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NUMERICAL ANALYSIS OF THE TWO BASIC COLLAPSE MECHANISM OF A TYPICAL COLONIAL FAÇADE

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Abstract. *The paper presents the computational modelling and the seismic damage assessment of a typology of façade that is recurring among the churches built during the colonial period in México, between 16^{th} and 18^{th} century. This typology is symmetric, with two relatively stubby bell towers on either side of the main façade itself, while the openings are usually aligned along the vertical axis of symmetry, including a portal that is much wider than the window underneath the tympanum. Even if these churches were built differently, according to the regional seismicity, this notwithstanding we considered This is an "archetype model" whose characteristics can be considered as a good basis to be representative of this typology in general, even if there is some local variability, mainly related to the regional level of seismicity.*

The main seismic collapse mechanisms of this typology tend to involve both the towers and the central part of the façade with damages that can be related both to in-plane shear actions and to the out-of-plane actions. The central part of the façade often presents damages and cracks along the symmetry axis and along the sides connecting the towers. These towers present very often a localized damage in the belfry due to flexural actions and also can present a damage mechanism related with the torsion in their middle part.

In order to study the complex nature of these damage mechanisms, we adopted two specific Rigid Body and Spring Models (RBSM) to analyse separately the in-plane and the out-of-plane dynamical behaviours. Great attention has been posed into selecting a set realistic constitutive rules to describe the main meso-scale material-damage mechanisms. The main characteristics are: i) orthotropy of the flexural and membranal response; ii) material response is strongly influenced by average vertical axial compression; iii) modelling the hysteretic energy dissipated by repeated cyclic loading.

Given a suitable discretization, these RBSM allowed us to describe realistic damage patterns and also the effects of the higher modes of vibration with a reasonable computational effort and using real accelerograms.

a) b)

Figure 1: Typical colonial churches built in: a) low seismic risk zone (central Mexico), b) high seismic risk zone (Pacific coast).

1 INTRODUCTION

Ancient masonry structures frequently show extensive damages after being subjected to earthquakes, even in the case of rather moderate seismic intensity. As an example, after the earthquake of magnitude 7.0 that happened in central México on 1999, 1542 colonial churches were detected as damaged, in a radius of more than 100 km from the epicenter. The careful survey and seismic assessment of this architectural heritage affected by such a severe earthquake was a very effective way to understand the structural weaknesses of these construction. These churches, that were mainly built during the colonial period between the 16^{th} to 18^{th} centuries, are quite variable in size and in architectural style, but it is possible to identify a general recurrent typology. Clearly, we should also mention that an important factor which influenced the architectural style was the local experience of the ancient builders due to the regional seismic activity, as shown, for example in Figure 1.

In this context, and for the present study, it seemed appropriate to establish a zone of interest and a typology of church that can be considered as typical. In particular, this typology is inspired by some churches that are present in the the State of Puebla, located around 250 km from the subduction zone. In this area the main earthquakes can be qualitatively considered of "medium" intensity and quite frequent. The survey after the 1999 earthquake showed that even if the number of full collapse was relatively low, this notwithstanding the large part of the hystorical churches suffered heavy damages which had a fairly typical and specific crack pattern. In fact, in these churches the out-of-plane mechanism is rarely preminent, somewhat differently from the experience made after recent Italian earthquakes [1]. On the other hand, it was often observed a complex influence of the presence of two large and relatively stubby bell towers built symmetrically on either side of the main façade.

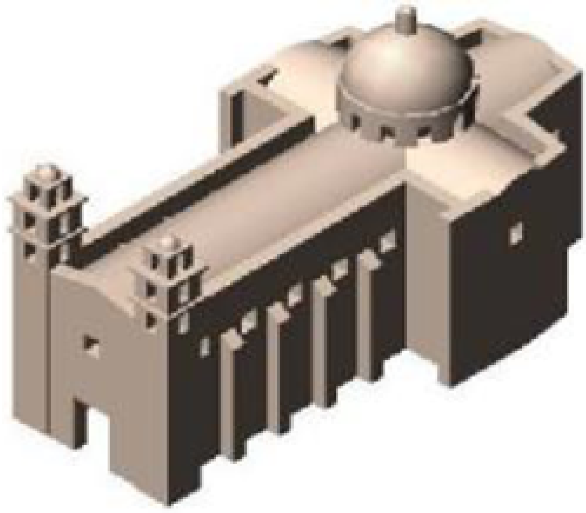

Figure 2: Typical colonial church of the central Mexico.

2 A TYPICAL COLONIAL CHURCH

Thousands of churches were built in Mexico from 16^{th} to 18^{th} century, and persist to date in rather good conditions; they vary in size and architectural sophistication, but follow some recurrent basic typologies. The simplest among them are rather small parochial churches which are found in every ”barrio” of Méxican towns and villages. One important factor that has influenced the evolution of their architectural features has been the experience of damage suffered from earthquake activity. In most areas of the Pacific Coast, where the recurrent destruction of the early colonial constructions produced an evolution towards low rise, heavily buttressed buildings with scarce external ornamentation. In other regions the lower concern for earthquake failures favored taller and more slender constructions, as for example in the area of Puebla, in which we are now focusing our attention.

