, it is shown that CMS allows the repeated computations to be carried out efficiently in a significantly reduced space of generalized coordinates, avoiding the solution of the fixed-interface/constrained modes and the assembling of reduced system matrices at each iteration. The final computational cost is associated with that of estimating the response characteristics of the reduced system at each iteration.*

1 INTRODUCTION

Structural model updating methods (e.g. [1]) have been proposed in the past to reconcile mathematical models, usually discretized finite element models, with experimental data. The estimate of the optimal model from a parameterized class of models is sensitive to uncertainties that are due to limitations of the mathematical models used to represent the behavior of the real structure, the presence of measurement and processing error in the data, the number and type of measured modal or response time history data used in the reconciling process, as well as the norms used to measure the fit between measured and model predicted characteristics. The optimal structural models resulting from such methods can be used for improving the model response and reliability predictions [2], structural health monitoring applications [3-6] and structural control [7].

Structural model parameter estimation problems based on measured data, such as modal characteristics (e.g. [3-6]) or response time history characteristics [8], are often formulated as weighted least-squares problems in which metrics, measuring the residuals between measured and model predicted characteristics, are build up into a single weighted residuals metric formed as a weighted average of the multiple individual metrics using weighting factors. Standard optimization techniques are then used to find the optimal values of the structural parameters that minimize the single weighted residuals metric representing an overall measure of fit between measured and model predicted characteristics. Due to model error and measurement noise, the results of the optimization are affected by the values assumed for the weighting factors.

The model updating problem has also been formulated in a multi-objective context [9,10] that allows the simultaneous minimization of the multiple metrics, eliminating the need for using arbitrary weighting factors for weighting the relative importance of each metric in the overall measure of fit. The multi-objective parameter estimation methodology provides multiple Pareto optimal structural models consistent with the measured data and the residuals used to measure the discrepancies between the measured and the finite element model predicted characteristics, in the sense that the fit each Pareto optimal model provides in a group of measured modal properties cannot be improved without deteriorating the fit in at least one other modal group. The Normal Boundary Intersection algorithm [11] is used to compute the Pareto optimal solutions.

Optimization algorithms used in the model updating methodology require repeated computations of the finite element model for various values of the model parameters. In addition, gradient-based algorithms require the estimation of the gradients of the residuals. For finite element models with a very high number of degrees of freedom, of the order of a few millions, the model updating methods require very high computational effort. Dynamic reduction techniques can be incorporated in the finite element model updating formulation to alleviate the computational burden.

In this work, a framework is presented for integrating component mode synthesis (CMS) [12,13] methods into existing finite element model updating methods in order to reduce the time consuming operations involved. The CMS allows the repeated computations to be carried out in a significantly reduced space of generalized coordinates. CMS techniques divide the structure into sub-structural components with mass and stiffness matrices that are reduced using fixed-interface and constrained modes. For structural components behaving linearly, an efficient model updating technique arises for component mass and stiffness matrices that depend linearly on only one of the free model parameters to be updated. In this case the reduced mass and stiffness matrices of a component also depends linearly on the free model parameter, allowing significant computational savings to be achieved during optimization by avoiding

the repeated computation of the fixed-interface and constrained modes of each component during the iterative process. Using the resulting linear representation of the assembled mass and stiffness matrices of the reduced system in terms of the model parameters, computationally efficient algorithms [14] can be used to further reduce the computational cost involved in estimating the gradients and Hessians of the objective functions representing the modal residuals.

