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# SIMPLIFIED SEISMIC PERFORMANCE ASSESMENT OVER THE LIFETIME OF A HIGHWAY BRIDGE SUBJECT TO PIER REINFORCEMENT CORROSION

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***Abstract.*** *The lifetime seismic performance of a typical segmental three-span (75+120+75) Egnatia Odos Highway bridge is assessed, considering the corrosive action of chloride ions that leads to pier strength degradation over time. The aim is to show the influence of corrosion on the seismic demand and capacity over the entire design life of the bridge, as well as the usefulness of simplified single-degree-of-freedom (SDOF) models to minimize the computational cost to non-prohibitive levels for contemporary capabilities. Five different time instants are chosen within the 120 year design life of the bridge. For each instant, the piers' reinforcement steel loss is calculated via a probabilistic model using Monte Carlo simulation to account for the uncertainty in the factors that affect the corrosion. While incremental dynamic analysis (IDA) of the complex model would be the method of choice for a comprehensive evaluation, we employ instead approximate IDA of equivalent SDOF systems with capacity curves derived by static pushover analysis. Such analyses are efficiently executed for different time moments of the design life of the bridge, taking into account the active pier rebar diameter due to corrosion effects. Thus, the usefulness of the equivalent SDOF system concept combined with nonlinear dynamic analysis or a powerful R-μ-T relationship such as SPO2IDA is shown. This usefulness turns to a necessity when more factors of epistemic uncertainty beyond the corrosion process are added in this already complex problem, disproportionately magnifying the required computational load.*

## 1 INTRODUCTION

In bridge design, the seismic load often constitutes the most critical loading case. Still, even though there is a relatively long inventory of bridges in Greece designed according to older guidelines, no significant damages have ever appeared due to earthquakes. On the contrary, the most common damage found can be attributed to the environmental conditions. Moisture, chlorides and $CO_2$ can impact the mechanical properties of structural materials, most importantly of structural steel, through the process of corrosion. Thus, the topic of ageing bridges subject to corrosion is a potentially dangerous situation in a seismically active region. The main question is whether, and at to extent, corrosion influences the seismic performance over the lifetime of a bridge.

Corrosion is a complex process that may affect a concrete bridge in a variety of ways, including, among others, cover spalling, loss of prestress, loss of steel-concrete bond strength and loss of steel itself. Since the seismic response is our topic of interest, we will focus exclusively on the effect of steel loss for the longitudinal and transverse pier reinforcement, while ignoring the potentially important impact of debonding. The reinforcement area losses lead to a direct decrease in the shear and moment capacity and secondarily in a reduction of the stiffness of piers. All the above contribute to a degradation of the bridge seismic performance. We aim to calculate this degradation over the 120 year lifetime of a well-designed modern bridge.

The corrosion in the case of reinforced concrete (RC) structures is a phenomenon that has received wide attention. Of immediate interest for the present study is the work of Stewart and Rosowsky [1] and Vu and Stewart [2] who have calculated RC bridge deck strength degradation against static loads as a function of time and distance from sea. More recently, Celarec et al. [3] have employed simplified single-degree-of-freedom models to calculate the increase of seismic risk in RC buildings due to corrosion. Finally, Ghosh and Padgett [4] have offered a holistic picture of ageing effects on the seismic fragility curves of bridges through time. Most importantly, researchers tend to agree that corrosion phenomena are subject to severe uncertainties thus necessitating the use of probabilistic models.

## 2 CASE STUDY BRIDGE

The G7 bridge is part of the Egnatia highway. It has a total length of 270m (Figure 1). The longitudinal axis of the bridge is shaped as a circular arc both in the horizontal and in the vertical plane, having radii of curvature equal to +320m and –1000m, respectively. For the construction of the bridge the segmental cantilever method has been chosen, where the piers function temporarily as pure cantilevers. These support the construction of the deck, short segments of which are extended in alternation or simultaneously from the two opposite sides of the piers to maintain a balanced load. Due to the specifics of the construction method, it is often the case that earthquakes do not constitute the critical loading for such bridges. Combined with the curvature of the bridge and the height of the piers which favor non-seismic loading cases, we do not expect to see integrity-threatening situations due to the combined corrosion and earthquake effect. Nevertheless, the relative change in the performance will help us understand the effect of corrosion for bridges where earthquake loads are dominant.

The bridge deck has a heavily post-tensioned single-cell box section (Figure 2a). The two piers M1 and M2 have box cross sections of 4.0x7.35m (Figure 2b) with a height of 41.70m and 49.50m, respectively. They are connected monolithically with the superstructure. At the abutments A1 and A2 the deck is supported on bearings of the elastic single-point type (pot bearings) moving freely. The mobility of the deck in the transverse direction is restrained by shear keys up to the point of the earthquake design loads (Figure 3). The pier foundations are deep, situated within the rocky substrate.

