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VIBRATION CONTROL OF STRUCTURES WITH A NOVEL SEMI-ACTIVE TUNED MASS DAMPER USING WAVELET TRANSFORM

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Abstract. *An innovative design for a semi-active tuned mass damper (SATMD) with variable stiffness has been proposed for structural vibration control. The stiffness of the device can be continuously varied within a certain range using the variations of moment of inertia of an area. This adaptive tuned mass damper (TMD) is capable of real-time retuning and operates effectively in broad band frequency excitations. Employing the numerical simulations, the responses of a SDOF model coupled with the SATMD subjected to a seismic record are obtained. Wavelet transform is used to detect the instantaneous frequency of the excitation. The control effectiveness of the proposed SATMD is evaluated by comparing the results with those of the system coupled with a TMD. It is found that, the vibration suppression by the proposed SATMD is significantly superior to that of a conventional passive TMD.*

1 INTRODUCTION

Semi-active tuned mass damper (SATMD) is a semi-active vibration control device which basically originates from a passive tuned mass damper (TMD). TMD is a simple, inexpensive and reliable device to attenuate the undesired vibrations of systems. The principal drawback of TMDs, however, is that they are only effective over a narrow frequency band and their performance can be highly degraded due to changes in excitation frequency.

In an effort to improve the effectiveness of TMDs, active tuned mass dampers (ATMDs) have been introduced [1]. Although the ATMDs demonstrate better performance than the TMDs, the active system is more costly, more complex and needs careful maintenance. Besides, injection of energy into the controlled system by active control mechanisms enhances the potential for system destabilization.

Considering the advantages and limitations of the active system, the concept of a semi-active vibration control has been introduced. SATMD is the semi-active counterpart of TMD which has been subsequently modified to allow for the adjustment of damping [2, 3] and/or stiffness properties [4-8]. Compared to ATMDs, a smaller amount of active energy is required to modulate the damping or stiffness of such devices.

Most of the variable stiffness devices which have been proposed, involve several moving parts and mechanical and electrical components, which may put limits on the usage of these devices. Moreover, they may do not allow for the quick adjustment of stiffness, and their complicated design may cause manufacturing and maintenance difficulties.

In present study, in an effort to overcome the disadvantages of the above-mentioned variable stiffness devices, an innovative semi-active variable stiffness (SAVS) has been proposed This beam-like device is capable of altering its stiffness in a smooth manner between minimum and maximum levels using the variations of the moment of inertia of an area as it rotates about a normal axis passing through its centroid. Theoretical expressions for the change of device stiffness as a function of rotation angle are developed underlying the concept of unsymmetrical bending of beams. A semi-active tuned mass damper (SATMD) is developed based on the proposed SAVS device. This adaptive vibration absorber is capable of real-time retuning and operates effectively in broad band frequency excitations. The excitation is assumed to be an earthquake record. Wavelet transform is employed to detect the instantaneous frequency of excitation and online tuning of the SATMD. The control effectiveness of the proposed SATMD is evaluated by comparing the results with those of the system coupled with a TMD.

2 THE INNOVATIVE SAVS DEVICE

The innovative variable stiffness device proposed in this paper is based on the variation of moment of inertia as an area rotates about a normal axis passing through its centroid. The formulations are developed using the well-known theory of skew bending or unsymmetrical bending of beams. The details are discussed in the following sub-sections.

2.1 Theory: Unsymmetrical bending of beams

Consider a cantilever prismatic elastic beam loaded at the end of the span with a concentrated force P as shown in Figure 1(a). The cross-sectional area of the beam is assumed to be rectangular with height h and width b, as shown in Figure 1(b). In this figure z and y are the principal axes passing through the centroid of the cross-sectional area. The downward deflection at the end of the beam is [9]

