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MASS, STIFFNESS AND DAMPING IDENTIFICATION OF A TWO-STORY BUILDING MODEL

D. Guida[1], **F. Nilvetti**[1], **C. M. Pappalardo**[1]

[1] University of Salerno
via Ponte don Melillo, Fisciano (Salerno), 84084, Italy
e-mail: {guida,nilvetti,cpappalardo}@unisa.it

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Abstract. *In this paper we propose the results of an experimental investigation addressed to identify mass, stiffness and damping matrices of a two-story building model. The goal of our research is to set up a quick procedure to design control laws for mitigating structural vibrations and/or detecting damage of structure itself. We placed on each floor a* 4507 *Bruel & Kjaer accelerometer connected to* 6160 *Bruel & Kjaer Pulse spectrum analyzer, then we excited the structure by* 8202 *Bruel & Kjaer impact hammer and through the recorded I/O data we identified the modal model. Once obtained the system modal parameters, we identified mass, stiffness and damping matrices of the structure. We are going to exploit these experimental results for designing a virtual passive controller. This apparatus is composed of an electric actuator placed on the top of the building connected to a NI-CompactRio System.*

1 INTRODUCTION

System identification is the art of determining a mathematical model of a physical system by combining information obtained from experimental data with that derived from an a priori knowledge. There are several types of system identification algorithms in relation to different goals one wants to pursue. In mechanical engineering, applied system identification allows to get modal parameters of a dynamical system using force and vibration measurements. These parameters are typically used to design optimal control laws whereas in the field of structural health monitoring they are used to detect and evaluate system damage. A very powerful algorithm to perform system identification is Eigensystem Realization Algorithm with Data Correlation using Observer/Kalman Filter Identification (ERA/DC OKID) [1, 2, 3, 4]. This numerical procedure is able to construct a state-space representation of a mechanical system starting from input and output measurements even in presence of process and measurement noise. On the other hand, when all degrees of freedom are instrumented with a force and/or an acceleration transducer, an efficient numerical procedure can be implemented to construct a second-order model of the mechanical system starting from state-space representation (MKR) [5, 6, 7]. Experimental investigations show that ERA/DC OKID correctly determines system natural frequencies and damping ratios whereas MKR method properly identifies mass and stiffness matrices but it fails in esteeming damping matrix because actual measurements are never noise-free. Nevertheless, if the real system is lightly damped, authors propose an efficient procedure [8, 9] for identifying in a direct way system damping matrix from state-space realization by assuming proportional damping hypothesis.

2 MATHEMATICAL BACKGROUND

2.1 System Modelling

Consider a multiple degrees of freedom mechanical system. Let $\mathbf{M} \in R^{n_2 \times n_2}$, $\mathbf{K} \in R^{n_2 \times n_2}$ and $\mathbf{R} \in R^{n_2 \times n_2}$ be the mass, stiffness and damping matrices, respectively. The system equations of motion can be expressed in matrix notation as:

$$\mathbf{M}\,\ddot{\mathbf{x}}(t) + \mathbf{R}\,\dot{\mathbf{x}}(t) + \mathbf{K}\,\mathbf{x}(t) = \mathbf{F}(t) \tag{1}$$

where $\mathbf{x}(t) \in R^{n_2}$, $\dot{\mathbf{x}}(t) \in R^{n_2}$, $\ddot{\mathbf{x}}(t) \in R^{n_2}$ are vectors of generalized displacement, velocity and acceleration, respectively, and $\mathbf{F}(t) \in R^{n_2}$ is the vector of forcing functions.

On the other hand, if the response of the dynamic system is measured by the $m \in N$ output quantities in the output vector $\mathbf{y}(t) \in R^m$, then the output equations can be written in a matrix form as follows:

$$\mathbf{y}(t) = \mathbf{C}_d\,\mathbf{x}(t) + \mathbf{C}_v\,\dot{\mathbf{x}}(t) + \mathbf{C}_a\,\ddot{\mathbf{x}}(t) \tag{2}$$

where $\mathbf{C}_d \in R^{m \times n_2}$, $\mathbf{C}_v \in R^{m \times n_2}$ and $\mathbf{C}_a \in R^{m \times n_2}$ are respectively the output influence matrices for displacement, velocity and acceleration. These output influence matrices simply describe the relation between the vectors $\mathbf{x}(t)$, $\dot{\mathbf{x}}(t)$, $\ddot{\mathbf{x}}(t)$ and the measurement vector $\mathbf{y}(t)$, which in general can be a linear combination of system generalized displacement, velocity and acceleration.