2 FINITE ELEMENT MODEL UPDATING FORMULATION

2.1 Modal residuals

Let $D=\{\hat{\omega}_r^{(k)},\hat{\underline{\phi}}_r^{(k)}\in R^{N_0},\ r=1,\cdots,m,\ k=1,\cdots,N_D\}$ be the measured modal data from a structure, consisting of modal frequencies $\hat{\omega}_r^{(k)}$ and mode shape components at N_0 measured DOFs, where m is the number of observed modes and N_D is the number of modal data sets available. Consider a parameterized class of linear structural models used to model the dynamic behavior of the structure and let $\underline{\theta}\in R^{N_\theta}$ be the set of free structural model parameters to be identified using the measured modal data. The objective in a modal-based structural identification methodology is to estimate the values of the parameter set $\underline{\theta}$ so that the modal data $\{\omega_r(\underline{\theta}),\ \underline{\phi}_r(\underline{\theta})\in R^{N_d}, r=1,\cdots,m\}$, where N_d is the number of model degrees of freedom (DOF), predicted by the linear class of models best matches, in some sense, the experimentally obtained modal data in D. For this, let

$$\varepsilon_{\omega_r}(\underline{\theta})=\frac{\omega_r^2(\underline{\theta})-\hat{\omega}_r^2}{\hat{\omega}_r^2} \tag{1}$$

and

$$\varepsilon_{\underline{\phi}_r}(\underline{\theta})=\frac{\left\|L\Phi(\underline{\theta})\underline{\beta}_r(\underline{\theta})-\hat{\underline{\phi}}_r\right\|}{\left\|\hat{\underline{\phi}}_r\right\|} \tag{2}$$

$r=1,\cdots,m$, be the measures of fit or residuals between the measured modal data and the model predicted modal data for the r-th modal frequency and modeshape components, respectively, where $\|\underline{z}\|^2=\underline{z}^{\mathrm{T}}\underline{z}$ is the usual Euclidian norm, the matrix $L\in R^{N_0\times N_d}$ is an observation matrix comprised of zeros and ones that maps the N_d model DOFs to the N_0 observed DOFs, $\Phi(\underline{\theta})\in R^{N_d\times m}$ is the matrix of the modeshapes predicted by the model, and $\underline{\beta}_r(\underline{\theta})=[\Phi^T(\underline{\theta})L^TL\Phi(\underline{\theta})]^{-1}[L\Phi(\underline{\theta})]^T\hat{\underline{\phi}}_r$ is a normalization vector that guaranties that the distance of the measured modeshape $\hat{\underline{\phi}}_r$ from the space spanned by the model predicted modeshapes in $L\Phi(\underline{\theta})\in R^{N_d\times m}$ is minimal.

It should be noted that for modes that are not closely spaced, the elements $\beta_{jr}(\underline{\theta})$ of the normalization vector $\underline{\beta}_r(\underline{\theta})$ are expected to have values close to zero for $j\neq r$ and so the measure of fit $\varepsilon_{\underline{\phi}_r}(\underline{\theta})$ is approximately the same as $\varepsilon_{\underline{\phi}_r}(\underline{\theta})=\left\|L\underline{\phi}_r(\underline{\theta})\beta_{rr}(\underline{\theta})-\hat{\underline{\phi}}_r\right\|/\left\|\hat{\underline{\phi}}_r\right\|$. However, for closely spaced modes, the measure of fit $\varepsilon_{\underline{\phi}_r}(\underline{\theta})$ in (2) is used to express the fact that any vector in the subspace spanned by the identified modeshapes for these closely spaced

modes is also a modeshape. Thus, any of the identified closely spaced modeshapes should be expected to be a linear combination of the model predicted modeshapes for the closely spaced modes. This fact is reflected in the use of the measure of fit $\varepsilon_{\underline{\phi}_r}(\underline{\theta})$ in (2).

In order to proceed with the model updating formulation, the measured modal properties are grouped into n groups. Each group contains one or more modal properties. The modal properties assigned in the ith group are identified by the set $g_i(k)$, $i=1,\cdots,n$ and $k=1,2$, with any element in the set $g_i(k)$ is an integer from 1 to m. An element in the set $g_i(k)$ with $k=1$ refer to the number of the measured modal frequency assigned in the group i, while the elements of the set $g_i(k)$ with $k=2$ refer to the number of the measured modeshape assigned in the group i. For the ith group, a norm $J_i(\underline{\theta})$ is introduced to measure the residuals of the difference between the measured values of the modal properties involved in the group and the corresponding modal values predicted from the model class for a particular value of the parameter set $\underline{\theta}$. The measure of fit in a modal group is the sum of the individual square errors in (1) for the corresponding modal properties involved in the modal group. Specifically, the measure of fit is given by

$$J_i(\underline{\theta}) = \sum_{r\in g_i(1)} \varepsilon^2_{\omega_r}(\underline{\theta}) + \sum_{r\in g_i(2)} \varepsilon^2_{\underline{\phi}_r}(\underline{\theta}) \tag{3}$$