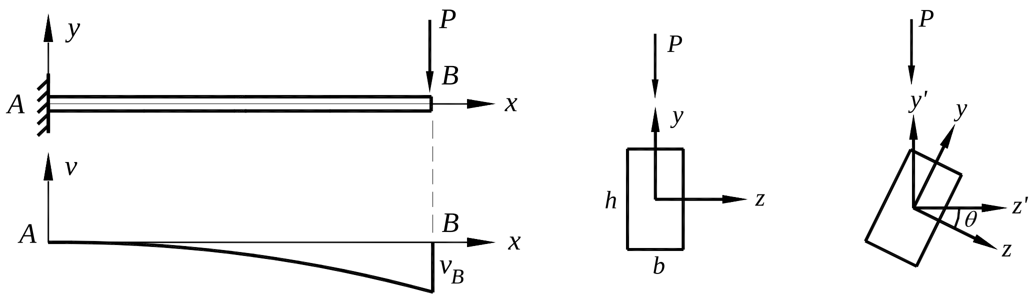

Figure 1: (a) Fixed-end beam loaded with a concentrated force at the end of the span and its deflected curve; (b) Cross-sectional area; (c) Rotated cross-section.

$$v_B = \frac{PL^3}{3EI_z} \tag{1}$$

where L, E and I_z are the length of the beam, modulus of elasticity of the beam material and moment of inertia of the cross-sectional area about principal axis z. It can be said that the effective stiffness at the end of the beam in y-direction is

$$k_{yB} = \frac{3EI_z}{L^3} \tag{2}$$

Now consider the case in which the beam is rotated clockwise about x-axis through an angle θ as shown in Figure 1(c). The new horizontal and vertical axes are denoted as z' and y'. The deflection at the end of the beam in y' and z'-directions can be expressed, respectively, as [10]

$$\bar{v}_B = \frac{PL^3 I_{y'}}{3E(I_{z'}I_{y'} - I_{z'y'}^2)} \tag{3a}$$

$$\bar{w}_B = \frac{PL^3 I_{z'y'}}{3E(I_{z'}I_{y'} - I_{z'y'}^2)} \tag{3b}$$

where I_y is the moment of inertia of the cross section about principal axis y and $I_{z'y'}$ is the product of inertia of the cross section in the new $z'y'$-coordinates. $\bar{v}_B$ is downward; and $\bar{w}_B$ is rightward for $0 < \theta < \pi/2$ and leftward for $\pi/2 < \theta < \pi$. It can be concluded that the equivalent stiffness at the end of the beam in y'-direction is

$$k_{y'B} = \frac{3E(I_{z'}I_{y'} - I_{z'y'}^2)}{L^3 I_{y'}} \tag{4}$$

Using the well-known equations which relate $I_{z'}$, $I_{y'}$ and $I_{z'y'}$ to I_z and I_y [11], gives

$$k_{y'B} = \frac{6EI_z I_y}{L^3[(I_z + I_y) - (I_z - I_y)\cos 2\theta]} \tag{5}$$

which shows the variation of the stiffness at the end of the rotated beam in vertical directions as a function of θ.

2.2 The proposed SAVS device

The proposed device is composed of a mass which is connected to the main structure by the use of two co-axial cantilever beams, located in two opposite sides of the mass as shown in Figure 2. The beams are hinged together at their free ends (point *B*) and their other ends are clamped to the main structure (points *A* and *C*). The rotational degrees of freedom at the clamped ends are released, so it is possible for the beams to be rotated about their neutral axes. The mass is perpendicularly connected to the hinge point through a rigid truss element. The beams are rotated through the same angle but in the opposite directions, Figure 2(b), as the result, point *B* and hence the mass will move just in horizontal direction. (the external excitation is assumed to be applied to the main structure horizontally). By rotating the beams, the effective stiffness of point *B* in horizontal direction will smoothly vary from its minimum value, Figure 2(a) to its maximum value, Figure 2(c). Electric motors mounted at two ends of the beams may rotate the beams about their neutral axes to the desired angle according to a signal which is supplied through control algorithm.