Let $\mathbf{z}(t) \in R^n$ be the state vector of the system:

$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix} \tag{3}$$

where $n \in N$ is the dimension of the system state vector. The forcing function $\mathbf{F}(t)$ over the period of interest at a certain specific location can be expressed using a vector $\mathbf{u}(t) \in R^r$ containing $r \in N$ input quantities according to this relation:

$$\mathbf{F}(t) = \mathbf{B}_2 \, \mathbf{u}(t) \tag{4}$$

where $\mathbf{B}_2 \in R^{n_2 \times r}$ is an input influence matrix characterizing the locations and type of inputs. The equations of motions and the output equations can both be respectively rewritten in terms of the state vector as follows:

$$\dot{\mathbf{z}}(t) = \mathbf{A}_c \, \mathbf{z}(t) + \mathbf{B}_c \, \mathbf{u}(t) \tag{5}$$

$$\mathbf{y}(t) = \mathbf{C} \, \mathbf{z}(t) + \mathbf{D} \, \mathbf{u}(t) \tag{6}$$

where $\mathbf{A}_c \in R^{n \times n}$ is the state transition matrix, $\mathbf{B}_c \in R^{n \times r}$ is the state influence matrix, $\mathbf{C} \in R^{m \times n}$ is the measurements influence matrix and $\mathbf{D} \in R^{m \times r}$ is the direct transmission matrix. These matrix can be computed in this way:

$$\mathbf{A}_c = \begin{bmatrix} \mathbf{O} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{R} \end{bmatrix} \tag{7}$$

$$\mathbf{B}_c = \begin{bmatrix} \mathbf{O} \\ \mathbf{M}^{-1}\mathbf{B}_2 \end{bmatrix} \tag{8}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_d - \mathbf{C}_a \mathbf{M}^{-1}\mathbf{K} & \mathbf{C}_v - \mathbf{C}_a \mathbf{M}^{-1}\mathbf{R} \end{bmatrix} \tag{9}$$

$$\mathbf{D} = \mathbf{C}_a \mathbf{M}^{-1} \mathbf{B}_2 \tag{10}$$

Equations (5) and (6) constitute a continuous-time state-space model of a multiple degrees of freedom dynamical system. Using the preceding definitions, the state-space complex eigenvalues problem can be stated as follows:

$$(\mathbf{A}_c - \lambda_c \mathbf{I})\psi = \mathbf{0} \tag{11}$$

where $\lambda_{c,j} \in C \;,\; j = 1, 2, \ldots, n$ and $\psi_j \in C^n \;,\; j = 1, 2, \ldots, n$ will be referred as system modal parameters. State-space model eigenvectors can be usefully grouped according to the following matrix notation:

$$\mathbf{\Psi} = \begin{bmatrix} \psi_1 & \psi_2 & \ldots & \psi_n \end{bmatrix} \tag{12}$$

where $\mathbf{\Psi} \in C^{n \times n}$ is a matrix constituted of system eigenvectors stacked by columns. Multibody model eigenvectors $\mathbf{W} \in C^{n_2 \times n}$ obtained from equation (1) and state-space model eigenvectors $\mathbf{\Psi} \in C^{n \times n}$ obtained from equation (5) are mathematically interconnected by the following formula:

$$\mathbf{\Psi} = \begin{bmatrix} \mathbf{W} \\ \mathbf{W}\mathbf{\Lambda}_c \end{bmatrix} \tag{13}$$

where $\mathbf{\Lambda}_c \in C^{n \times n}$ is a diagonal matrix whose elements are system eigenvalues.

2.2 Eigensystem Realization Algorithm with Data Correlation (ERA/DC) using Observer/Kalman Filter Identification (OKID)

The basic development of the state-space realization is attributed to Ho and Kalman. The Ho-Kalman procedure uses the generalized Hankel matrix to construct a state-space representation of a linear system form noise-free data. This methodology has been modified and substantially extended by Juang [1, 2] to develop the Eigensystem Realization Algorithm with Data Correlation (ERA/DC) to identify modal parameters from noisy measurement data. Recently, a method named Observer/Kalman Filter Identification (OKID) has been developed by Juang [3, 4] to compute the Markov parameters of a linear system from which the state-space model and a corresponding observer are determined simultaneously. This method is entirely formulated in time-domain and it is capable of handling general response data.

Conventional time-domain system identification methods use only the system Markov parameters [4] to determine $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$. On the other hand, OKID uses the combined system and observer gain Markov parameters [4]. These parameters are computed directly from time-domain input and output measurements and are used to identify $\mathbf{A}$, $\mathbf{B}$, $\mathbf{G}$, $\mathbf{C}$ and $\mathbf{D}$ by the time-domain method named ERA/DC [4].

Basically, the ERA/DC OKID procedure consists in three steps: 1) computation of Markov parameters; 2) realization of state-space model; 3) modal parameters identification.

2.3 Modal Parameters Identification

The ERA/DC OKID procedure is a time-domain identification method which compute a minimum realization of system and the observer gain matrix starting from the combined system and observer gain Markov parameters [4]. A realization is a triplet of matrices $\left\{\hat{\mathbf{A}}\,,\hat{\mathbf{B}}\,,\hat{\mathbf{C}}\right\}$ that satisfies the discrete-time state-space equations. Obviously, the same system has an infinite set of realizations which will predict the identical response for any particular input. Minimum realization means a model with the smallest state space dimension among all the realizable systems that have the same input-output relations.