The grouping of the modal properties $\{\omega_r(\underline{\theta}),\ \underline{\phi}_r(\underline{\theta}), r=1,\cdots,m\}$ into n groups and the selection of the measures of fit (residuals) $J_1(\underline{\theta}),\cdots,J_n(\underline{\theta})$ are usually based on user preference. The modal properties assigned to each group are selected by the user according to their type and the purpose of the analysis.

2.2 Multi-objective identification

The problem of identifying the model parameter values $\underline{\theta}$ that minimize the modal or response time history residuals can be formulated as a multi-objective optimization problem stated as follows [10]. Find the values of the structural parameter set $\underline{\theta}$ that simultaneously minimizes the objectives

$$\underline{y} = \underline{J}(\underline{\theta}) = (J_1(\underline{\theta}),\cdots,J_n(\underline{\theta})) \tag{4}$$

subject to inequality constrains $\underline{c}(\underline{\theta}) \le \underline{0}$ and parameter constrains $\underline{\theta}_{low} \le \underline{\theta} \le \underline{\theta}_{upper}$, where $\underline{\theta} = (\theta_1,\cdots,\theta_{N_\theta}) \in \Theta$ is the parameter vector, Θ is the parameter space, $\underline{y} = (y_1,\cdots,y_n) \in Y$ is the objective vector, Y is the objective space, $\underline{c}(\underline{\theta})$ is the vector function of constrains, and $\underline{\theta}_{low}$ and $\underline{\theta}_{upper}$ are respectively the lower and upper bounds of the parameter vector . For conflicting objectives $J_1(\underline{\theta}),\cdots,J_n(\underline{\theta})$, there is no single optimal solution, but rather a set of alternative solutions, known as Pareto optimal solutions, that are optimal in the sense that no other solutions in the parameter space are superior to them when all objectives are considered.

Using multi-objective terminology, the Pareto optimal solutions are the non-dominating vectors in the parameter space Θ, defined mathematically as follows. A vector $\underline{\theta} \in \Theta$ is said to be non-dominated regarding the set Θ if and only if there is no vector in Θ which dominates $\underline{\theta}$. A vector $\underline{\theta}$ is said to dominate a vector $\underline{\theta}'$ if and only if

$$J_i(\underline{\theta}) \le J_i(\underline{\theta}') \quad \forall i \in \{1,\cdots,n\} \quad \text{and} \quad \exists\ j \in \{1,\cdots,n\}\ :\ J_j(\underline{\theta}) < J_j(\underline{\theta}') \tag{5}$$

The set of objective vectors $\underline{y} = \underline{J}(\underline{\theta})$ corresponding to the set of Pareto optimal solutions $\underline{\theta}$ is called Pareto optimal front. The characteristics of the Pareto solutions are that the residuals cannot be improved in any group without deteriorating the residuals in at least one other group. The multiple Pareto optimal solutions are due to modelling and measurement errors. The level of modelling and measurement errors affect the size and the distance from the origin of the Pareto front in the objective space, as well as the variability of the Pareto optimal solutions in the parameter space.