2.1 Geometric and material description

The typical colonial church in the central Mexico has a Latin-cross floor plan. The Figure 2 shows the isometric projection of the building that we have taken as a representative example of the typology to be studied. In this example, the main nave of the church measures 20×58 m^2, while the height of the vaults is 18 m. Its two bell towers are 28 m high. The roof is constituted by a quadripartite vaulting system. A hemispherical dome is placed over the transept bay and it is supported by the drum. Small buttresses are placed along the main nave. In the first bay there is an intermediate floor for the chorus.

The façade is composed by the ”main wall”, which is connected with the towers and the frontispiece. A schematic view, in elevation and with a section in plan is shown in Figure 3. The façade has, generally, two openings: the main door and a window for illuminate the chorus. The height of the door is between five to eight meters. The main material of construction is a heterogeneous masonry constituted by stones agglutinated by lime-sand mortar. Frequently, broken clay bricks or lightweight volcanic stones were added to the masonry; this heterogeneous masonry constitutes a kind of concrete whose composition varies according to the structural element; it is lighter than normal stone masonry, and has a tensile strength bigger than brick masonry, due mainly to the absence of weak planes constituted by the mortar layers. On the

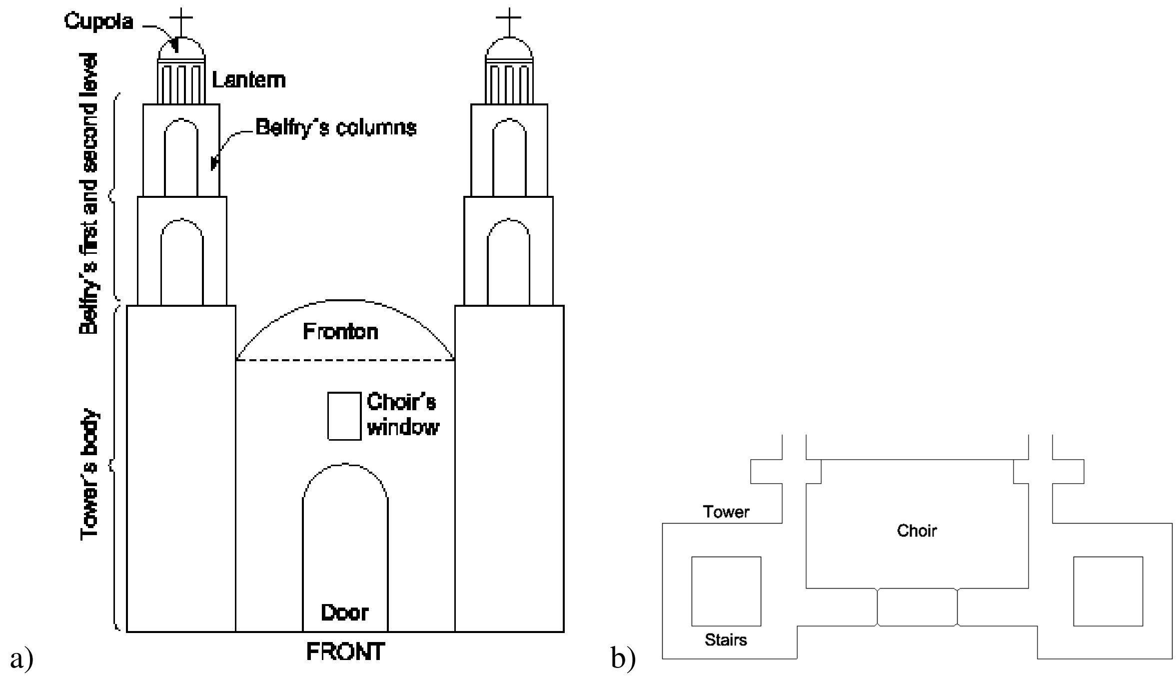

Figure 3: Schematical representation of the façade: a) Front view, b) Plan view.

basis of these characteristics, we can consider the material as homogeneous at the macroscopic scale [2]. From the mechanical point of view, the response can be considered almost isotropic, or orthotropic as a function of different factors, such as the type of loading (e.g. in-plane or out-of-plane) and the stress condition (e.g. in the case of prevaliling compression load along vertical direction).

The bell tower can be divided in three different parts: the main body of the tower which comprises the base; the belfry which can be made as a combination of superimposed bodies; and the cover, that is often a small hemispheric dome at the top of the bell tower. In plant, the body of the towers can be treated as a square pipe, which is attached to the walls of the façade and to the lateral wall of the nave. Small openings can be found in the main body, which have the function to illuminate and ventilate the stairwell. In general, it is common that the belfry is made by two or three superimposed bodies; being the most top one smaller than the other ones. Sometimes the third body is more similar to a "lantern like" element.

2.2 Typical damages

The observation and evaluation of seismic damage suffered by typical masonry historic churches have allowed to identify their basic modes of failure and the main characteristics of their seismic response: the behaviour is governed by the low tensile and shear strength of constituting materials, which makes it difficult to guarantee the full continuity within and between structural members, in the event of strong ground motions, and leads to specific mechanisms for resisting seismic actions [1, 3].