All minimum realizations have the same set of eigenvalues and eigenvectors, which are the modal parameters of the system itself. Assume that the state matrix $\hat{\mathbf{A}}$ has a complete set of linearly independent eigenvectors $\hat{\psi}_j\ ,\ j=1,2,\ldots,n$ with corresponding eigenvalues $\hat{\lambda}_j\ ,\ j=1,2,\ldots,n$:

$$\hat{\mathbf{A}}\,\hat{\mathbf{\Psi}}=\hat{\mathbf{\Psi}}\,\hat{\mathbf{\Lambda}} \tag{14}$$

where $\hat{\mathbf{\Lambda}}\in R^{n\times n}$ is the diagonal matrix of the eigenvalues and $\hat{\mathbf{\Psi}}\in C^{n\times n}$ is a matrix formed by the eigenvectors stacked per columns. The realization $\left\{\hat{\mathbf{A}}\,,\hat{\mathbf{B}}\,,\hat{\mathbf{C}}\right\}$ can be transformed in the realization $\left\{\hat{\mathbf{\Lambda}},\hat{\mathbf{\Psi}}^{-1}\,\hat{\mathbf{B}}\,,\hat{\mathbf{C}}\,\hat{\mathbf{\Psi}}\right\}$ by using spectral decomposition. The diagonal matrix $\hat{\mathbf{\Lambda}}$ contains the informations of modal damping rates and damped natural frequencies. The matrix $\hat{\mathbf{\Psi}}^{-1}\,\hat{\mathbf{B}}$ defines the initial modal amplitudes and the matrix $\hat{\mathbf{C}}\,\hat{\mathbf{\Psi}}$ the mode shapes at the sensor points. All the modal parameters of a dynamic system can thus be identified by the unique triplet $\left\{\hat{\mathbf{\Lambda}},\hat{\mathbf{\Psi}}^{-1}\,\hat{\mathbf{B}}\,,\hat{\mathbf{C}}\,\hat{\mathbf{\Psi}}\right\}$. This discrete-time realization can be transformed to its continuous-time counterpart $\left\{\hat{\mathbf{\Lambda}}_c,\hat{\mathbf{\Psi}}^{-1}\,\hat{\mathbf{B}}_c\,,\hat{\mathbf{C}}\,\hat{\mathbf{\Psi}}\right\}$ by using the zero-order-hold assumption. Finally, assuming that all the identified system modes are underdamped, modal damping rates and damped natural frequencies can be computed from the diagonal matrix $\hat{\mathbf{\Lambda}}_c\in C^{n\times n}$ as follows:

$$\left\{\begin{array}{ll}\hat{\omega}_{n,i}=\sqrt{\hat{\varepsilon}_i^2+\hat{\omega}_{d,i}^2} & ,\quad i=1,2,\ldots,n_2\\ \hat{\xi}_i=\frac{-\hat{\varepsilon}_i}{\sqrt{\hat{\varepsilon}_i^2+\hat{\omega}_{d,i}^2}} & ,\quad i=1,2,\ldots,n_2\end{array}\right. \tag{15}$$

where $\hat{\varepsilon}_i\ ,\ i = 1, 2, \dots, n_2$ and $\hat{\omega}_{d,i}\ ,\ i = 1, 2, \dots, n_2$ are respectively the real and imaginary part of the system eigenvalues $\hat{\lambda}_{c,j}\ ,\ j = 1, 2, \dots, n$.

In many practical applications the hypothesis of proportional damping can be assumed as satisfied, especially in the case of structural systems in which damping is small and no a priori informations about its nature are available. Proportional damping assumption is the following:

$$\mathbf{R} = \alpha\,\mathbf{M} + \beta\,\mathbf{K} \tag{16}$$

where $\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrices, respectively, whereas α and β are proportional coefficients. If the system is lightly damped, authors propose a simple and efficient method to identify damping matrix starting from identified state space representation [8, 9]. The proportional damping assumption implies that the identified modal damping are related to the identified natural frequencies according to the following equations:

$$\hat{\xi}_i = \frac{\hat{\alpha}}{2\hat{\omega}_{n,i}} + \frac{\hat{\beta}\,\hat{\omega}_{n,i}}{2} \quad , \quad i = 1, 2, \dots, n_2 \tag{17}$$

where $\omega_{n,i}\ ,\ \ i = 1, 2, \dots, n_2$ are the identified natural frequencies. These equations can be grouped in a matrix from to yield:

$$\begin{bmatrix} \frac{1}{2\hat{\omega}_{n,1}} & \frac{\hat{\omega}_{n,1}}{2} \\ \frac{1}{2\hat{\omega}_{n,2}} & \frac{\hat{\omega}_{n,2}}{2} \\ \vdots & \vdots \\ \frac{1}{2\hat{\omega}_{n,n_2}} & \frac{\hat{\omega}_{n,n_2}}{2} \end{bmatrix} \begin{bmatrix} \hat{\alpha} \\ \hat{\beta} \end{bmatrix} = \begin{bmatrix} \hat{\xi}_1 \\ \hat{\xi}_2 \\ \vdots \\ \hat{\xi}_{n_2} \end{bmatrix} \tag{18}$$

At this point the proportional coefficients α and β that optimal fits the identified natural frequencies $\omega_{n,i}\ ,\ \ i = 1, 2, \dots, n_2$ in the least-square sense can be computed taking the pseudo-inverse matrix:

$$\begin{bmatrix} \hat{\alpha} \\ \hat{\beta} \end{bmatrix} = \begin{bmatrix} \frac{1}{2\hat{\omega}_{n,1}} & \frac{\hat{\omega}_{n,1}}{2} \\ \frac{1}{2\hat{\omega}_{n,2}} & \frac{\hat{\omega}_{n,2}}{2} \\ \vdots & \vdots \\ \frac{1}{2\hat{\omega}_{n,n_2}} & \frac{\hat{\omega}_{n,n_2}}{2} \end{bmatrix}^{\dagger} \begin{bmatrix} \hat{\xi}_1 \\ \hat{\xi}_2 \\ \vdots \\ \hat{\xi}_{n_2} \end{bmatrix} \tag{19}$$

This approximation represents a simple and useful mathematical tool to deals with real experimental data.