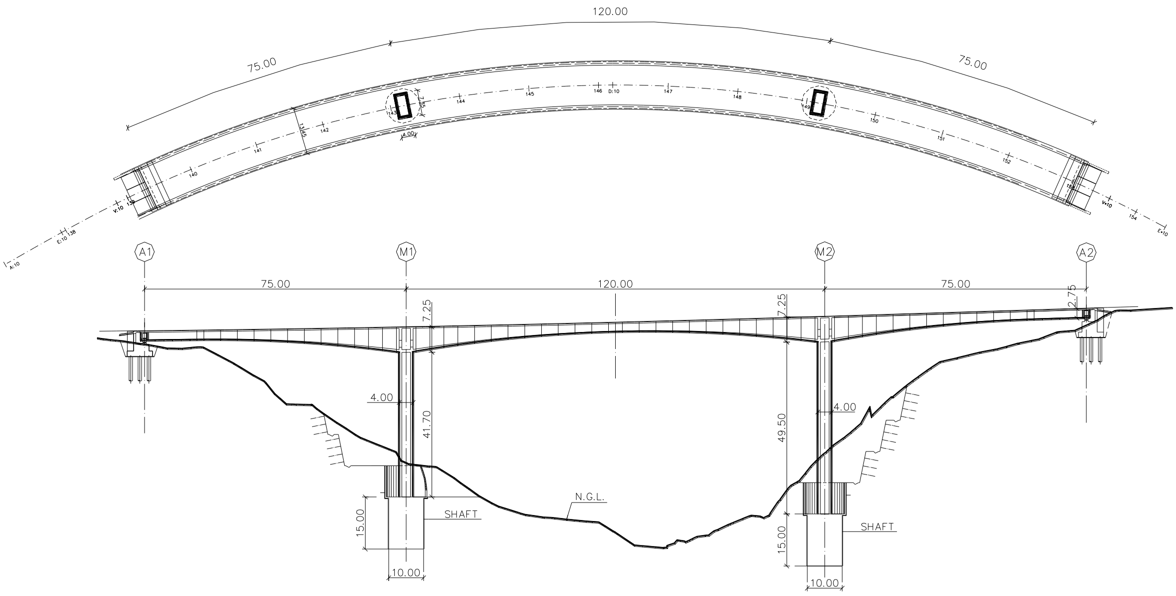


Figure 1. (a) plan view and (b) Longitudinal section.

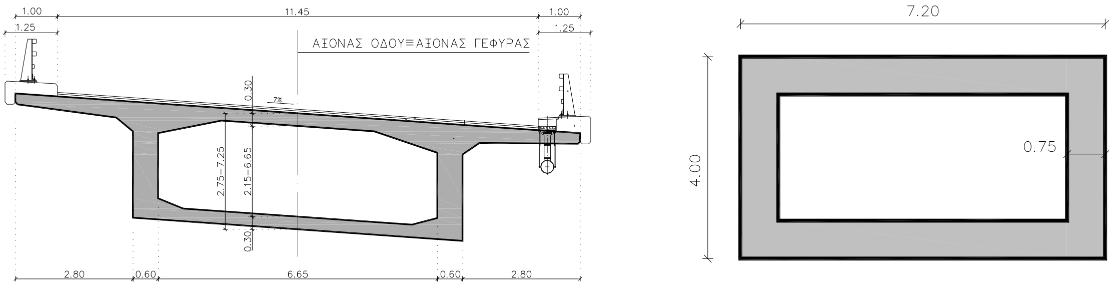


Figure 2. (a) Deck cross section and (b) Pier cross section.

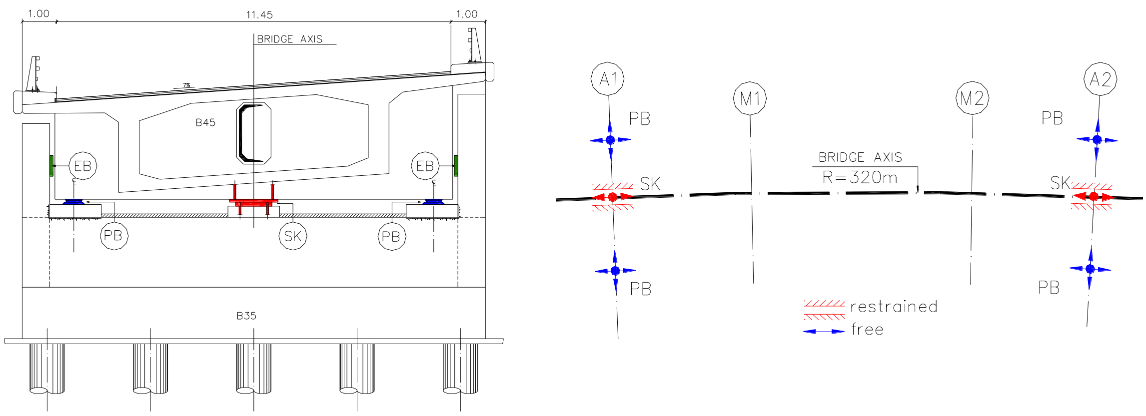


Figure 3. Deck support conditions.

A three dimensional finite-element model (Figure 4a) has been developed using the Opensees platform [5]. It is a powerful program that offers the possibility to create realistic nonlinear models with elements that approach with accuracy the real behavior of reinforced concrete structures. In addition it allows reliable analyses even under numerically unstable conditions that approach global collapse.

The bridge deck has been modeled with linear beam-column elements, closely following the centroidal axis of the tapering cross sections. The superstructure and the abutments are connected to the bearings via stiff zero-mass elements. The piers are modeled using force-based beam-column elements with fiber sections. Therein, different material properties have been used (Figure 4b) to distinguish the confined core concrete from the unconfined cover and the steel reinforcement. The effect of confinement allows higher strength and ductility at the core, an effect that is taken into account according to the Mander et al. [6] model.

