A COMPARATIVE STUDY OF FORCE-DEFORMATION RELATIONSHIP OF FRP-CONFINED CONCRETE COLUMNS

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Keywords: FRP-Confined Concrete, Force-Deformation, Strengthening

Abstract. In this study, the FRP-confined concrete models for prismatic members given in the ACI440.2R-08 and Turkish Earthquake Code (TEC2007) are compared with the experimental data taken from the literature. It is found that TEC2007 assumes only hardening behavior for concrete strengthened with FRP. Although ACI440.2R-08 assumes a hardening and softening behavior depending on the confinement-effectiveness, the limit separating the hardening and softening must be improved to predict the behavior well. The assumptions in these codes cannot predict the post-peak region of force-deformation curves because of the high factor of safeties. It is essential to understand the degree of confinement, confinement effectiveness and the resulting stress-strain relationship of a concrete strengthened with FRP.
1 INTRODUCTION

Reinforced concrete structures should have enough capacity to resist lateral forces. Most buildings in high seismic zones in Turkey do not have the desired capacity and their situations are getting worse by the environmental effects, such as temperature variations, humidity, corrosion etc. It is obvious that vulnerable structures should be strengthened against the potential high magnitude earthquakes. Since the number of deficient buildings is high, strengthening method should be easy to apply in a short period, i.e. occupant friendly techniques must be preferred.

Fiber reinforced polymer (FRP) can be one of the optimal choice because of its unique properties. It has high strength and modulus. It is durable when compared to steel and concrete. It is easy to apply which enables to strengthen more buildings in a shorter period. Moreover, strengthening can be done without disturbing the residents via evacuation.

Considering the abovementioned advantages, codes start to recommend the use of FRP as one of the alternative strengthening technique. However, because of some uncertainties related to either material behavior or strengthening application, codes define equations that are more conservative. Since the material and application result in a high initial investment, the equations in the codes make it difficult to utilize.

In this study, the force-deformation characteristics of square columns are searched via analytical study.

2 MATERIAL MODELS

To generate a force-deformation curve, material models should be defined first. For FRP-confined concrete, the models available in the ACI440.2R-08 and Turkish Earthquake Code (TEC2007) are used. For steel model, a simple model is proposed and used.

2.1 FRP-Confined Concrete Model

2.1.1. TEC2007 Model

In TEC2007-Section 7E, evaluation of strength and ductility enhancement using FRP-confined concrete columns, equations related to the strength, strain and shear calculations are introduced.

Strength Enhancement:

In this section, it is stated that in order to increase the axial load capacity of a column using FRP material, the ratio of long side to short side of the column should not be greater than 2.0. If corners of rectangular sections rounded and the section turns to be an elliptical section, than the ratio can be 3.0. Dimensions of each section are given in Figure 1. $f_{cc}$ can be calculated as follows:

\[
f_{cc} = f'_{co} \left[ 1 + 2.4 \left( \frac{f_i}{f_{co}} \right) \right] \geq 1.2f'_{co} \quad (1)
\]

\[
f_i = \frac{1}{2} \kappa_{a} \rho_{f} \varepsilon_{f} E_{f} \quad (2)
\]

\[
\varepsilon_{f} \leq \begin{cases} 0.004 \\ 0.5\varepsilon_{fu} \end{cases} \quad (3)
\]
\[
\kappa_a = \begin{cases} 
\frac{1}{b} & Cylindrical sections \\
\frac{1}{h} & Ellipsoid sections \\
1 - \frac{(b - 2r_c)^2 + (h - 2r_c)^2}{3bh} & Rectangular sections 
\end{cases} 
\]

where \( \kappa_a \) is section effectiveness factor, \( \rho_f \) is the volumetric ratio of FRP, \( \varepsilon_{fu} \) is the ultimate strain capacity of FRP, \( \varepsilon_f \) is the effective strain capacity of FRP, \( E_f \) is the modulus of elasticity of FRP, \( b \) is the short dimension of prism and \( h \) is the long dimension of the prism.