After the earthquake of 1999, damage surveys of churches from the state of Puebla was made by different institutions, and it was possible to recover reliable data from a total of 30 churches from Puebla. It must to bear in mind that this number is not the total of churches damaged by the 1999 earthquake, especially in the State of Puebla.

The large part of these buildings presented damages on the bell towers, being one of the most damaged elements, both in the belfry and in the main body of the tower, whereas only few cases presented damages in the small dome. The bell towers are rather weak elements, in which the ground motion tends to be amplified in the upper levels. Even if they are relatively low and sturdy, damages are rather common, especially in the vertical elements and in the arches surrounding the belfry. Additionally, their bending and torsion motion tends to separate them from the rest of the church, or to generate shear cracking in the basement of the tower. The façade is typically an heavy wall, poorly connected to the rest of the temple, and the out-of-plane vibrations tend to separate it from the towers. Horizontal cracking in the frontispiece, or in the lower part of the façade weakened by large openings, is rather common, sometimes giving rise to the partial or total overturning of the façade.

3 RIGID BODY AND SPRINGS NUMERICAL MODELS

In order to cover a wide spectrum of possible mechanisms of damage two specific *rigid body spring models* (RBSM) has been considered: one for the *in-plane* [4, 5], and another one for *out-of-plane* numerical analyses [6, 7]. These two numerical models has been implemented with the aim to perform full dynamical analyses by considering non linear behaviour and the mechanical degradation of the masonry material. In the present investigation real recorded ground motions have been adopted as forcing actions.

A procedure of calibration has been performed in order to obtain a good approximation of the fundamental natural modes of vibration of the church, as it resulted from a previous research in which the complete three-dimensional building was investigated in the linear-elastic by a detailed finite element model [8]. In this process, it was also necessary to investigate and consider with some attention the influence of the lateral walls that affects the values of natural periods and modal shapes of the façade. The essential parameters assumed to describe the macroscopic masonry material behaviour are reported in Table 3. Both models were analyzed

Property	Magnitude	Unite
Elasticity modulus	1962	MPa
Shear modulus	817	MPa
Poisson coefficient	0.2	
Mass density	1600	kg/cm^3
Compressive strength	2943	kPa
Tensile strength	147	kPa
Cohesion	200	kPa
Friction angle	15	degree

Table 1: Mechanical properties of the masonry [9, 10]

using as a forcing action the strong ground motion recorded at Ciudad Serdad during the Mexico earthquake on 15 June, 1999. The time history in terms of ground acceleration of the NS and EW components are shown in Figure 4.

3.1 In-plane dynamical analysis

The model implemented for the in-plane dynamic analysis is based on a RBSM approach that was specifically developed for modelling the non-linear behaviour and damage of the ma-

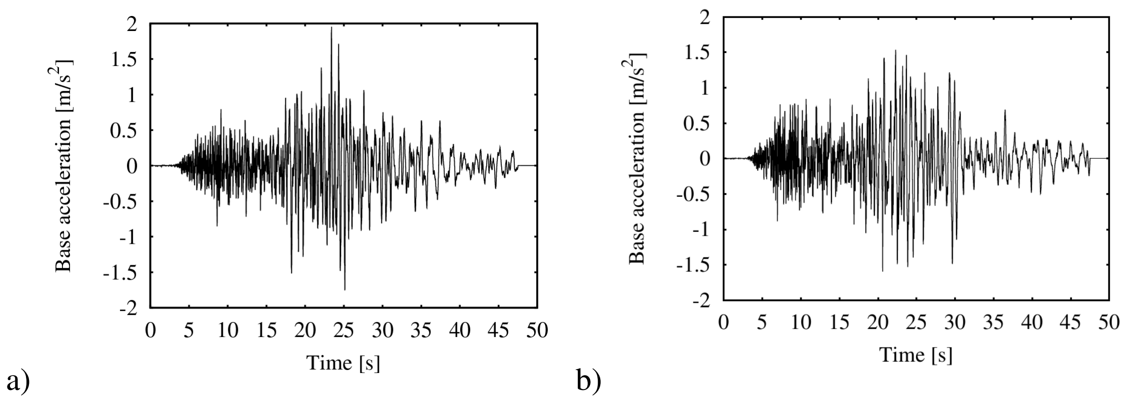

Figure 4: Acceleration time hystoris of the recording used in the analyses: a) PueCSER990615-EW component, b) PueCSER990615-NS component.

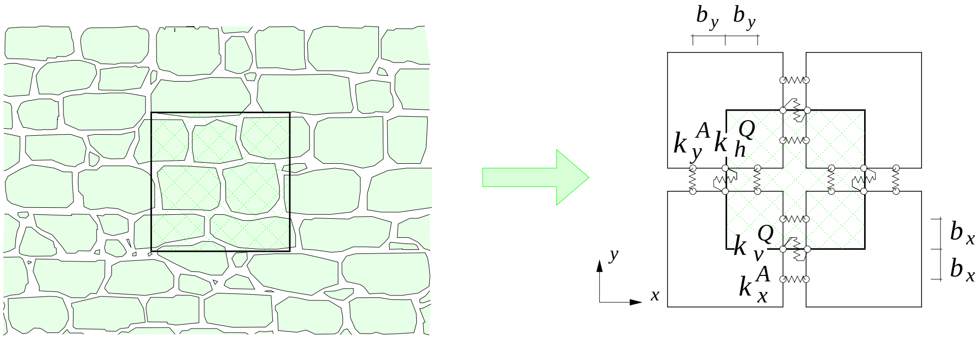

Figure 5: Schematic view of an irregular masonry and the corresponding RBSM "unit cell" made by four rigid elements [5].