2.3 Weighted modal residuals identification

The parameter estimation problem is traditionally solved by minimizing the single objective

$$J(\underline{\theta};\underline{w}) = \sum_{i=1}^{n} w_i J_i(\underline{\theta}) \tag{6}$$

formed from the multiple objectives $J_i(\underline{\theta})$ using the weighting factors $w_i \geq 0$, $i = 1,\cdots,n$, with $\sum_{i=1}^{n} w_i = 1$. The objective function $J(\underline{\theta};\underline{w})$ represents an overall measure of fit between the measured and the model predicted characteristics. The relative importance of the residual errors in the selection of the optimal model is reflected in the choice of the weights. The results of the identification depend on the weight values used. Conventional weighted least squares methods assume equal weight values, $w_1 = \cdots = w_n = 1/n$.

2.4 Computational issues

The proposed single and multi-objective identification problems are solved using available single- and multi-objective optimization algorithms. The optimization of $J(\underline{\theta};\underline{w})$ in (6) with respect to $\underline{\theta}$ for given $\underline{w}$ can readily be carried out numerically using any available gradient-based algorithm for optimizing a nonlinear function of several variables. The set of Pareto optimal solutions can be obtained using available multi-objective optimization algorithms. A very efficient algorithm for solving the multi-objective optimization problem is the Normal-Boundary Intersection (NBI) method [11]. Each Pareto optimal solutions is obtained by solving a single-objective optimization problem using gradient based optimization algorithms. Thus, the computational time is of the order of the number of points used to represent the Pareto front multiplied by the computational time required to solve a single-objective problem for computing each point on the front.

It is obvious that the search for the Pareto optimal models require repeated computations of the finite element model for various values of the model parameters. In addition, gradient-based algorithms require the estimation of the gradients of the residuals. For finite element models with a very high number of degrees of freedom, of the order of a few millions, the computational time involved for repeatedly solving the large-scale eigen-problems may be excessive, especially if the number of iterations is high. The objective of this work is to examine the conditions under which substantial reductions in the computational effort can be achieved using dynamic reduction techniques such as component mode synthesis methods. Dividing the structure into components and reducing the number of physical coordinates to a much smaller number of generalized coordinates certainly alleviates part of the computational effort. However, at each iteration one needs to re-computed the eigen-problem for each component which can be a very time consuming operation. It is shown that for certain parameterization schemes, often encountered in finite element model updating formulations, the repeated

solutions of the component eigen-problems are avoided, reducing substantially the computational demands in finite element model updating formulations.

3 COMPONENT MODE SYNTHESIS

In component mode synthesis [12,13] a structure is divided into several components. Reduction techniques are applied on a number of these components, while the rest are the non-reduced parts of the structure which could be left un-altered. For each component, the unconstrained DOFs are divided into the boundary DOFs, denoted by the subscript b and the internal DOFs, denoted by the subscript i. The boundary DOFs of a component are common with the DOFs of adjacent components, while the internal DOFs of a component are not shared with any adjacent component.

The stiffness and mass matrices $K^{(s)}$ and $M^{(s)}$ of a component s are partitioned to blocks related to the internal and boundary DOFs, with the undamped equation of motion of a component given by

$$\begin{bmatrix} M_{ii}^{(s)} & M_{ib}^{(s)} \\ M_{bi}^{(s)} & M_{bb}^{(s)} \end{bmatrix} \begin{Bmatrix} \ddot{\underline{u}}_i^{(s)} \\ \ddot{\underline{u}}_b^{(s)} \end{Bmatrix} + \begin{bmatrix} K_{ii}^{(s)} & K_{ib}^{(s)} \\ K_{bi}^{(s)} & K_{bb}^{(s)} \end{bmatrix} \begin{Bmatrix} \underline{u}_i^{(s)} \\ \underline{u}_b^{(s)} \end{Bmatrix} = \begin{Bmatrix} \underline{f}_i^{(s)} \\ \underline{f}_b^{(s)} + \underline{r}_b^{(s)} \end{Bmatrix} \tag{7}$$

where $\underline{f}_i^{(s)}$ and $\underline{f}_b^{(s)}$ are the external forces, while $\underline{r}_b^{(s)}$ are the reaction forces on the component from the adjacent components at the boundary DOFs. The indices i and b are sets containing the internal and boundary DOFs of the component s.