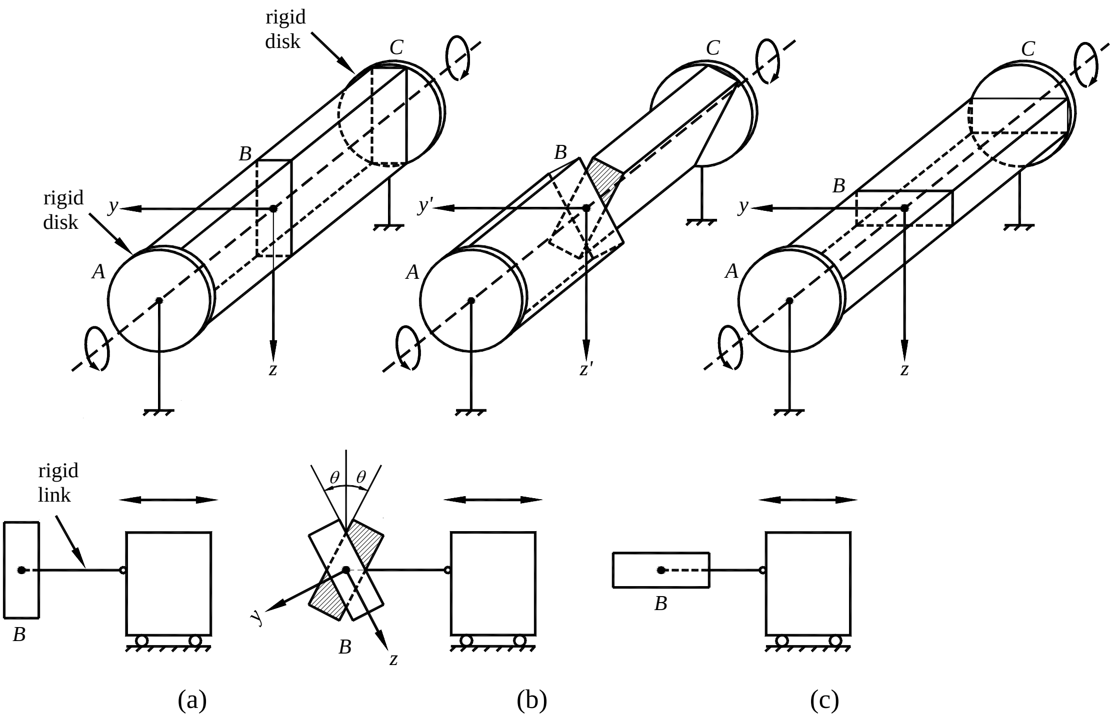

Figure 2: Variable stiffness device (a) θ = 0; (b) $0 < \theta < 90^{\circ}$; (c) $\theta = 90^{\circ}$.

Since the beams behave like parallel springs, the effective stiffness at point *B* equals the sum of each beam's stiffness at their end points. Employing the same procedure as [7], it can be shown that

$$\bar{k}_{y'B} = \frac{6EI_{z'}}{L^3} \tag{6}$$

or

$$\bar{k}_{y'B} = \frac{3E}{L^3}[(I_z + I_y) + (I_z - I_y)\cos 2\theta] \tag{7}$$

It should be noted that due to opposite rotation of the beams through the same angle, their vertical deflections will cancel out.

3 VIBRATION CONTROL OF A SDOF SYSTEM BY THE PROPOSED SATMD

As a case study, the efficacy of the proposed SATMD for suppression of vibrations of a SDOF system under earthquake excitation is investigated.

3.1 Analysis of the model

Consider a SDOF system equipped with a SATMD. This integrated system can be modeled as a 2DOF system, as schematically shown in Figure 3. The governing equations of motion of the system are

$$\begin{aligned} m_1\ddot{x}_1(t) + (c_1 + c_2)\dot{x}_1(t) - c_2\dot{x}_2(t) + (k_1 + k_2(t))x_1(t) - k_2(t)x_2(t) &= -m_1\ddot{x}_g \\ m_2\ddot{x}_2(t) - c_2\dot{x}_1(t) + c_2\dot{x}_2(t) - k_2(t)x_1(t) + k_2(t)x_2(t) &= -m_2\ddot{x}_g \end{aligned} \tag{8}$$

where m_1, c_1 and k_1 are the mass, damping capacity and stiffness of the primary system, respectively; m_2, c_2 and $k_2(t)$ are the mass, damping capacity and adjustable stiffness of the mass damper, respectively; $x_1(t)$ and $x_2(t)$ represent, respectively, primary and mass damper displacement relative to the ground; and $\ddot{x}_g$ is the ground acceleration.