For the pot bearings, a set of one-dimensional non-interacting springs have been grouped into a single macroelement. They simulate accurately the freedom of the horizontal movement under friction and the transverse connection of the bridge with the central shear key up to the load of 250 tn by employing a bilinear force-deformation relationship. Still, they do not take into account the possibility of a contact with the side walls of the abutments when the shear key collapses. At the two outer bearings of each abutment, a link with stiffness of 1MN/m has been used in the vertical direction to reduce the deck uplift during earthquake loading.

The first two mode shapes of the bridge are mainly translational for the deck (Figure 5). The first is a displacement along the longitudinal axis; the second is a displacement along the transverse axis; the third is a symmetric deformation in the vertical direction. The corresponding periods are $T_1$=1.59s, $T_2$= 1.48s and $T_3$= 0.85s. It must be noted that due to the curvature of the bridge and the existence of transverse links at the abutments, the transverse and longitudinal displacements are partially interacting [7]. As a result, the two prime mode shapes combine both transverse and longitudinal displacement of the piers.

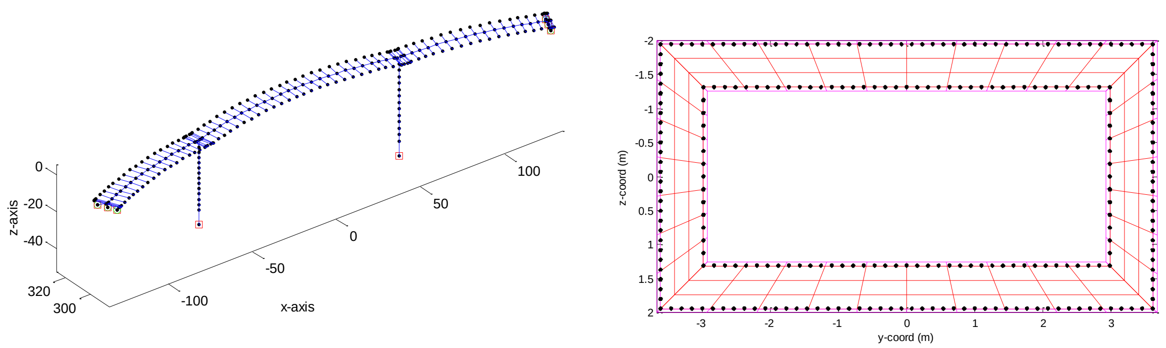


Figure 4. (a) finite-element model and (b) fiber model of the pier section.

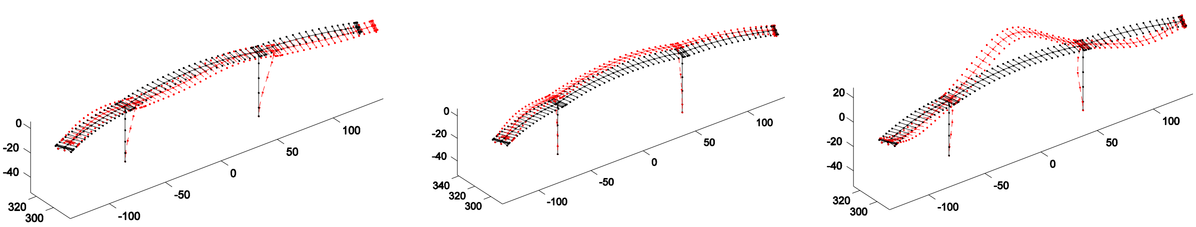

Figure 5. (a) first mode shape, (b) second mode shape and (c) third mode shape.

## 3 PIER DEGRADATION DUE TO CORROSION

Corrosion may be initiated either due to carbonation in dry-air conditions or due to the action of chlorides and moisture. The latter constitute the main corrosive hazard for this bridge, arising from the application of de-icing salts and due to sea spray exposure in a coastal environment. Reinforcement corrosion is a complex process that can attack the pier integrity via multiple vectors, e.g. spalling, debonding and steel material loss. In our case, we will only focus on the latter and will assume that corrosion leads to a uniform reduction in reinforcing steel along the pier height, equally affecting the entire external row of longitudinal and transverse reinforcement bars. A probabilistic model is used to take into account the aleatory and epistemic uncertainty inherent in the process while Monte Carlo simulation is performed to propagate the effects to the estimated structural performance.

### 3.1 Corrosion Model

The aim is to estimate the corrosion initiation time $T_i$ and the respective rebar diameter loss $\Delta D_b$ for every time instant. The corrosion deterioration model proposed by Stewart and Rosowsky [1], as amended later by Vu and Stewart [2], is used as our basis. It leads to a uniform reduction in the bar diameter of the reinforcing steel due to general (non-pitting) corrosion. For reasons of completeness, the fundamentals of the model are repeated here.