3 CONSTRUCTION OF SECOND ORDER MODEL FROM IDENTIFIED STATE-SPACE REPRESENTATION

Consider the following matrices:

$$\mathbf{V}_c = \begin{bmatrix} \mathbf{R} & \mathbf{M} \\ \mathbf{M} & \mathbf{O} \end{bmatrix} \tag{20}$$

$$\mathbf{S}_c = \begin{bmatrix} -\mathbf{K} & \mathbf{O} \\ \mathbf{O} & \mathbf{M} \end{bmatrix} \tag{21}$$

$$\mathbf{B}_3 = \begin{bmatrix} \mathbf{B}_2 \\ \mathbf{O} \end{bmatrix} \tag{22}$$

Using these definitions, a symmetric formulation of system continuous-time state-space model can be developed:

$$\mathbf{V}_c\,\dot{\mathbf{z}}(t) = \mathbf{S}_c\,\mathbf{z}(t) + \mathbf{B}_3\,\mathbf{u}(t) \tag{23}$$

$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{z}(t) + \mathbf{D}\,\mathbf{u}(t) \tag{24}$$

where the matrices $\mathbf{V}_c \in R^{n\times n}$, $\mathbf{S}_c \in R^{n\times n}$ and $\mathbf{B}_3 \in R^{n\times r}$ are all symmetric matrices. The symmetric formulation of system state-space model can be easily reconnected to the typical one (5) noting that the output equations are unchanged and that system transition matrix and state influence matrix can be computed in this way:

$$\mathbf{A}_c = \mathbf{V}_c^{-1}\,\mathbf{S}_c \tag{25}$$

$$\mathbf{B}_c = \mathbf{V}_c^{-1}\,\mathbf{B}_3 \tag{26}$$

The advantages of reformulating system state-space model in this way is that now the associated eigenvalues problem is kept symmetric. Indeed:

$$\mathbf{S}_c\,\mathbf{\Psi} = \mathbf{V}_c\,\mathbf{\Psi}\,\mathbf{\Lambda}_c \tag{27}$$

In general, these eigenvectors can be arbitrarily scaled but if the scaling is chosen such that:

$$\mathbf{\Psi}^T\,\mathbf{V}_c\,\mathbf{\Psi} = \mathbf{I} \tag{28}$$

$$\mathbf{\Psi}^T\,\mathbf{S}_c\,\mathbf{\Psi} = \mathbf{\Lambda}_c \tag{29}$$

then, for a proportionally damped system, the real and imaginary parts of the components of these complex eigenvectors are equal in magnitude. Once that the symmetric eigenvalues problem has been solved, it can be proved [5, 6, 7] that a transformation matrix can be computed in order to extract multibody model eigenvectors matrix $\hat{\mathbf{W}}$ from state-space realization $\left\{\hat{\mathbf{A}}\,,\hat{\mathbf{B}}\,,\hat{\mathbf{C}}\right\}$. Finally, using this method it is possible to construct a second-order model of the mechanical system by using the following formulae:

$$\begin{cases} \hat{\mathbf{M}} = \left(\hat{\mathbf{W}}\,\hat{\mathbf{\Lambda}}_c\,\hat{\mathbf{W}}^T\right)^{-1} \\ \hat{\mathbf{K}} = -\left(\hat{\mathbf{W}}\,\hat{\mathbf{\Lambda}}_c^{-1}\,\hat{\mathbf{W}}^T\right)^{-1} \\ \hat{\mathbf{R}} = -\hat{\mathbf{M}}\,\hat{\mathbf{W}}\,\hat{\mathbf{\Lambda}}_c^2\,\hat{\mathbf{W}}^T\,\hat{\mathbf{M}} \end{cases} \tag{30}$$

This numerical procedure is referred as MKR algorithm [5, 6, 7].

4 SYSTEM IDENTIFICATION OF A TWO-STORY BUILDING MODEL

We have set up a two-story building model composed of four steel pillars and two aluminum beams as showed in figure (**??**). The first floor pillars have a section $1mm$ x $35mm$ and are $300mm$ long while the second floor pillars have a section $1mm$ x $35mm$ and are $350mm$ long. The two beams are $200mm$ long with a square section $45mm$ x $45mm$. On the first and on the second floor there are two piezoelectric accelerometers as showed in figure (1). The 4507 Bruel & Kjaer accelerometers are connected to 6160 Bruel & Kjaer Pulse spectrum analyzer. The excitation signal is produced by the 8202 Bruel & Kjaer impact hammer, which is connected to the spectrum analyzer too.