sonry material [11, 4, 12, 13]. This plane model considers the heterogeneous solid material as a mechanism consisting of rigid masses connected by simple elastic-plastic springs, in the spirit of the rigid body spring model (RBSM) [14, 15, 16], and also other discrete element methods [17]. The core of the present model is the *unit cell* defined by four rigid elements connected to each other by two normal springs plus one shear spring at each side, as shown in Figure 5, on the right. Since the model is designed to work at the macro-scale, the size of the unit cell should be equal or larger than the representative volume element of the masonry. Potentially, this model allows to define the elastic characteristics of the springs by a specific procedure of identification with the objective to transfer some characteristics of the internal texture to the macro-scale model [13]. For the present case, as a first approach, and considering the observsations written in Section 2.1, the we have hypothesized a masonry texture composed by quite irregular blocks. This implies that this material presents limited effects of microstructure and the description of the global behaviour at the macro-scale can be approximated by assuming a simplified isotropic material based on the parameters reported in Table 3. On the other hand, a simplified phenomenological approach, based on the experimental tests available in the technical literature has been followed in order to model the response under the cyclic loading [18, 19, 20]. Different hysteretic rules are assumed for the axial springs and for the shear springs, as sketched in Figure 6.

A resume of the results obtained by the discrete model made by 237 rigid elements, for a

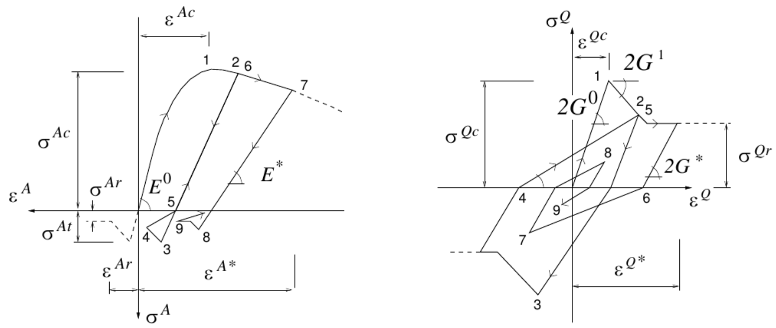

Figure 6: Hysteretic rules for axial and shear springs [5].

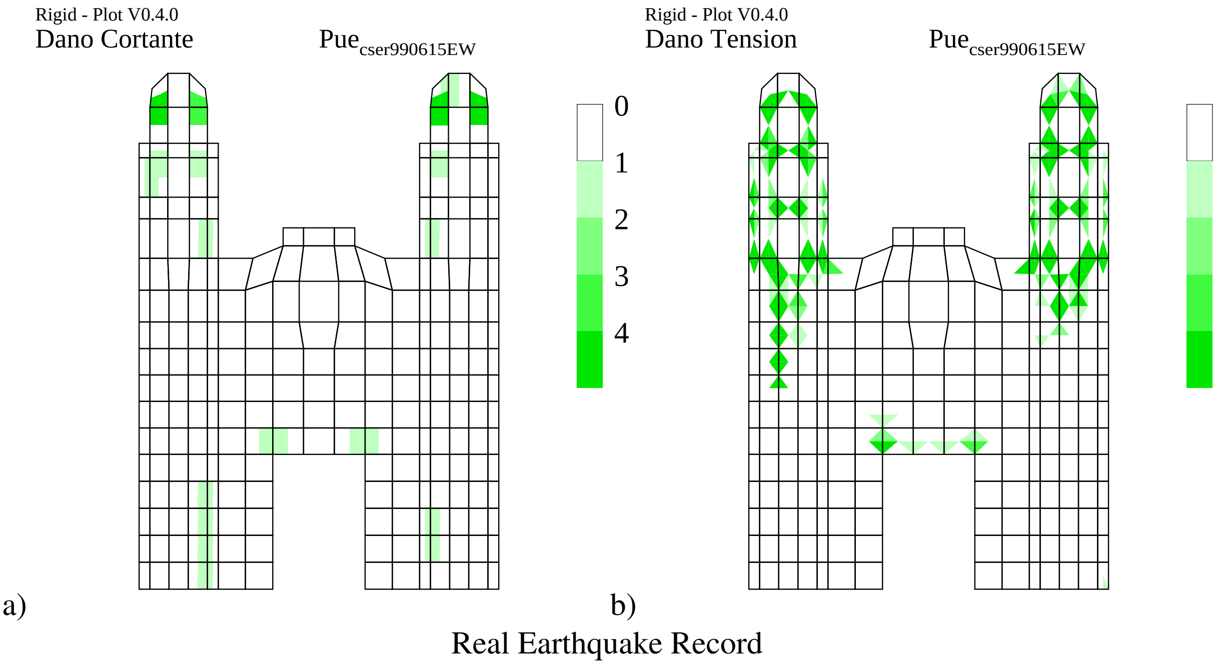

Figure 7: Damages for the in-plane model for real earthquake record: the EW component shown in Figure 4. a) Shear damage, b) Tensile damage

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total amount of 711 degrees of freedom, is shown in Figure 7 that reports the damage patterns obtained by a full-dynamic non-linear analysis. A damage index scale $[0-4]$ has been created, as a funtion of the degradation of the mechanical characteristics of the springs of the numerical model. The main damage was related to the tensile stresses due to the belfry flexion. The belfries were the most vulnerable elements in the towers, since they present an appendix-like behaviour. A vertical cracking along the body of the tower is also present, while a slight damage zone due to shear forces is obtained in the main wall of the façade, as well as in the body of the tower. There is no compression damage in any element.