**Figure 1. Sectional properties**

\( a) \) Cylindrical Section \hspace{1cm} \( b) \) Rectangular Section \hspace{1cm} \( c) \) Elliptical Section

**Ductility Enhancement:**
Ultimate strain capacity can be calculated as follows:

\[
\varepsilon_{cc} = 0.002 \left[ 1 + 15 \left( \frac{f_i}{f'_{co}} \right)^{0.75} \right] 
\]

\( 2.1.2. \text{ACI440.2R-08 Model} \)

In ACI440.2R-08-Chapter 12, strengthening of members subjected to axial force or combined axial and bending forces, equations related to strength and strain are introduced. It is stated that “The provisions are not recommended for members having noncircular section featuring side aspect ratios \( h/b \) greater than 2.0, or face dimensions \( b \) or \( h \) exceeding 900 mm unless testing demonstrates their effectiveness”.

In pure axial compression section, the maximum confined concrete compressive strength \( (f_{cc}) \) and the maximum confinement pressure \( (f_i) \) are calculated using Lam and Teng’s equation [9] with an additional reduction factor \( \psi = 0.95 \).

For ultimate strain, the code limits the strain value to prevent excessive cracking and resulting loss of concrete integrity.

\[
f_{cc} = f'_{co} + \psi 3.3 \kappa_a f_i \\
f_i = \frac{2E_f n t_f \varepsilon_{fe}}{D} \\
\varepsilon_{fe} = \begin{cases} 
\kappa e \varepsilon_{fu} & \text{axial compression (} \kappa_e = 0.55 \text{)} \\
0.004 \leq \kappa e \varepsilon_{fu} & \text{combined axial compression and bending} 
\end{cases} \\
D = \sqrt{b^2 + h^2}
\]
\[ \varepsilon_{cc} = \varepsilon_{co} \left( 1.5 + 12 \kappa_b \frac{f_t}{f_{co}} \left( \frac{\varepsilon_{fe}}{\varepsilon_{co}} \right)^{0.45} \right) \leq 0.01 \]  
(10)

\[ \kappa_a = \frac{A_e}{A_c} \left( \frac{b}{h} \right)^2 \]  
(11)

\[ \kappa_b = \frac{A_e}{A_c} \left( \frac{h}{b} \right)^{0.5} \]  
(12)

\[ \frac{A_e}{A_c} = 1 - \left[ \left( \frac{b}{h} \right) \left( h - 2 \tau_c \right)^2 + \left( \frac{h}{b} \right) \left( b - 2 \tau_c \right)^2 \right] \frac{3A_g}{\rho_g} \]  
(13)

In these equations, \( \kappa_a, \kappa_b \) are the efficiency factor (for strength and strain, respectively) which takes the shape of the section into account, \( \varepsilon_{fe} \) is the effective strain level in FRP, \( \kappa_e \), is the FRP strain efficiency factor, \( \frac{A_e}{A_c} \), is the effective confinement area ratio, \( \rho_g \) is the longitudinal steel ratio, \( n \) is the number of FRP layers and \( t_f \) is the thickness of the FRP material.

### 2.1.3. Comparison of the models

It is found out that FRP usage does not result in a hardening behavior all the time. Depending on some parameters such as, unconfined concrete strength, number of FRP layers and corner radius in prismatic members, a softening behavior is possible if the FRP effectiveness is not adequate.

Mirmiran et. al. [6] proposed “Modified Confinement Ratio (MCR)” to determine if a section shows hardening or softening behavior. The equation for MCR is given in Eqn. 14. It is seen that parameters affecting MCR are confinement effectiveness and sectional properties. It is stated by Mirmiran et. al. that if MCR is greater than 0.15 then it is possible to see a hardening behavior. To validate whether this limit is well-defined or not, 132 experimental data (76 tests having hardening behavior and 56 having softening behavior) are collected from the literature and their MCR factors are calculated. MCR values are compared with the experimental data as seen in Table 1 and it is realized that MCR factor can be used to predict the behavior.

\[ MCR = \frac{2r_r f_t}{D_f f_{co}} \]  
(14)

\[ f_t = \frac{2A_{FRP} f_{FRP}}{D} \]  
(15)