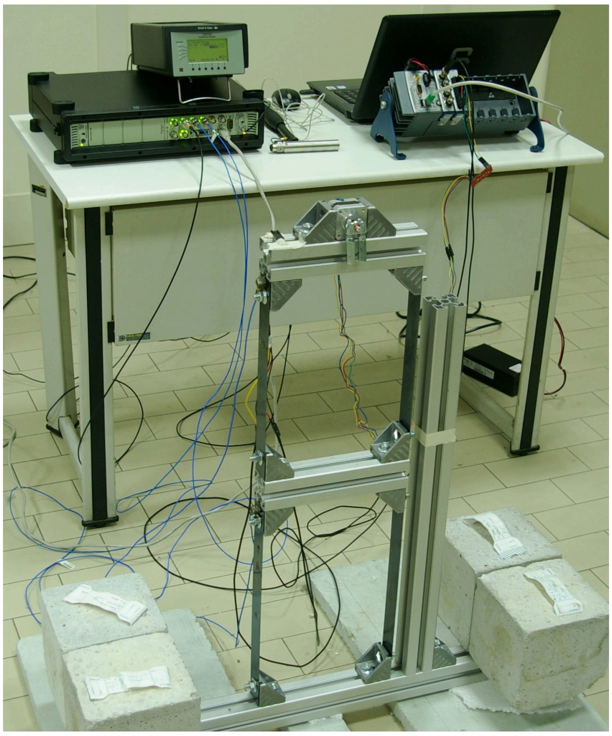

Figure 1: Experimental Apparatus

4.1 CASE-STUDY 1

We have studied two experimental configuration. In the first case, we analyzed the two-story building frame. In the second case, we placed an additional mass on the first floor. In figure (2) is showed the force measurement applied on the first floor and in figure (3) there is the system response corresponding to the input. In order to get statistically meaningful results, we repeated the experimental acquisition ten times but in the figures (2), (3) is showed only the first test.

Once the acquisition has been performed, we used ERA/DC OKID to get a state-space representation of the system. The following matrices represent the realization corresponding to the input and output measurements, figures (2), (3):

$$\hat{\mathbf{A}} = \begin{bmatrix} 0.6220 & -0.7789 & -0.0003 & -0.0021 \\ 0.7839 & 0.6191 & 0.0031 & -0.0012 \\ -0.0036 & 0.0061 & 0.9623 & 0.2616 \\ 0.0041 & 0.0044 & -0.2676 & 0.9634 \end{bmatrix} \tag{31}$$

$$\hat{\mathbf{B}} = \begin{bmatrix} 0.0541 \\ -0.0268 \\ 0.0412 \\ 0.0363 \end{bmatrix} \tag{32}$$

$$\hat{\mathbf{C}} = \begin{bmatrix} -37.8710 & -5.0124 & -5.8513 & 1.8479 \\ 26.5700 & 3.2700 & -7.2966 & 2.2001 \end{bmatrix} \tag{33}$$

$$\hat{\mathbf{D}} = \begin{bmatrix} 3.6387 \\ 0.0739 \end{bmatrix} \tag{34}$$

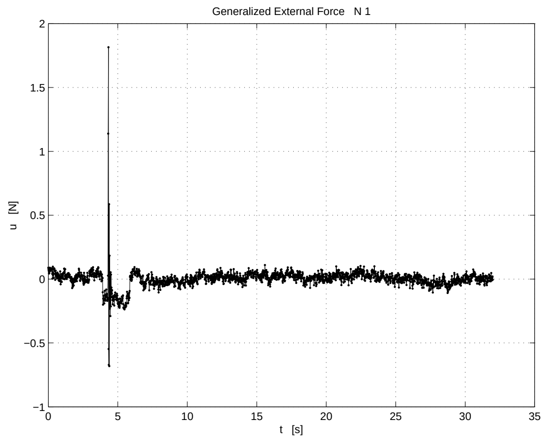

Figure 2: CASE 1 - Force Measurement

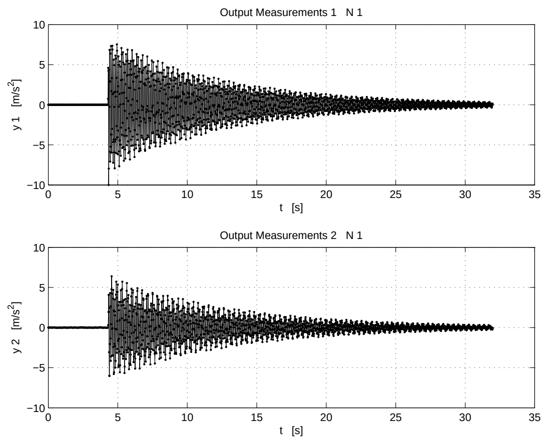

Figure 3: CASE 1 - Acceleration Measurements

Examining the singular value $\bar{\Sigma}_n$ of the Hankel matrix $\bar{H}(0)$ showed in figure (4) it is possible to determine the order of the system. Indeed, there are only 4 singular values whose magnitude is not negligible: it means that the system state has dimension $\hat{n} = 4$. Obviously, the same system has an infinite set of realizations which will predict the identical response for any particular input. Minimum realization means a model of the smallest state space dimensions among all realizable systems that have the same input-output relation. All minimum realizations have the same set of eigenvalues and eigenvectors, which are the modal parameters of the system itself:

$$\hat{\mathbf{\Lambda}} = diag(0.6205 + 0.7814\mathbf{i},\, 0.6205 - 0.7814\mathbf{i},\, 0.9629 + 0.2646\mathbf{i},\, 0.9629 - 0.2646\mathbf{i}) \quad (35)$$

$$\hat{\mathbf{\Psi}} = \begin{bmatrix} 0.0013 + 0.7059\mathbf{i} & 0.0013 - 0.7059\mathbf{i} & -0.0007 - 0.0019\mathbf{i} & -0.0007 + 0.0019\mathbf{i} \\ 0.7082 & 0.7082 & 0.0016 + 0.0008\mathbf{i} & 0.0016 - 0.0008\mathbf{i} \\ -0.0069 - 0.0030\mathbf{i} & -0.0069 + 0.0030\mathbf{i} & -0.0015 + 0.7031\mathbf{i} & -0.0015 - 0.7031\mathbf{i} \\ 0.0016 - 0.0071\mathbf{i} & 0.0016 + 0.0071\mathbf{i} & -0.7111 & -0.7111 \end{bmatrix} \quad (36)$$