First of all, chloride content $C(x,t)$ at a distance $x$ from the concrete surface at time $t$ follows Fick's second law:

$$C(x,t) = C_o\left[1 - \operatorname{erf}\left(\frac{x}{2\sqrt{tD}}\right)\right] \tag{1}$$

where $C_o$ is the surface chloride content (kg/m$^3$), $D$ is the chloride diffusion coefficient (cm$^2$ /sec) and erf is the error function. The surface chloride concentration $C_o$ is the sum due to the application of de-icing salts $C_{oi}$ = 3.5 and air-born chlorides $C_{od}$. The latter is assumed to depend only on the distance from the coastline:

$$C_o = C_{oi} + C_{od} \tag{2}$$

The surface chloride concentration $C_{od}$ as a function of distance from the coast ($d$ in km) is:

$$C_{od} = \begin{cases} 2.95 & d \le 0{,}1\text{km} \\ 1.15 - 1.81\log_{10}(d) & 0.1\text{km} < d \le 2.84\text{km} \\ 0.03 & d > 2.84\text{km} \end{cases} \tag{3}$$

The chloride diffusion coefficient ($D$) is influenced mainly by the mix proportions (water-cement ratio):

$$D = D_{H_2O}\, 0.15 \frac{1 + \rho_c \dfrac{c}{w}}{1 + \rho_c \dfrac{w}{c} + \dfrac{\rho_c}{\rho_a}\dfrac{a}{c}} \left( \frac{\rho_c \dfrac{w}{c} - 0.85}{1 + \rho_c \dfrac{w}{c}} \right)^3 \ (cm^2/\text{sec}) \tag{4}$$

The terms $\rho_a$ = 2.7g/cm$^3$ and $\rho_c$ = 3.12g/cm$^3$ are the mass densities of cement and limestone aggregates, respectively. The ratio $a/c$ = 0.66/0.11 is the aggregate-cement ratio, $D_{H2O}$ =1.6x10$^{-5}$ cm$^2$/sec is the chloride diffusion coefficient in an infinite solution and the water-cement ratio is estimated as

$$w/c = \frac{27}{f'_{cyl} + 13.5} \tag{5}$$

where $f'_{cyl}$ is the mean concrete compressive strength in MPa, assumed to be approximately 7.5MPa above the characteristic strength.

The corrosion rate is a function of concrete quality, cover thickness and time. The time-dependency appears due to the formation of rust on the steel surface, which tends to reduce the corrosion rate:

$$i_{corr}(t_p) = \frac{32.13}{b}(1 - w/c)^{-1.64}\, t_p^{-0.29} \qquad (\mu\mathrm{A}/cm^2) \tag{6}$$

where the concrete cover $b$ is given in cm and $t_p$ is the time since corrosion initiation.

The corrosion initiation time $T_i$ is calculated from Equation (1) to coincide with the time that the chloride concentration reaches a critical value $C_r$ of 0.9kg/m$^3$ on average. Thus, if the inverse of the error function is erf$^{-1}$, we have:

$$T_i = \frac{b^2}{4D}\left[\mathrm{erf}^{-1}(1 - C_r/C_0)\right]^{-2}. \tag{7}$$

Finally the deterioration process caused by reinforcement corrosion will lead to a uniform reduction $\Delta D_b(t_p)$ in the bar diameter of the reinforcing steel as follows:

$$\Delta D_b(t_p) = \min\left(0.0232\int_{T_i}^{t_p} i_{corr}(\tau)\,\mathrm{d}\tau\;,\;D_{b0}\right). \tag{8}$$

The reduction is zero for times earlier than $T_i$, $D_{b0}$ is the initial bar diameter and the instantaneous corrosion rate (at the surface) in mm/yr is taken as $\lambda(\tau)=0.0116\cdot R\cdot i_{corr}(\tau)$, where $R$=1 for general corrosion. The parameters $f'_{cyl}$, $C_{od}$, $C_{oi}$, $C_r$, D and $i_{corr}$ are randomly distributed as described in [2]. The latter two quantities, in addition to the randomness induced by their parameters, also incorporate model epistemic uncertainty.

### 3.2 Impact of corrosion on the longitudinal pier reinforcement

The effects of corrosion on the longitudinal reinforcement are presented in Figure 6. The rebar diameter decreases throughout the lifetime of bridge from 25 mm to a median value of 22.7 mm which corresponds to a 9% reduction. At each of the five time instants selected, Monte Carlo simulation is performed for 100 realizations of the bridge. Since nonlinear dynamic analysis is prohibitive for such numbers, we instead chose to employ static pushover analysis (SPO). Thus, the bridge is pushed along its longitudinal axis where it is most vulnerable to collapse. Figure 7 shows the resulting 500 SPO curves together with their corresponding medians. Although the effect is not overwhelming, there is still a 10% median reduction in terms of the maximum base shear capacity after 120 years; it drops from 17720 kN to 15880 kN. The low impact of corrosion is mainly attributed to the large diameter bars, the sufficient concrete cover and the good quality of the concrete. While the latter two factors are intuitive, the former is perhaps slightly more difficult to understand. The reason is that corrosion uniformly attacks the entire surface area of the rebars. At least for general corrosion, the diameter reduction is the same under the same conditions, regardless of the bar size. Thus, if the same amount of steel is distributed among few large diameter bars or many small diameter ones, corrosion will always impact more heavily the latter configuration. For the above reasons, the longitudinal reinforcement will not suffer much from the chloride attack.