These types of damages are quite similar to the damages observed during the strong ground motions of 1999. The tower tends to separate from the rest of the structure, due to a vertical crack. Thus, this vertical crack is not due to vertical crushing, but it is due to a combination of shear actions and horizontal tensile forces. As in this particular case, it is rather common that this vertical crack appears along the body of the tower. However in some cases, the vertical crack can appear at the union of the tower with the main wall. Shear cracks are also present in the walls of the central part of the façade as well as in at the base of the tower.

3.2 Out-of-plane dynamical analysis

The out-of-plane behaviour of the masonry façade is modelled at the macro-scale level as an assemblage of quadrilateral rigid elements connected by elastic-plastic joints at which only bending and twisting movements are possible. This model was originally intended to plan out the analysis of relatively slender walls, remaining within the scope of the Kirchhoff plate model [6, 21]. In this sense, as a general rule, the wall should be regarded as a special case of a three-dimensional body $\mathcal{B} \subset \Re^3$ with one dimension - the thickness t - small compared to the other characteristic lengths. Hence, the model describes the whole structure in terms of the mid-surface $\mathcal{M} \subset \mathcal{B}$, and only the transverse displacements of this reference mid-surface are considered. For the present research, the model has been enhanced in order to be employed also in the presence of the towers, even if this case cannot be considered, at a first sight, as coherent with the terms of the Kirchhoff model. Nevertheless, we believe that the application of this model is acceptable because it is still able to describe the dynamics of these towers when their response is a mix of flexural and torsional-type. Clearly this analysis should be complemented by the in-plane analysis and morover it cannot describe the shear damages of the tower out-of the plane the façade.

Thus, a Cartesian reference frame $\{O, x, y, z\}$ is then defined so as the plane $z = 0$ is coincident with the mid-plane mapping that is parametrized by only employing the global coordinates $(x, y) \in \Omega$. The plane domain Ω is then discretized by a partition into m quadrilateral sub-domains ω_i (with $i = 1, m$) such that no vertex of one sub-domain lies on the edge of another one. The mid-point of the sides of these sub-domains is the joint between the elements and is modeled as a spherical hinge from the kinematic point of view. This spherical joint allows small rotations whose axis is in the plane $z = 0$. We distinguish two components of rotation: one in which the axis is parallel to the edge of the abutting elements (flexural deformation), and the

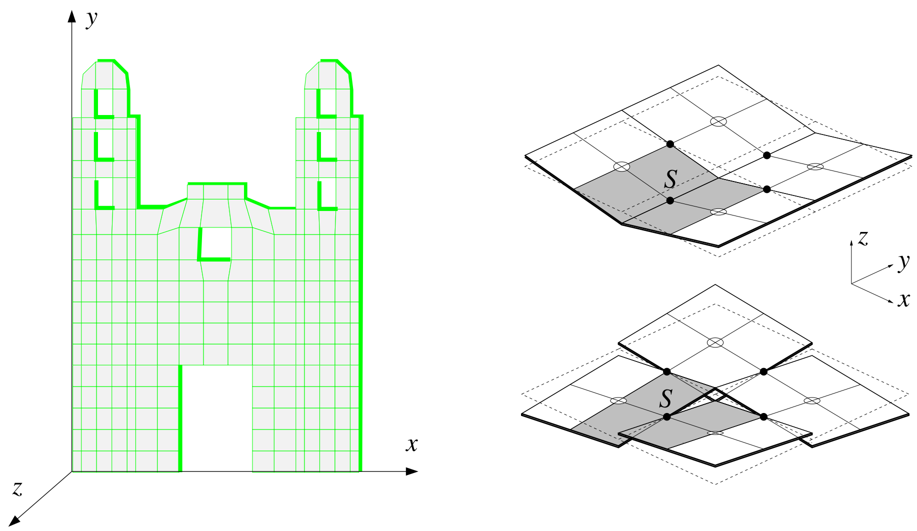

Figure 8: RBSM mesh for the out-of-plane analysis (left), and sketch of an assemblage of 4 rigid elements subjected to flexural and torsional deformation [7].