System eigenvectors are graphically showed in figure (5). The discrete-time realization can

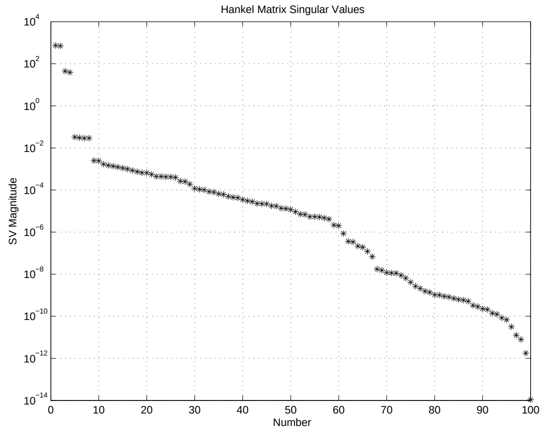

Figure 4: CASE 1 - Hankel Matrix Singular Values

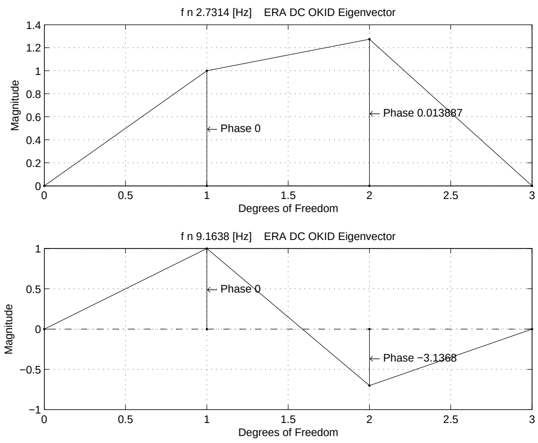

Figure 5: CASE 1 - System Eigenvalues and Eigenvectors

be transformed to its continuous-time counterpart by using the zero order hold assumption and subsequently modal damping rates and natural frequencies can be computed from the diagonal matrix $\hat{\mathbf{\Lambda}}_c \in C^{n \times n}$ to yield:

$$\hat{f}_{n,1} = 2.7314 \quad [Hz] \tag{37}$$

$$\hat{f}_{n,2} = 9.1638 \quad [Hz] \tag{38}$$

$$\hat{\xi}_1 = 0.0054 \quad [\backslash] \tag{39}$$

$$\hat{\xi}_2 = 0.0025 \quad [\backslash] \tag{40}$$

The optimally damping coefficients $\hat{\alpha}$ and $\hat{\beta}$ that fit identified natural frequencies in the least-square sense can be computed according to equations (19) to yield:

$$\hat{\alpha} = 0.1755 \tag{41}$$

$$\hat{\beta} = 3.2283 \cdot 10^{-5} \tag{42}$$

Finally, using the MKR algorithm a mechanical model of system mass, stiffness and damping matrices can be computed:

$$\hat{\mathbf{\Phi}} = \begin{bmatrix} -0.1317 + 0.0758\mathbf{i} & 0.1582 - 0.1057\mathbf{i} \\ 0.0927 - 0.0527\mathbf{i} & 0.2034 - 0.1319\mathbf{i} \end{bmatrix} \quad (43)$$

$$\hat{\mathbf{M}} = \begin{bmatrix} 0.2913 & 0.0153 \\ 0.0153 & 0.3387 \end{bmatrix} \quad [kg] \quad (44)$$

$$\hat{\mathbf{K}} = \begin{bmatrix} 630.44 & -425.83 \\ -425.83 & 439.01 \end{bmatrix} \quad [kg/s^2] \quad (45)$$

$$\hat{\mathbf{R}} = \begin{bmatrix} 6.9208 & -3.6833 \\ -3.6833 & 5.6759 \end{bmatrix} \quad [kg/s] \quad (46)$$

where $\hat{\mathbf{\Phi}}$ is system eigenvectors matrix scaled according to equations (28), (29). While in the case of mass $\hat{M}$ and stiffness $\hat{K}$ matrices the experimental results of the MKR algorithm are acceptable, the identified damping matrix $\hat{R}$ appears to be incongruous. Authors propose a different estimation of damping matrix (**??**optimal damping)) based on the identified proportional coefficients $\hat{\alpha}$ and $\hat{\beta}$. The resulting $\hat{R}$ matrix is the following:

$$\hat{\mathbf{R}} = \begin{bmatrix} 0.0715 & -0.0111 \\ -0.0111 & 0.0736 \end{bmatrix} \quad [kg/s] \quad (47)$$

this damping matrix is a better estimation of actual system damping.