In component mode synthesis, the physical displacement coordinates $\underline{u}^{(s)}$ of a component are represented in terms of the generalized coordinates $\underline{p}^{(s)}$ of the component by the Ritz coordinate transformation

$$\underline{u}^{(s)} = \Psi^{(s)} \underline{p}^{(s)} \tag{8}$$

According to the Craig-Bampton fixed-interface-mode method, the physical coordinates of the component are related to the generalized coordinates using the fixed-interface normal modes and the interface constrained modes as follows

$$\underline{u}^{(s)} = \begin{Bmatrix} \underline{u}_i^{(s)} \\ \underline{u}_b^{(s)} \end{Bmatrix} = \Psi^{(s)} \underline{p}^{(s)} = \begin{bmatrix} \Phi_{ik}^{(s)} & \Psi_{ib}^{(s)} \\ 0_{bk}^{(s)} & I_{bb}^{(s)} \end{bmatrix} \begin{Bmatrix} \underline{p}_k^{(s)} \\ \underline{p}_b^{(s)} \end{Bmatrix} \tag{9}$$

where $\Phi_{ik}^{(s)}$ is the interior partition matrix of kept fixed-interface modes satisfying the eigen-problem

$$K_{ii}^{(s)} \Phi_{ik}^{(s)} = M_{ii}^{(s)} \Phi_{ik}^{(s)} \Lambda_{kk}^{(s)} \tag{10}$$

with all boundary DOFs for the considered component restrained, and the $\Psi_{ib}^{(s)}$ is the interior partition matrix of the constrained-modes given by $\Psi_{ib}^{(s)} = -[K_{ii}^{(s)}]^{-1} K_{ib}^{(s)}$. The matrix $\Lambda_{kk}^{(s)} = diag(\omega_j^{2(s)})$ is diagonal containing the eigenvalues $\omega_j^{2(s)}$ of the kept fixed-interface normal modes. The fixed-interface modes $\Phi_{ik}^{(s)}$ are considered mass normalized, satisfying

$$\Phi_{ik}^{(s)T} M_{ii}^{(s)} \Phi_{ik}^{(s)} = I_{kk}^{(s)}, \qquad \Phi_{ik}^{(s)T} K_{ii}^{(s)} \Phi_{ik}^{(s)} = \Lambda_{kk}^{(s)} \tag{11}$$

The reduced set of component equations of motion in generalized coordinates is

$$\hat{M}^{(s)}\ddot{\underline{p}}^{(s)} + \hat{K}^{(s)}\underline{p}^{(s)} = \hat{\underline{f}}^{(s)} + \hat{\underline{r}}^{(s)} \tag{12}$$

where the reduced mass, stiffness matrix and force vectors for the component are given in terms of the original matrices and force vectors in the form

$$\hat{M}^{(s)} = \Psi_{ik}^{(s)T} M^{(s)} \Psi_{ik}^{(s)}, \quad \hat{K}^{(s)} = \Psi_{ik}^{(s)T} K^{(s)} \Psi_{ik}^{(s)}, \quad \hat{\underline{f}}^{(s)} = \Psi_{ik}^{(s)T}\ \underline{f}^{(s)} \quad \text{and} \quad \hat{\underline{r}}^{(s)} = \Psi_{ik}^{(s)T}\underline{r}^{(s)} \tag{13}$$

with the partitions for the mass and stiffness matrices given respectively by

$$\begin{aligned}
\hat{M}_{kk}^{(s)} &= I_{kk}^{(s)} \\
\hat{M}_{kb}^{(s)} &= \hat{M}_{bk}^{(s)T} = \Phi_{ik}^{(s)T} M_{ii}^{(s)} \Psi_{b}^{(s)} + \Phi_{ik}^{(s)T} M_{ib}^{(s)} \\
\hat{M}_{bb}^{(s)} &= (\Psi_{b}^{(s)T} M_{ii}^{(s)} + M_{bi}^{(s)})\Psi_{b}^{(s)} + \Psi_{b}^{(s)T} M_{ib}^{(s)} + M_{bb}^{(s)}
\end{aligned} \tag{14}$$