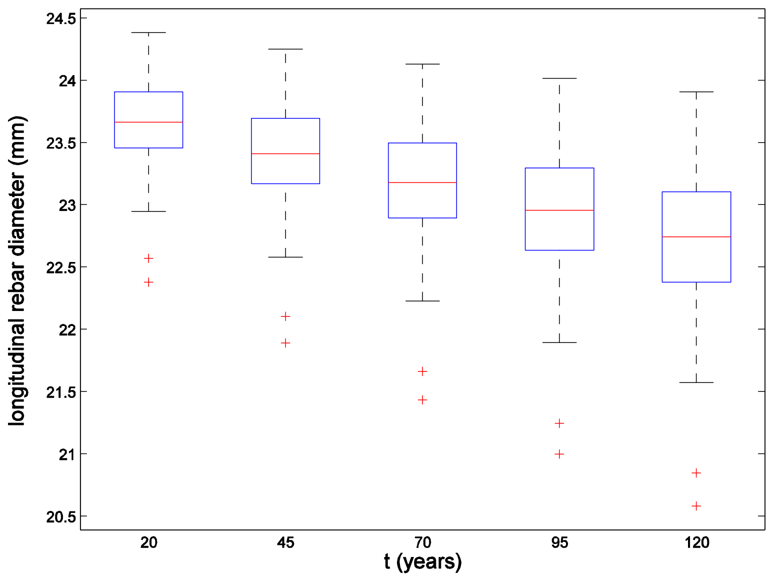


Figure 6. Boxplots of the longitudinal rebar diameter at 25 year intervals. The three horizontal lines defining each box represent the 25, 50 and 75% values of the sample while the whiskers show the gross extent of the sample distribution, out of which the outliers (red crosses) are.

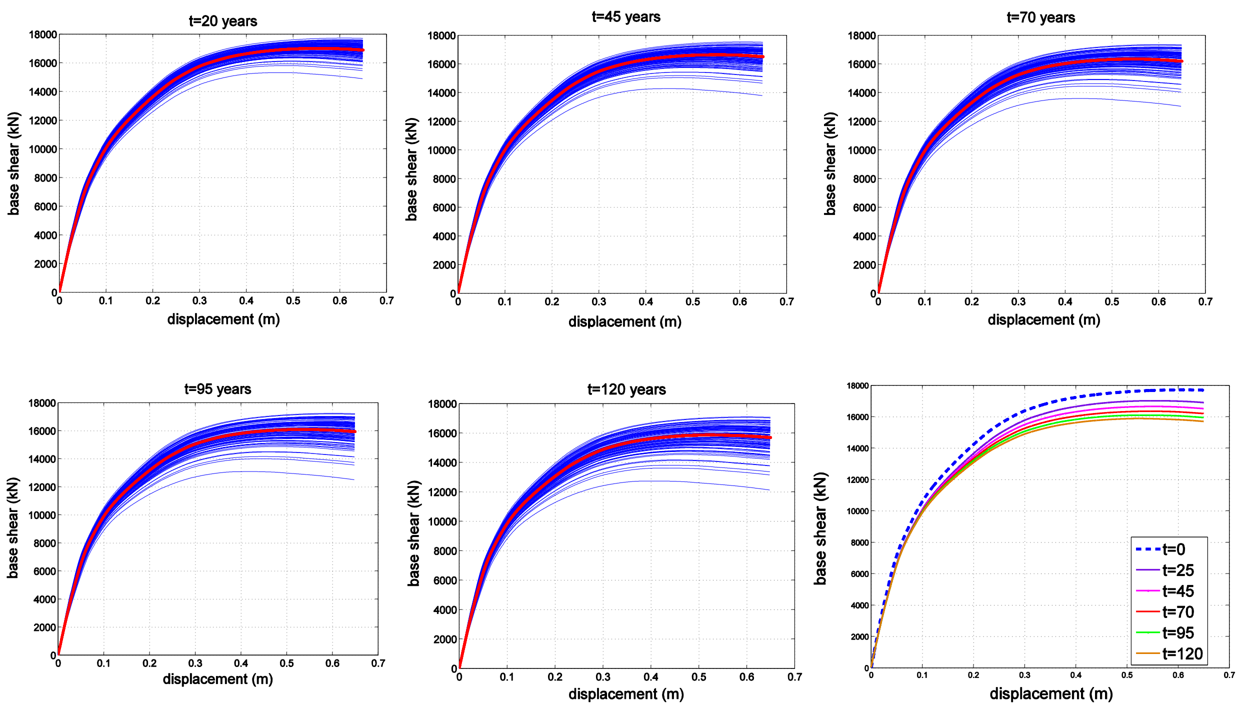


Figure 7. (a),(b),(c),(d),(e) SPOs of 100 samples (blue) at 25 year intervals together with their median (red); (f) the median SPOs for every time instant.

### 3.3 Impact of corrosion on the transverse pier reinforcement

While longitudinal reinforcement is well protected from corrosion, the same is not true for transverse reinforcement. Actually, the shorter distance to the surface and the significantly smaller bar diameter are both heavily aggravating factors that work against the integrity of the stirrups.

The shear strength of concrete members is given by

$$V_R = V_s + V_c \tag{9}$$

where $V_c$ and $V_s$ are the nominal strength of concrete and transverse reinforcement shear-resisting mechanisms, respectively. According to Directive E39/99 [8], within the plastic end regions the concrete contribution is removed from the shear strength. The reinforcement contribution (truss mechanism) according to DIN1072 is given by

$$V_s = \frac{A_{sv}}{s} f_y \frac{7}{8} d \tag{10}$$

where $A_{sv}$ is the total area of transverse reinforcement in a layer in the direction of the shear force, $s$ is the spacing along the member axis of the layers of stirrups or hoops, and $d$ the effective depth of the section.