ACI440.2R-08 also takes the degree of confinement into account and states that for minimum confinement ratio \( \frac{f_t}{f_{co}} \geq 0.08 \) should be used. As it is seen from Table 1 the limit can predict if a section shows hardening or softening behavior but it is not as successful as the MCR factor.
Table 1. Verification of the hardening limits

<table>
<thead>
<tr>
<th>Experimental Data</th>
<th>MCR Prediction</th>
<th>ACI440.2R-08 Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td># of data</td>
<td>Accuracy, %</td>
<td># of data</td>
</tr>
<tr>
<td>76 Hardening</td>
<td>62 Hardening</td>
<td>73 Hardening</td>
</tr>
<tr>
<td></td>
<td>82</td>
<td>96</td>
</tr>
<tr>
<td>56 Softening</td>
<td>52 Softening</td>
<td>17 Softening</td>
</tr>
<tr>
<td></td>
<td>93</td>
<td>30</td>
</tr>
<tr>
<td>Total 132 data</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>86 % of the data is accurately predicted</td>
<td>68 % of the data is accurately predicted</td>
</tr>
</tbody>
</table>

In Figure 2, typical behaviors of low strength square concrete specimens confined with FRP are given together with the analytical predictions. As seen in the figures, ACI440.2R-08 and TEC2007 models assume lower strain and strength values, which yields higher factor of safety (Figure 2a). If the confinement is inadequate as shown in Figure 2b both procedures give higher strength and low strains. From the figures, it can be seen that TEC2007 assumes hardening behavior in both cases, but ACI440.2R-08 can predict the change in the behavior.

![Figure 2: FRP-confined concrete](image)

2.2 Steel Model

For steel model, it is assumed that steel does not behave similar in both tension and compression. It is assumed that, in tension steel shows a trilinear behavior, i.e., steel yields and then shows strain hardening behavior. However, in compression it is assumed that steel shows softening behavior after yielding (Figure 3).

In Figure 3, at the tension zone, yield strength is denoted by $f_y$ and yield strain by $\varepsilon_y$. $\varepsilon_{sh} (=10\varepsilon_y)$ represents strain at the point where the hardening starts, $f_u$ is the ultimate strength of the steel (and can be taken as 1.25$f_y$) whereas $\varepsilon_u (=50\varepsilon_y)$ is the ultimate strain at $f_u$. In the compression zone, since there exists no hardening, $f_u$ stands for the stress at 0.2$f_y$ and $\varepsilon_u$ is taken as 50$\varepsilon_y$. 
Figure 3. Model for the reinforcing steel

3 FORCE-DEFORMATION RELATIONSHIP

Moment-curvature relationship based on the FRP-confined concrete models and the reinforcing steel model is used to evaluate the force-deformation characteristic of a column. The assumptions made in the moment curvature analysis are as follows:

1) Plane sections remain plane
2) Concrete does not take any tension force. Tension forces are only carried by steel reinforcement.
3) FRP is only active in compression part.

Deformation consists of two parts: first part is calculated from the fixed-end rotation caused by the bar slip and second part is calculated from the curvature.

\[ \Delta = \Delta_{bar\ slip} + \Delta_{curvature} \] (16)

Deformation due to the fixed end rotation caused by bar slip can be found using the equations proposed by Ozcan et al [5] as follows.

\[ u = 0.4 \sqrt{f_{co}} \] (17)

\[ s = \begin{cases} \frac{\varepsilon_s f_s d_b}{8u}, & \varepsilon_s \leq \varepsilon_y \\ \frac{d_b}{8u} \left[ \varepsilon_s f_s + 2(\varepsilon_s + \varepsilon_y)(f_s - f_y) \right], & \varepsilon_s > \varepsilon_y \end{cases} \] (18)

\[ \Delta_{bar\ slip} = \frac{s}{d - d_L} \] (19)

In these equations, \( \varepsilon_s \) is the steel strain, \( f_s \) is the corresponding steel stress, \( d_b \) is the bar diameter, \( u \) is the maximum bond stress, \( \varepsilon_t \) is the yield strain for steel, \( f_y \) is the yield stress for steel, \( d \) is the effective depth of the section and \( s \) is the bar slip.