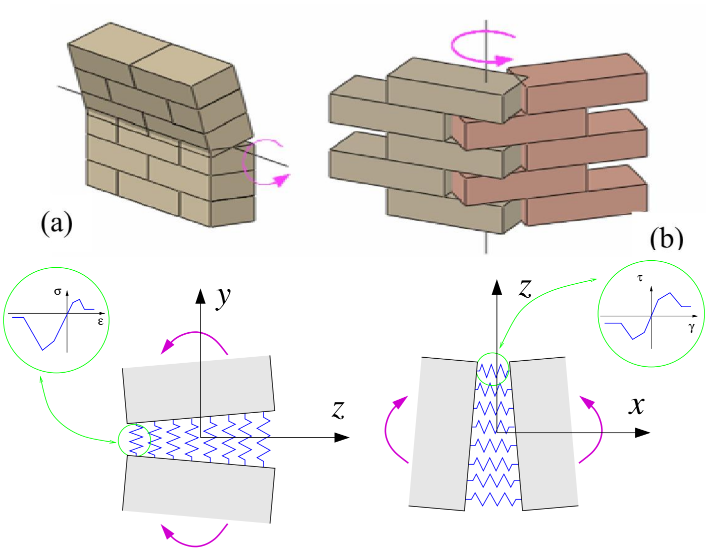

Figure 9: Heuristics of the mechanical connection between a couple of elements; corresponding layers of non-linear line-springs.

other whose axis is perpendicular to the edge (twisting deformation), as shown in Figure 8 on the right side. The allowed displacements w of the points on the reference xy plane are parallel to z; they are small when compared to the thickness and are called *deflections*. The mass is lumped into these r connection nodes, as shown in Figure 8 where the mass of the shaded area A_s is given to node S. Then, the mean bending and twisting curvature is measured at each connection joint as a function of the angle between the rigid elements.

The constitutive behavior of joints between the elements is defined on the basis of the one-to-one moment-curvature relations assigned to the bending and twisting springs placed in correspondence of each connection joint S. In the linear-elastic field, the out-of-plane constitutive behavior could be properly described by referring to the homogeneous orthotropic Kirchhoff plate model. [1]

Out-of the linear elastic field, the moment-curvature relationship assigned to each connection joint follows a piecewise linear function that is defined following an heuristic procedure in which the behavior of the wall is modelled at a sectional level by a combination of line-springs. In the Figure 9 (top) the cases of flexural damage due to rotation along an horizontal hinge (on the left side) and along a vertical hinge (on the right side) are shown. In the Figure 9 (bottom), we have sketched the different non-linear constitutive laws that are assigned to the line-spring along the vertical direction (for modelling a rotation along an horizontal hinge) and to the line-spring along the horizontal direction (for modelling a rotation along a vertical hinge).

Another aspect that must be considered in the definition of the flexural and torsional out-of-plane behaviour, is the very different effect of the in-plane vertical pressure on the response when considering the horizontal bending or the vertical bending. The geometric texture of the masonry material has also an important role in the definition of the moment-curvature rela-

[1] the orthotropic plate rigidities can be related to the Young moduli and Poisson coefficient according to the following relation, also recommended by M.T. Huber [22]:

$$D_{xx} = \frac{E_x J_{yy}}{1-\nu^2} \quad , \quad D_{yy} = \frac{E_y J_{xx}}{1-\nu^2} \quad , \quad D_{xy} = \nu\sqrt{D_{xx} D_{yy}} \quad , \quad D_{ss} = \frac{1-\nu}{2}\sqrt{D_{xx} D_{yy}} \tag{1}$$

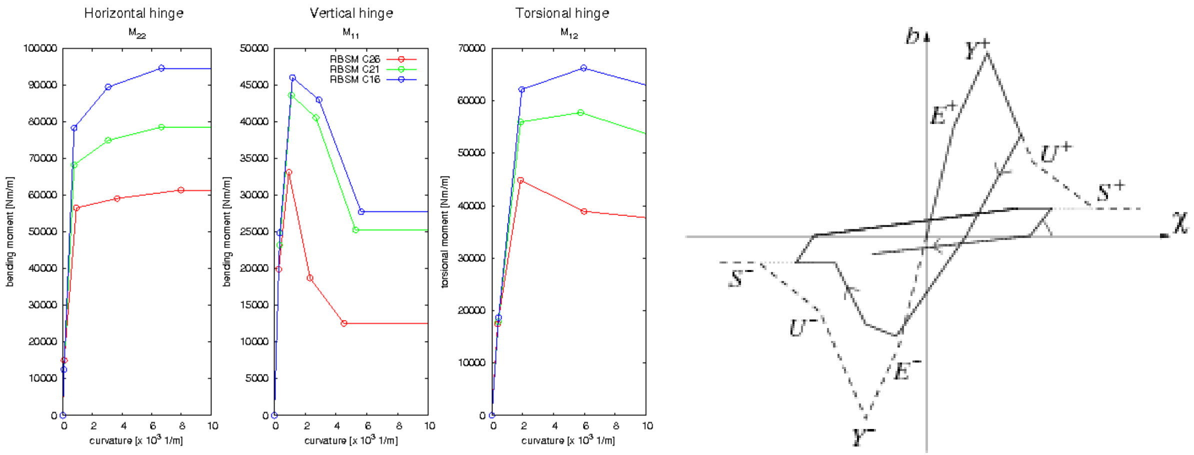

Figure 10: Skeleton curves for the relationship between bending moment and curvature in correspondence of a different vertical pressure: red=low, green=average, blue=high, (on the left side of the drawing). Scheme of hysteretic rules under cyclic loading (on the right side of the drawing).