The results of corrosion impact to the transverse reinforcement and finally to the piers' shear strength are presented in Figure 8. The base shear strength throughout the lifetime of the bridge decreases from 21770 kN to 17140 kN. This corresponds to a 21% reduction in the median value, most of which happens at an early stage: After the first 20 years of the bridge's life, shear strength decreases from 21770 kN to 18950 kN, having a 13% median reduction. While still not threatening for the bridge, which retains enormous amounts of shear capacity due to the overdesign mandated by the code minima for reinforcement, such reductions are not to be ignored.

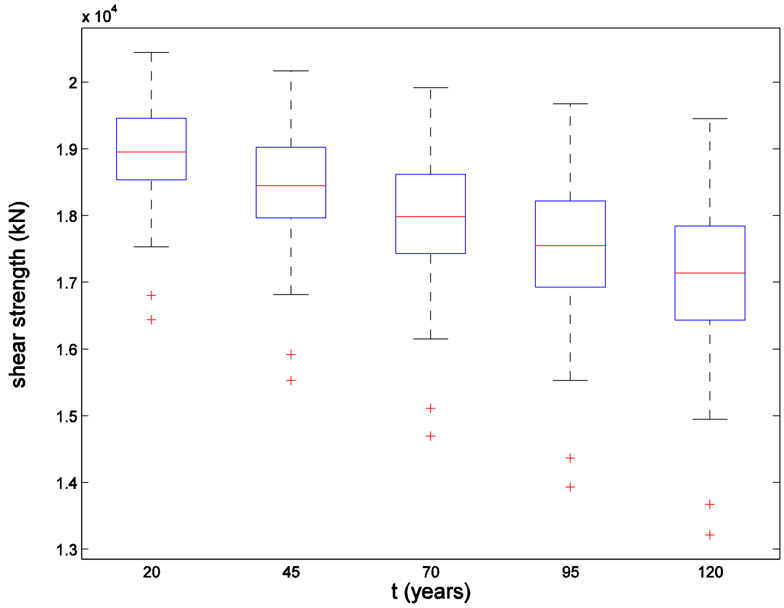


Figure 8. Boxplots of the pier shear strength at 25 year intervals. The three horizontal lines defining each box represent the 25, 50 and 75% values of the sample while the whiskers show the gross extent of the sample distribution, out of which the outliers (red crosses) are.

## 4 DYNAMIC PERFORMANCE OF CORRODED BRIDGE

Perhaps the most comprehensive method for estimating seismic performance is Incremental Dynamic Analysis (IDA) [9]. It utilizes numerous nonlinear time-history analyses of the system under a suite of multiply-scaled ground motion records. In our case the aim is to produce the distribution of seismic demand and capacity, ideally represented by summarized 16, 50 and 84% IDA curves, for each of the five time instants (20, 45, 70, 95 and 120 years) within the 120 years design life of the bridge. The seismic performance of the bridge is investigated for loads mainly along its longitudinal axis, where it is most vulnerable to collapse. In this direction, the bridge has a general elastoplastic response as shown by the SPO curve (Figure 7). For every time instant 100 model realizations are used in a Monte Carlo simulation in order to take into account the epistemic uncertainty inherent in corrosion.

Assuming that 30 records are used at seven intensity levels for IDA for each corroded bridge realization, then 30x7x5x100=105.000 dynamic time-history analyses should be executed on a multi-degree-of-freedom (MDOF) model, requiring unreasonable computational resources. In order to reduce the computational load to realistic levels, simplified methods must be employed. Thus, following the example of Celarec et al. [3], we will use a suitable equivalent SDOF instead of the MDOF model, and replace the expensive IDA with an SPO analysis together with SPO2IDA [10,11]. The latter is a powerful $R$-$\mu$-$T$ (reduction factor-ductility-period) relationship which allows instantaneous estimation of summarized IDA curves of SDOF systems, offering significant savings in computations at the cost of a small error. Thus, weeks of analysis are reduced to less than 48hrs on a single computer.

### 4.1 Simplified single-degree-of-freedom (SDOF) models

The applicability of equivalent SDOF models to estimate the MDOF response largely depends on the regularity of the structure (Isakovic et al. [12]). In irregular structures such a substitution may underestimate significantly the seismic response. According to the eigenvalue analysis, the studied bridge responds in each direction predominantly in a single mode. Moreover the primary mode shapes are mainly translational with only low coupling between them. On the contrary the rotational mode shapes are of secondary importance. Thus, the bridge can be characterized as regular and the use of equivalent SDOF models is appropriate.

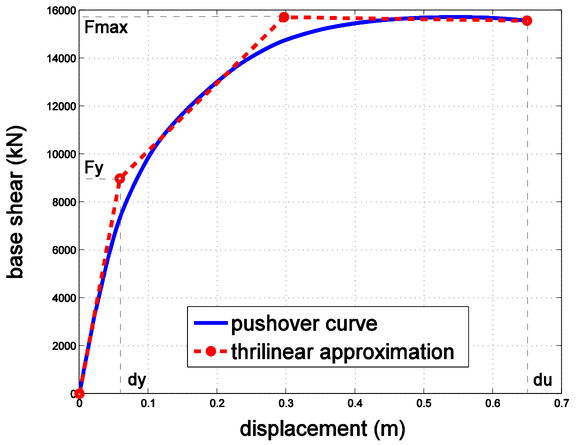


Figure 9. The SPO curve and its trilinear approximation.