Deformation due to curvature is found as follows:

1) The column is divided into a number of segments
2) For each segment, two moment values are calculated. First moment is for the base of the segment and the second one is for the top of the segment (Figure 4)
3) Curvatures for each segment corresponding to the moments are found
4) The area surrounded by curvatures are calculated
5) The area is then multiplied by the distance to find the corresponding deformation
6) Total deformation due to curvature is evaluated by summing each deformations found for each segment:

\[ \Delta_{\text{curvature}} = \sum K_i x_i d_i \] \hspace{1cm} (20)

7) After reaching the maximum moment at the base, because of the FRP rupture it is assumed that a plastic hinge develops.
8) Plastic hinge length is calculated as follows [4].

\[ \frac{l_p}{h} = \left[ 0.3 \left( \frac{N}{N_0} \right) + 3 \left( \frac{A_s}{A_g} \right) - 0.1 \right] \left( \frac{L}{h} \right) + 0.25 \geq 0.25 \] \hspace{1cm} (21)

where \( N \) is the axial load on the column, \( N_0 \) is the axial load carrying capacity of the column, \( A_s \) is the total longitudinal steel area and \( A_g \) is the gross area of the column.
9) For the plastic hinge region, it is assumed that curvature values are the same for the entire region (Figure 4b).

After establishing the procedure for force-deformation relationship, the models are compared with the experimental data taken from Ozcan et al [6].

Ozcan et al. tested 4 reinforced concrete columns strengthened with CFRP material. Columns were square in cross-section with 350x350 mm and they were 2000 mm tall. Sections were rounded to 30 mm in order to increase the FRP effectiveness. Although target strength was 15 MPa, concrete strength was found to be between 11 MPa and 19.4 MPa.

They used 1 or 2 layers of CFRP to understand the effect of CFRP thickness. Longitudinal reinforcement was of eight 18 mm diameter plain bars, which correspond to a reinforcement ratio of 1.66%. 10 mm diameter plain bars were used as lateral reinforcements and they were spaced at every 200 mm. Lateral reinforcements had 90° hooks to simulate the deficient case in Turkey.

From Figures 5 and 6, analytical curves are compared with the experimental data. It can be seen from the figures that neither ACI440.2R-08 nor TEC2007 captures the post peak behavior because of limiting the strain capacity of FRP-confined concrete. The arrows in the figures point the end of each curve. Since TEC2007 assumes that FRP application results in a hardening behavior depending on the degree of confinement, it results in a hardening behavior in the force-deformation relationship also. However, it is well known that hardening may be observed when the concrete member is adequately confined, that is, when the MCR value is above a certain limit as discussed above.
DISCUSSIONS

In this study, the axial stress-strain estimations and models given in ACI440.2R-08 and TEC2007 are compared with the experimental data to investigate the differences and the estimates that codes make. It is found that TEC2007 assumes only hardening behavior for concrete strengthened with FRP. Although ACI440.2R-08 assumes a hardening and softening behavior depending on the confinement-effectiveness, the limit defining the behavior as hardening or softening is not good to predict the behavior because in that equation no corner radius for rectangular sections is taken into account. The assumptions in these codes result in a more conservative prediction at the post-peak region of the force-deformation curves. It is...
essential to understand the degree of confinement, effectiveness of the confinement and the resulting stress-strain relationship of a concrete strengthened with FRP. Assuming a hardening behavior in all the case is not reasonable.

Since the material and application result in a high initial investment, the equations in the codes make it difficult to utilize. Codes may decrease the factor of safety used in FRP-confined concrete members because the reliability of the material is verified and it is still being studied. This will make the material easy to reach and easy to apply in strengthening applications.

5 REFERENCES

[1] ACI440.2R-08, Guide for the design and construction of externally bonded FRP systems for strengthening concrete structures, ACI 2008


[4] S. Bae, O. Bayrak, Plastic hinge length of reinforced concrete columns, 