4.2 CASE-STUDY 2

In the second case, we placed an additional mass on the first floor. In figure (6) is showed the force measurement applied on the first floor and in figure (7) there is the system response corresponding to the input. Even in this case, in order to get statistically meaningful results, we repeated the experimental acquisition ten times but in the figures (6), (7) is showed only the first test.

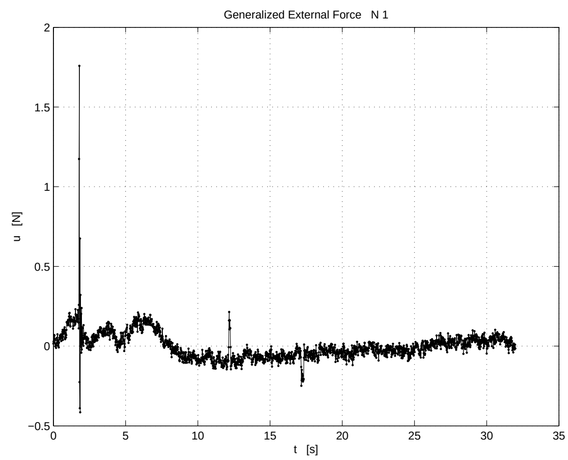

Figure 6: CASE 2 - Force Measurement

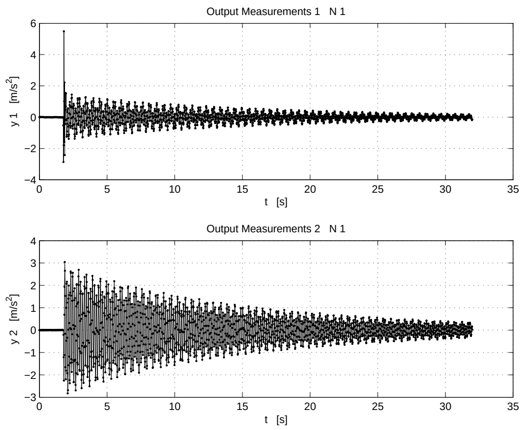

Figure 7: CASE 2 - Acceleration Measurements

Once the acquisition has been performed, we used ERA/DC OKID to get a state-space representation of the system. The following is the realizations corresponding to the input and output measurements, figures (6), (7):

$$\hat{\mathbf{A}} = \begin{bmatrix} 0.7776 & -0.6220 & 0.0041 & -0.0044 \\ 0.6205 & 0.7848 & 0.0014 & -0.0011 \\ -0.0116 & 0.0064 & 0.9799 & 0.1867 \\ 0.0072 & -0.0023 & -0.1848 & 0.9824 \end{bmatrix} \tag{48}$$

$$\hat{\mathbf{B}} = \begin{bmatrix} -0.0524 \\ -0.0317 \\ -0.0561 \\ 0.0124 \end{bmatrix} \tag{49}$$

$$\hat{\mathbf{C}} = \begin{bmatrix} -5.4178 & -3.7972 & -1.9582 & 1.7973 \\ 12.0353 & 8.4148 & -1.8316 & 1.9635 \end{bmatrix} \tag{50}$$

$$\hat{\mathbf{D}} = \begin{bmatrix} -2.7906 \\ -0.0621 \end{bmatrix} \tag{51}$$

Indeed, examining the singular value $\bar{\Sigma}_n$ of the Hankel matrix $\bar{H}(0)$ showed in figure (8) it is possible to determine the order of the system. In fact, there are only 4 singular values whose magnitude is not negligible: it means that the system state has dimension $\hat{n} = 4$. Now system modal parameters can be computed to yield:

$$\hat{\mathbf{\Lambda}} = diag(0.7812 + 0.6213\mathbf{i}, 0.7812 - 0.6213\mathbf{i}, 0.9812 + 0.1857\mathbf{i}, 0.9812 - 0.1857\mathbf{i}) \tag{52}$$

$$\hat{\mathbf{\Psi}} = \begin{bmatrix} 0.7075 & 0.7075 & 0.0015 + 0.0008\mathbf{i} & 0.0015 - 0.0008\mathbf{i} \\ -0.0041 - 0.7065\mathbf{i} & -0.0041 + 0.7065\mathbf{i} & 0.0044 - 0.0057\mathbf{i} & 0.0044 + 0.0057\mathbf{i} \\ -0.0046 + 0.0161\mathbf{i} & -0.0046 - 0.0161\mathbf{i} & 0.7089 & 0.7089 \\ -0.0047 - 0.0081\mathbf{i} & -0.0047 + 0.0081\mathbf{i} & 0.0049 + 0.7053\mathbf{i} & 0.0049 - 0.7053\mathbf{i} \end{bmatrix} \tag{53}$$

System eigenvectors are graphically showed in figure (9). The discrete-time realization can be transformed to its continuous-time counterpart by using the zero order hold assumption and