and

$$\begin{aligned}
\hat{K}_{kk}^{(s)} &= \Lambda_{kk}^{(s)} \\
\hat{K}_{kb}^{(s)} &= \hat{K}_{bk}^{(s)T} = 0_{kb}^{(s)} \\
\hat{K}_{bb}^{(s)} &= K_{bb}^{(s)} - K_{bi}^{(s)}[K_{ii}^{(s)}]^{-1} K_{ib}^{(s)} = K_{bb}^{(s)} + \Psi_{b}^{(s)T} K_{ib}^{(s)}
\end{aligned} \tag{15}$$

In the substructure assembly process, the vector $\underline{p} = [\underline{p}^{(1)T}, \cdots, \underline{p}^{(S)T}]^T$ of all generalized coordinates for each components is introduced. Letting $\underline{q} = [\underline{p}_k^{(1)T}, \cdots, \underline{p}_k^{(S)T}, \underline{u}_b^T]^T$ be the vector of independent generalized coordinates formed from the generalized coordinates (fixed-interface modal and boundary coordinates) of all components, the following transformation holds

$$\underline{p} = S\underline{q} \tag{16}$$

where the component coupling matrix S couples the independent generalized coordinates with the generalized coordinates of each component. Using the continuity of displacement/rotations and the equilibrium of forces along the shared boundaries of the components, the equations of motion in the reduced space of independent generalized coordinates takes the final form

$$\hat{M}^{CB}\ddot{\underline{q}} + \hat{K}^{CB}\underline{q} = S^T \hat{\underline{f}} \tag{17}$$

where the assembled mass and stiffness matrices for the reduced system are given by

$$\hat{M}^{CB} = S^T \begin{bmatrix} \hat{M}^{(1)} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \hat{M}^{(S)} \end{bmatrix} S \qquad \text{and} \qquad \hat{K}^{CB} = S^T \begin{bmatrix} \hat{K}^{(1)} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \hat{K}^{(S)} \end{bmatrix} S \tag{18}$$

Solving the eigen-problem

$$\hat{K}^{CB} Q\ = \hat{M}^{CB} Q\ \Lambda \tag{19}$$

associated with the reduced mass and stiffness matrices $\hat{M}^{CB}$ and $\hat{K}^{CB}$, respectively, one obtains the modal frequencies in $\Lambda = diag(\omega_i^2)$ and the mode shapes Q of the reduced system. The mode shapes of the original structure are assembled from the reduced ones. Specifically, using (8) and (16), the physical mode shapes are recovered as follows

$$\Phi = \begin{bmatrix} \Psi^{(1)} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \Psi^{(s)} \end{bmatrix} SQ \tag{20}$$

4 MODEL UPDATING USING COMPONENT MODE SYNTHESIS

Next, the component mode synthesis procedure is integrated into the finite element formulation. We limit the presentation for the case for which the stiffness and mass matrices depend linearly on the model parameters to be estimated using the measured data. Specifically, it is assumed that the mass and stiffness matrix takes the form

$$\begin{aligned} K &= K_0 + \sum_{i=1}^{N_\theta} K_{,j} \theta_j \\ M &= M_0 + \sum_{j=1}^{N_\theta} M_{,j} \theta_j \end{aligned} \tag{21}$$

This implies that the component mass and stiffness matrix as well as their partitions admit a similar representation. Specifically, the following is true for a component stiffness and mass matrix

$$\begin{aligned} K^{(s)} &= K_0^{(s)} + \sum_{i=1}^{N_\theta} K_{,j}^{(s)} \theta_j \\ M^{(s)} &= M_0^{(s)} + \sum_{j=1}^{N_\theta} M_{,j}^{(s)} \theta_j \end{aligned} \tag{22}$$

Let $J = \{j_1, \cdots, j_s\}$ be a set of index values with $j_1 \cup \cdots \cup j_s = \{1, \cdots, N_\theta\}$, where j_s is a subset of the set J, with integer elements, denoting the parameters in the set $\underline{\theta}$ on which the structural component s depends on. Note that more than one components are allowed to depend on a parameter in the set $\underline{\theta}$.