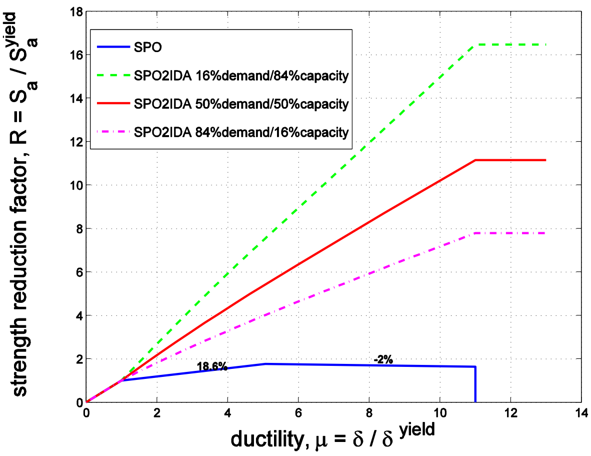


Figure 10. The summarized IDA curves as predicted by SPO2IDA for the adopted trilinear fit to the SPO.

## 4.2 Approximate Incremental Dynamic Analysis via SPO2IDA

The Static Pushover to IDA (SPO2IDA) software [10,11] is used to approximate the IDA 16, 50 and 84% curves by taking advantage of the results of the static pushover (SPO). In our case, the capacity curves are closely approximated with a trilinear shape. This starts elastically, followed by a hardening positive-stiffness part and then by a lightly negative-stiffness segment that we choose to terminate at a reasonable maximum attainable displacement dictated by the abutment geometry (Figure 9).

The SPO2IDA software is based on the fitted response of multiple SDOF systems with a wide range of periods, moderately pinching hysteresis and 5% viscous damping. It can effortlessly generate the statistics of the normalized intensity $R$ versus normalized response $\mu$ curves (Figure 10) for complex quadrilinear force-deformation backbone shapes, unlike simpler $R$-$\mu$-$T$ relationships that are restricted to the simplest of shapes and just the mean response. $R$ is the reduction factor, defined as the ratio $S_a(T_1,5\%)/S_a^{yield}(T_1,5\%)$ of the spectral acceleration over its value at yield. Similarly, $\mu$ is the ductility, i.e., the ratio of displacement response over the displacement at yield $\delta/\delta^y$

To obtain proper IDA curves in terms of $S_a(T_1,5\%)$ and for any desired structural response, a denormalization must take place. We employ the procedure described by Vamvatsikos and Cornell [12] but allowing for the simplifications of Fragiadakis and Vamvatsikos [13]. Thus, the initial elastic slope of the IDA fractile curves for the deck displacement is set to $4\pi^2/(C_0 T_1^2 g)$, where $C_0$ is the first-mode participation factor, e.g., as defined by FEMA-356 [14]. The approximate 16, 50 and 84% IDA curves are then produced for every realization of the bridge and every time instant considered, shown in Figure 11. According to the results, the corrosion causes the displacement of bridge deck to increase by 10% on average for any given level of $S_a(T_1,5\%)$, closely following our earlier SPO-based conclusions for this simple SPO shape. While not a threatening condition for this bridge, where seismic loads were not critical for design, it still provides a useful standard of comparison for other, less-protected cases.

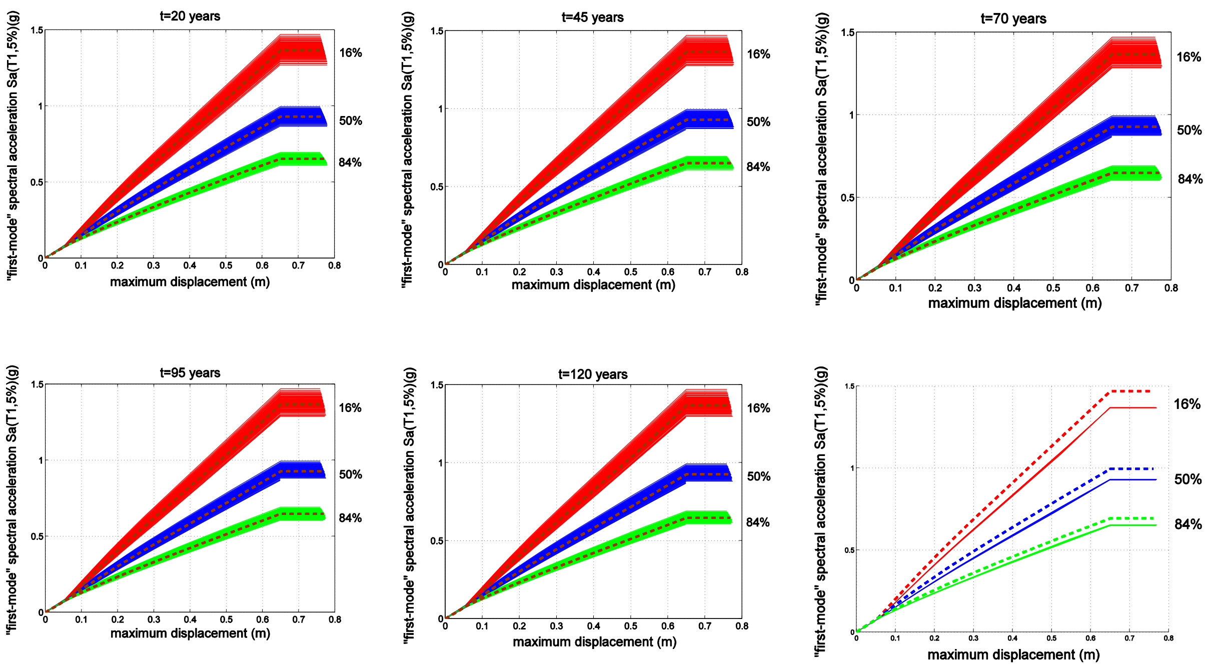


Figure 11. (a),(b),(c),(d),(e) 16, 50 and 84% fractile IDA curves for 100 samples together with the overall median curve for each fractile (in black) ; (f) the overall median IDA curves for all time instants and each fractile.