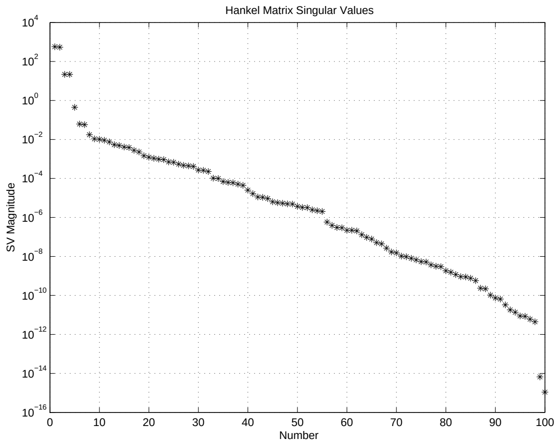

Figure 8: CASE 2 - Hankel Matrix Singular Values

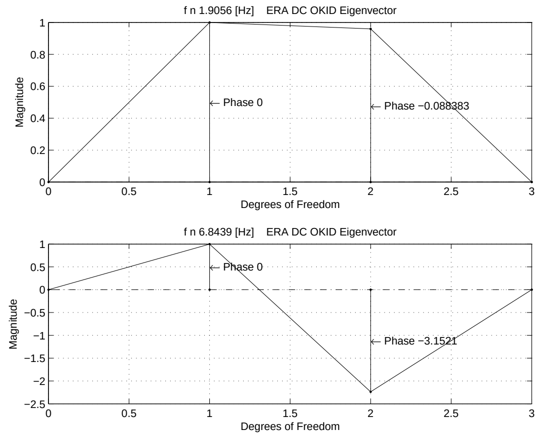

Figure 9: CASE 2 - System Eigenvalues and Eigenvectors

subsequently modal damping rates and natural frequencies can be computed from the diagonal matrix $\hat{\mathbf{\Lambda}}_c \in C^{n\times n}$ to yield:

$$\hat{f}_{n,1} = 1.9056 \quad [Hz] \tag{54}$$

$$\hat{f}_{n,2} = 6.8439 \quad [Hz] \tag{55}$$

$$\hat{\xi}_1 = 0.0075 \quad [\backslash] \tag{56}$$

$$\hat{\xi}_2 = 0.0028 \quad [\backslash] \tag{57}$$

Now it is straightforward to note that the effect of the additional mass is the reduction of system natural frequencies whereas the damping ratios are roughly unaffected. At this point the optimal damping coefficients $\hat{\alpha}$ and $\hat{\beta}$ that fits in the least-square sense the identified natural frequencies can be computed according to equations (19) to yield:

$$\hat{\alpha} = 0.1752 \tag{58}$$

$$\hat{\beta} = 3.4355 \cdot 10^{-5} \tag{59}$$

This parameters have almost the same magnitude compared to the preceding case. Finally, using the MKR algorithm a mechanical model of system mass, stiffness and damping matrices can be computed to yield:

$$\hat{\mathbf{\Phi}} = \begin{bmatrix} 0.0168 + 0.0824\mathbf{i} & -0.0579 - 0.1785\mathbf{i} \\ -0.0395 - 0.1840\mathbf{i} & -0.0705 - 0.1657\mathbf{i} \end{bmatrix} \tag{60}$$

$$\hat{\mathbf{M}} = \begin{bmatrix} 1.3771 & 0.0119 \\ 0.0119 & 0.5566 \end{bmatrix} \quad [kg] \tag{61}$$

$$\hat{\mathbf{K}} = \begin{bmatrix} 630.44 & -425.83 \\ -425.83 & 439.01 \end{bmatrix} \quad [kg/s^2] \tag{62}$$

$$\hat{\mathbf{R}} = \begin{bmatrix} 59.9062 & -34.1418 \\ -34.1418 & 38.4897 \end{bmatrix} \quad [kg/s] \tag{63}$$

where $\hat{\mathbf{\Phi}}$ is system eigenvectors matrix scaled according to equations (28), (29). Even in this case, while the identified mass $\hat{\mathbf{M}}$ and stiffness $\hat{\mathbf{K}}$ matrices are satisfactory acceptable, the identified damping matrix $\hat{\mathbf{R}}$ appears to be in some way incongruous. On the other hand, by using the proposed formulae (19), the result is the following:

$$\hat{\mathbf{R}} = \begin{bmatrix} 0.2739 & -0.0223 \\ -0.0223 & 0.1231 \end{bmatrix} \quad [kg/s] \tag{64}$$

this damping matrix is a better estimation of actual system damping. Note that there is a marked difference between results of case-study 1 and case-study 2. Indeed, the introduction of the additional mass on the first floor increases the magnitude of the first element of identified mass matrix $\hat{\mathbf{M}}$.

5 CONCLUSIONS

In this paper we performed an experimental investigation on a two-story frame in order to identify a second-order mechanical model, that is to derive system mass, stiffness and damping matrices. First, we identified system modal parameters through Eigensystem Realization Algorithm with Data Correlation using Observer/Kalman Filter Identification (ERA/DC OKID) [4]. Then we obtained mass, stiffness and damping matrices using a numerical method (MKR) proposed by [5, 6, 7]. Authors also proposed a new method to identify damping matrix from modal parameters [8, 9]. The structure was excited by an impulse yielded by 8202 Bruel & Kjaer impact hammer and the response was recorded by 4507 Bruel & Kjaer accelerometers connected to 6160 Bruel & Kjaer Pulse spectrum analyzer. The identification procedure was carried out several times, changing system mass and stiffness, and the results obtained are in good agreement with our FEM simulations. This work is the first step of our research project aimed at setting up a new virtual passive controller in order to regulate structural vibrations.

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