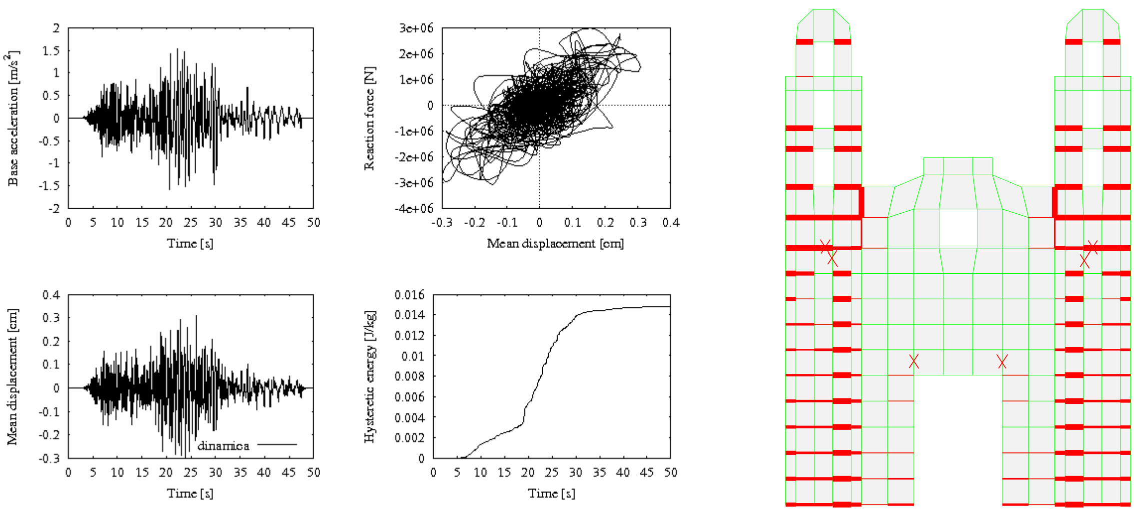

Figure 11: Scheme of damage after PueCSER990615-NS component. Map of energy dissipated.

tionship for the bending along a vertical hinge. As a consequence, the out-of-plane flexural response is strongly orthotropic and variable in the different connection points of the model [23]. In the present model, the simplified out-of-plane homogenization gives skeleton curve that are strongly related to the local vertical compression stress, as shown in Figure 10. As a first approximation, a very simple criterion was adopted here, in which the mean compression stress is assumed as directly proportional to the vertical distance of the connection joint from the upper edge of the specimen.

The non-linear dynamical analysis is performed at the macro-scale by solving the elastic-plastic problem by means of a standard full Newton-Raphson algorithm [24] until the attainment of a prefixed numerical convergence.

The Figure 11 shows a summary of the response to the NS component of the strong ground motion recorded at Ciudad Serdad during the Mexico earthquake on 15 June, 1999. On the left side of the figure, some time histories have been reported, while the drawing on the right side shows the distribution of the histeretic energy dissipated during the dynamic analysis, and it can be considered as a reasonable first indication of damage and mechanical degradation. We can observe that the damage pattern is not too much different with respect to the in-plane case already shown in Figure 7. In fact, the most vulnerable parts of the façade are again the belfries (due to out-of-plane flexure) and the joint areas between the towers and central façade. In this particular case we also note that the main body of the façade seems not to suffer from the typical out-of-plane damages that affected in many cases the Italian façades [3, 21].

4 FINAL REMARKS

The first remark of this research is that the behavior of this type of façade, very typical of the colonial churches in México, has some special features that were hardly found in previous studies that were based mainly on the surveys of the masonry churches affected by earthquakes in Italy, for which the out-of-plane collapses were often the main threat. In the present case, the presence of the towers, built in an integrated manner with the façade and not simply constructed in compliance, substantially alter the situation. In fact, on the one hand these towers provide a constraint that reduces the movement off-plan, but also they increase the in-plane actions, and are also committed themselves to a combined bending and torsion stress. We find a framework in which the shear actions have a significant role in the damage patterns, especially in the joints between the facade and towers, and moreover we observed damage due to bending-torsion in the main body of the towers. The complete picture of this behavior must therefore be studied by means of a modelling that should be able to capture all these different aspects. The application of the two models proposed here seems suitable for this purpose and it is considered promising for a subsequent and more detailed investigation of this typology.

5 ACKNOWLEDGMENTS

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REFERENCES

[1] F. Doglioni, A. Moretti, V. Petrini, Le Chiese e il Terremoto, Lint press, Trieste, 1994, in Italian.

[2] R. Meli, R. Sánchez-Ramírez, Criteria and experiences on structural rehabilitation of stone masonry buildings in mexico city, International Journal of Architectural Heritage 1 (2007) 3–28. doi:10.1080/15583050601123118.

[3] V. Petrini, S. Casolo, F. Doglioni, Models for vulnerability analysis of monuments and strengthening criteria, in: Bisch, P and Labbe, P and Pecker, A (Ed.), Proceedings of the Eleventh European Conference on Earthquake Engineering - Invited lectures , A A Balkema Publishers, Schipholweg 107C, PO BOX 447, 2316 XC Leiden, Netherlands, 1999, pp. 179–198, 11th European Conference on Earthquake Engineering, Paris, FRANCE, Sep 06-11, 1998.