Consider the following three special cases of structural component parameterization. In the first case, the mass and stiffness matrix of a component s do not depend on the model parameters $\underline{\theta}$. In this case the set j_s is empty. The component fixed-interface and constrained modes are independent of the parameter values. Only a single analysis is required to estimate the fixed-interface and constrained modes. Within the model updating iteration scheme, these component modes are computed once and are then used in the iterations involved, thus reducing the computational effort for assembling the reduced system from these components at each iteration.

In the second case, the mass or stiffness matrix of a structural component s depends only on one model parameter in the set $\underline{\theta}$. In this case the subset j_s contains only one element and the stiffness or mass matrices take the form

$$K^{(s)} = \bar{K}_{,j_s}^{(s)} \theta_{j_s} \qquad \text{or} \qquad \mathrm{M}^{(s)} = \bar{M}_{,j_s}^{(s)} \theta_{j_s} \tag{23}$$

Equivalently, the partitions of the component mass and stiffness matrices take the form

$$K_{ii}^{(s)} = \bar{K}_{ii,j_s}^{(s)} \theta_{j_s}, \qquad K_{ib}^{(s)} = \bar{K}_{ib,j_s}^{(s)} \theta_{j_s}, \qquad K_{bb}^{(s)} = \bar{K}_{bb,j_s}^{(s)} \theta_{j_s} \tag{24}$$

or

$$\mathrm{M}_{ii}^{(s)} = \bar{M}_{ii,j_s}^{(s)}\theta_{j_s}, \qquad \mathrm{M}_{ib}^{(s)} = \bar{M}_{ib,j_s}^{(s)}\theta_{j_s}, \qquad \mathrm{M}_{bb}^{(s)} = \bar{M}_{bb,j_s}^{(s)}\theta_{j_s} \tag{25}$$

In the case of stiffness dependence of the parameter θ_{j_s}, it can be readily shown that the matrix of the kept eigenvalues and eigenvectors of the component fixed-interface modes are given with respect to the parameter θ_{j_s} in the form

$$\Lambda^{(s)} = \bar{\Lambda}_{,j_s}^{(s)}\theta_{j_s}, \qquad \omega_j^{2(s)} = \bar{\omega}_j^{2(s)}\theta_{j_s} \qquad \text{and} \qquad \Phi_{ik} = \bar{\Phi}_{ik,j_s} \tag{26}$$

where the matrices $\bar{\Lambda}^{(s)}$ and $\bar{\Phi}_{ik}$ are solutions of the following eigen-problem

$$\bar{K}_{ii,j_s}^{(s)}\bar{\Phi}_{ik,j_s}^{(s)} = \bar{M}_{ii,j_s}^{(s)}\bar{\Phi}_{ik,j_s}^{(s)}\bar{\Lambda}_{kk,j_s}^{(s)} \tag{27}$$

which is independent of the values of θ_{j_s}. Also the constrained modes, given by $\Psi_{ib}^{(s)} = -[K_{ii}^{(s)}]^{-1}K_{ib}^{(s)} = -[\bar{K}_{ii,j_s}^{(s)}]^{-1}\bar{K}_{ib,j_s}^{(s)}$, are constant independent of the values of the parameter θ_{j_s}. It should be noted that even in this case only a single component analysis is required to estimate the fixed-interface and constrained modes, independent of the values of θ_{j_s}.
Substituting the stiffness matrices (24) and the eigenproperties (26) of a component into the reduced stiffness matrix (15), one obtains