## 5 CONCLUSIONS

The seismic performance of a typical cantilever-construction three-span bridge under the effect of corrosion has been examined. Instead of performing IDA on the MDOF bridge model, we produce instead the 16, 50 and 84% IDA capacity curves approximately, for each of the five time instants within the 120 years design life of the bridge, using the SPO2IDA software on the equivalent SDOF bridge model. The latest simplification is generally applicable as the examined structure is a first-mode dominated system as most similar bridges tend to be. Thus, the equivalent of almost 100,000 dynamic time-history analyses can be executed in less than two days rather than several weeks. This allows us to consider a fully probabilistic model of steel reinforcement corrosion and accurately assess its effects on the seismic response of the bridge. It is a general method that can be efficiently applied to reinforced concrete structures and leads to useful, quantifiable results.

In the case examined, since only the external layer of pier reinforcement is exposed to corrosion, a median degradation of 10% for flexural capacity and 20% for shear capacity is expected to occur within the design life of 120 years. In both cases, the degradation rate is at its highest at the initiation of corrosion and it slowly declines with time. Thus, more than half of the reduction is typically reached within the first twenty years. Still, the effects are not overwhelming for this bridge. For example, although the shear capacity loss seems substantial, it poses no threat for the bridge as the code-minimum transverse reinforcement has led to large amounts of reserve shear capacity.

Bridge G7 is a contemporary bridge designed by the latest codes. It enjoys good quality concrete and a thick reinforcement cover which are the principal factors acting against corrosion. Added to the fact that earthquakes are not the critical loading case, it becomes clear that corrosion will never be catastrophical for the piers. This conclusion might be very different for older or contemporary bridges where earthquake loads are dominant. In such cases, performing assessment or design while neglecting the substantial effect of corrosion may not be a wise choice.

## REFERENCES

[1] M.G. Stewart, D.V. Rosowsky, Time-dependent reliability of deterioration reinforced concrete bridge decks, *Structural Safety*, **20**, 91–109, 1998.

[2] K.A.T. Vu, M.G. Stewart, Structural reliability of concrete bridges including improved chloride-induced corrosion models, *Structural Safety*, **22**, 313–333, 2000.

[3] D. Celarec, D. Vamvatsikos, M. Dolsek, Estimation of the seismic risk with consideration of capacity degradation over time for RC buildings, *Bulleting of Earthquake Engineering*, 2011 (to appear).

[4] J. Ghosh, J.E. Padgett, Aging Consideration in the Development of time-dependent seismic fragility curves, *ASECE Journal of structural engineering,* **136**(12),1497-1511, 2010.

[5] F. McKenna, G. Fenves, *Open system for earthquake engineering simulation,* Pacific Earthquake Engineering research Center, Berkley, California.

[6] J.B. Mander, M.J.N. Priestley, R. Park, Theoretical stress-strain model for confined concrete, *ASCE Journal of structural engineering,* **114**(8), 1804-1826, 1988.

[7] C. Giannelos, D. Vamvatsikos, Simplified Methods of Nonlinear Dynamic Analysis of a Bridge under Bi-Directional Seismic Load, *Proceedings of the* 3*th* *Greek Conference on Earthquake Engineering and Engineering Seismology*, Athens, November 5-7, 2008.

[8] Greek Ministry for the Environment, Physical Planning and Public Works, Guidelines for the seismic design of bridges, Directive 39, 1999.

[9] D. Vamvatsikos, C.A. Cornell, Incremental Dynamic Analysis. *Earthquake Engineering and Structural Dynamics*, **31**(3), 491–514, 2002.

[10] D. Vamvatsikos, *SPO2IDA software for short, moderate and long periods*, 2002. http://users.ntua.gr/divamva/software/spo2ida-allt.xls.

[11] D. Vamvatsikos, C.A. Cornell, Direct estimation of the seismic demand and capacity of MDOF systems through Incremental Dynamic Analysis of an SDOF Approximation. *ASCE Journal of Structural Engineering*, **131**(4), 589–599, 2005.

[12] T. Isakovic, M. Fischinger, P. Kante, *Bridges: when is single mode seismic analysis adequate?*, Proceedings of the Institution of Civil Engineers-Structures and Buildings, 156(2), 165-173, 2003.

[13] M. Fragiadakis, D. Vamvatsikos, Fast performance uncertainty estimation via pushover and approximate IDA. *Earthquake Engineering and Structural Dynamics*, **39**(6), 683–703, 2010.

[14] FEMA-356. *Prestandard and Commentary for Seismic rehabilitation of buildings.* Federal Emergency management Agency: Washington, DC, 2000.