[4] S. Casolo, Modelling in-plane micro-structure of masonry walls by rigid elements, International Journal of Solids and Structures 41 (13) (2004) 3625–3641. doi:10.1016/j.ijsolstr.2004.02.002.

[5] S. Casolo, F. Pena, Rigid element model for in-plane dynamics of masonry walls considering hysteretic behaviour and damage, Earthquake Engineering & Structural Dynamics 36 (8) (2007) 1029–1048. doi:10.1002/eqe.670.

[6] S. Casolo, Rigid element model for non-linear analysis of masonry façades subjected to out-of-plane loading, Communications in Numerical Methods in Engineering 15 (7) (1999) 457–468. doi:10.1002/(SICI)1099-0887(199907)15:7<457::AID-CNM259>3.0.CO;2-W.

[7] S. Casolo, Modelling the out-of-plane seismic behaviour of masonry walls by rigid elements, Earthquake Engineering & Structural Dynamics 29 (12) (2000) 1797–1813. doi:10.1002/1096-9845.

[8] R. Meli, F. Peña, On elastic models for evaluation of the seismic vulnerability of masonry churches., in: Lourenço, Roca, Modena (Eds.), Proceeding of Structural Analysis of Historical Constructions, Padova, Italia, 2004, pp. 1121–1131.

[9] M. Chávez, Estudio experimental de las propiedades mecánicas de mampostería de piedra natural, Master thesis, Universidad Nacional Autónoma de México, Mexico, 2005, in Spanish.

[10] F. Peña, J. Meza, Seismic assessment of bell towers of mexican colonail churches, Advanced Materials Research 133-134 (2010) 585–590. doi:10.4028/www.scientific.net/AMR.133-134.585.

[11] S. Casolo, F. Pena, Modelling micro-structure aspects of masonry walls by a simplified approach, in: Brebbia, CA (Ed.), Structural Studies, Repairs and Maintenance of Heritage Architecture VIII, Vol. 16 of Advances in Architecture Series, Wessex Inst Technol, WIT press, Ashurst Lodge, Southampton SO40 7AA, Ashurst, England, 2003, pp. 337–346, 8th International Conference on Structural Studies, Repairs and Maintenance of Heritage Architecture, Halkidiki, Greece, May 07-09, 2003.

[12] S. Casolo, V. Petrini, A specific model for in-plane non-linear dynamics of masonry walls including some texture effects, in: Jones, N and Brebbia, CA (Ed.), Structures Under Shock and Impact VIII, Vol. 15 of Structures and Materials, Wessex Inst Technol, WIT press, Ashurst Lodge, Southampton SO40 7AA, Ashurst, England, 2004, pp. 283–292, 8th International Conference on Structures Under Shock and Impact, Iraklion, Greece, Mar 2004.

[13] S. Casolo, Macroscale modelling of microstructure damage evolution by a rigid body and spring model, Journal of Mechancis of materials and Structures 4 (3) (2009) 551–570. doi:10.2140/jomms.2009.4.551.

[14] D. V. Griffiths, G. G. W. Mustoe, Modelling of elastic continua using a grillage of structural elements based on discrete element concepts, International Journal for Numerical Methods in Engineering 50 (7) (2001) 1759–1775. doi:10.1002/nme.99.

[15] T. Kawai, New discrete models and their application to seismic response analysis of structures, Nuclear Engineering and Design 48 (1978) 207–229.

[16] M. Ostoja-Starzewski, Lattice models in micromechanics, Applied Mechanics Review ASME 55 (1) (2002) 35–60. doi:10.1115/1.1432990.

[17] J. V. Lemos, Discrete element modeling of masonry structures, International Journal of Architectural Heritage 1 (2) (2007) 190–213. doi:10.1080/15583050601176868.

[18] G. Calvi, M. Priestley, Post collapse analyses of a medioeval masonry tower, in: Fifth North American Masonry Conference, University of Illinois at Urbana-Champaign, 1990.

[19] A. Anthoine, G. Magenes, G. Magonette, Shear compression testing and analysis of brick masonry walls, in: Proceeding of the 10th European Conference on Earthquake Engineering, Wien, 1994, pp. 1657–1662.

[20] G. Calvi, G. Magenes, Experimental research on response of urm building systems, in: U.S.-Italian Workshop on guidelines for seismic evaluations and rehabilitations of unreinforced masonry buildings., Pavia, 1994.

[21] S. Casolo, S. Neumair, M. A. Parisi, V. Petrini, Analysis of seismic damage patterns in old masonry church facades, Earthquake Spectra 16 (4) (2000) 757–773. doi:10.1193/1.1586138.

[22] S. P. Timoshenko, S. Woinowsky-Krieger, Theory of Plates and Shells, McGraw-Hill, 1959.

[23] S. Casolo, G. Milani, A simplified homogenization-discrete element model for the non-linear static analysis of masonry walls out-of-plane loaded, ENGINEERING STRUCTURES 32 (8) (2010) 2352–2366. doi:10.1016/j.engstruct.2010.04.010.

[24] K.-J. Bathe, Finite Element Procedures in Engineering Analysis, Prentice-Hall Inc., New Jersey, U.S.A., 1996.