$$\hat{K}^{(s)} = \hat{\bar{K}}_{,j_s}^{(s)}\theta_{j_s} \tag{28}$$

where $\hat{\bar{K}}_{,j_s}^{(s)}$ is given by

$$\begin{aligned}
\hat{\bar{K}}_{kk,j_s}^{(s)} &= \bar{\Lambda}_{kk,j_s}^{(s)} \\
\hat{\bar{K}}_{kb,j_s}^{(s)} &= \hat{\bar{K}}_{bk,j_s}^{(s)T} = 0_{kb}^{(s)} \\
\hat{\bar{K}}_{bb,j_s}^{(s)} &= \bar{K}_{bb,j_s}^{(s)} - [\bar{K}_{ii,j_s}^{(s)}]^{-1}\bar{K}_{ib,j_s}^{(s)}\bar{K}_{ib,j_s}^{(s)}
\end{aligned} \tag{29}$$

and it is independent on values of the the model parameters. Finally, substituting (28) into (18), the stiffness matrix of the reduced system admits the representation

$$\begin{aligned}
K^{CB} &= K_0^{CB} + \sum_{i=1}^{N_\theta} K_{,j}^{CB}\theta_j \\
M^{CB} &= M_0^{CB} + \sum_{j=1}^{N_\theta} M_{,j}^{CB}\theta_j
\end{aligned} \tag{30}$$

where the matrices K_0^{CB} and $K_{,j}^{CB}$ are given by

$$\hat{K}_0^{CB} = S^T \begin{bmatrix} \ddots & 0 & 0 \\ 0 & \hat{\bar{K}}^{(0)} & 0 \\ 0 & 0 & \ddots \end{bmatrix} S \qquad \text{and} \qquad \hat{K}_{,j}^{CB} = S^T \begin{bmatrix} \ddots & 0 & 0 \\ 0 & \hat{\bar{K}}^{(s)} & 0 \\ 0 & 0 & \ddots \end{bmatrix} S \tag{31}$$

It is important to note that the matrices $\hat{K}_0^{CB}$ and $\hat{K}_{,j}^{CB}$ are independent of $\underline{\theta}$. In order to save computational time, these constant matrices are computed and assembled once and, therefore,

there is no need this computation to be repeated during the iterations involved in optimization algorithms for model updating. This aforementioned procedure saves significant computational time since it avoids (a) re-computing the fixed-interface and constrained modes and (b) assembling the reduced matrices from these components, at each iteration step involved in model updating.

In the third case, the mass and stiffness matrices of a component depend on more than one model parameters. In this case the subset j_s contains more than one element. The stiffness and mass matrices are given by

$$\begin{aligned} K^{(s)} &= K_0^{(s)} + \sum_{j \in j_s} K_{,j}^{(s)} \theta_j \\ M^{(s)} &= M_0^{(s)} + \sum_{j \in j_s} M_{,j}^{(s)} \theta_j \end{aligned} \tag{32}$$

The fixed-interface and constrained modes have to be recomputed in each iteration involved in the model updating procedure and used to form the reduced stiffness and mass matrices of the components. This iterative computation, however, is usually confined a small number of components with sets j_s involving more than one elements.

5 CONCLUSIONS

Component mode synthesis methods were presented to substantially reduce the computational effort required in the iterative optimization algorithms used for finite element model updating. Exploiting certain schemes often encountered in finite element model parameterization, the mass and stiffness matrices of the reduced system are shown to depend linearly on the model parameters with the mass and stiffness sensitivity matrices to be assembled once and to remain constant during the iteration process. The only time consuming operation left is associated with the solution of the eigen-problem of the reduced system, avoiding the expensive estimation of the component eigen-problems at each iteration. In the proposed model updating formulation, the division of the structure into components is controlled by the parameterization scheme. The methodology is particularly efficient for large-scale finite element models where the solution of the component eigen-problem may be a computationally demanding operation. The methodology described in this work is also applicable to damage detection methods based on finite element model updating methods. According to available damage detection methods [6], the structure is subdivided into one components with properties that remain unchanged during the search for the damaged sub-structure and components with mass and stiffness properties that depend linearly on the parameter indicative of damage.

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