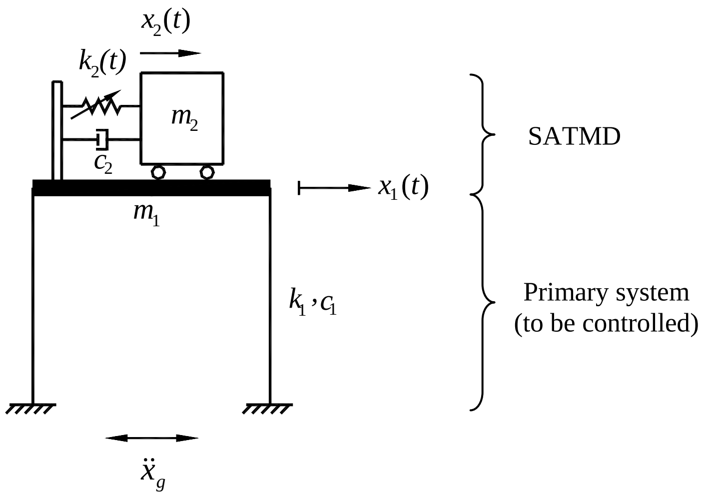

Figure 3: SDOF system equipped with a SATMD.

3.2 Tuning algorithm

In order to attenuate undesirable vibrations during earthquake, the stiffness of the mass damper is varied to tune the damper to the excitation frequency. However, in practice the semi-active stiffness coefficient is limited by both an upper bound, k_{max}, and lower bound, k_{min}. The instantaneous frequency of the excitation is detected using complex morlet wavelet transform and the stiffness of SATMD is altered to keep it tuned to the excitation in real time.

When the calculated tuning stiffness for the mass damper, $m_2\omega^2(t)$, lies within the range specified by the lower and upper limit parameters, the damper stiffness is set to this value. When the calculated stiffness is outside these bounds, the damper stiffness is clipped to the upper or lower bound. The stiffness is then related to an angle θ according to Equation (7).

3.3 Developed model and parameter values used

In the system considered, the damper’s stiffness varies nonlinearly with time. Therefore, the system response is to be obtained through numerical integration of equations of motion, in which the damper's stiffness is to be calculated at each step. For this purpose a SIMULINK model is developed. The assumed parameters of the primary system and SATMD are summarized in Table 1. In present study, the variable spring is assumed to be massless and there is thus no kinetic energy associated with its motion.

The results are obtained and compared to those of the system equipped with a conventional TMD. For this case, the mass and damping coefficient of the TMD are the same as SATMD and the stiffness has a constant value such that the natural frequency of the TMD equals the steady-state excitation frequency.

Table 1: Primary system and SATMD parameters**.**

Primary system		SATMD	
Parameter	Value	Parameter	Value
m_1	10000 kg	b	4 mm
f_1	1 Hz	h	22 mm
ξ_1	0.02	L	48 cm
		E	2×10^{11} Pa
		ξ_2	0.01
		m_2	100 kg

3.4 Numerical results

In this study, 1968 Hachinohe earthquake signal is used as the excitation (Fig 4). Fig. 5 shows the instantaneous frequency of the earthquake which is detected by wavelet transform. The system's response is obtained numerically using the SIMULINK model. To evaluate the control system performance, maximum displacement, velocity and acceleration of the primary system for the system controlled by TMD (passive control) and by SATMD (semi-active control) are calculated. Since the performance of the controlled system may not be provided solely by the peak responses, normed system responses (RMS) are obtained as well. The norm, denoted $\|\cdot\|$, is defined as

$$\|\cdot\| = \sqrt{\frac{1}{t_f}\int_0^{t_f} [\cdot]^2 dt} \tag{9}$$

where t_f is the simulation time.

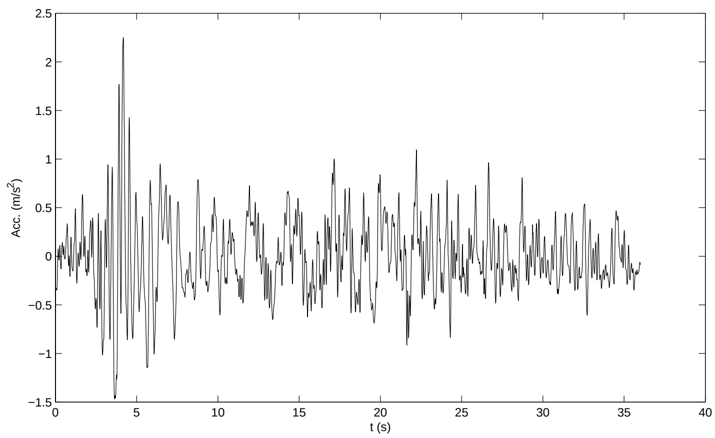

Figure 4: Hachinohe earthquake signal.

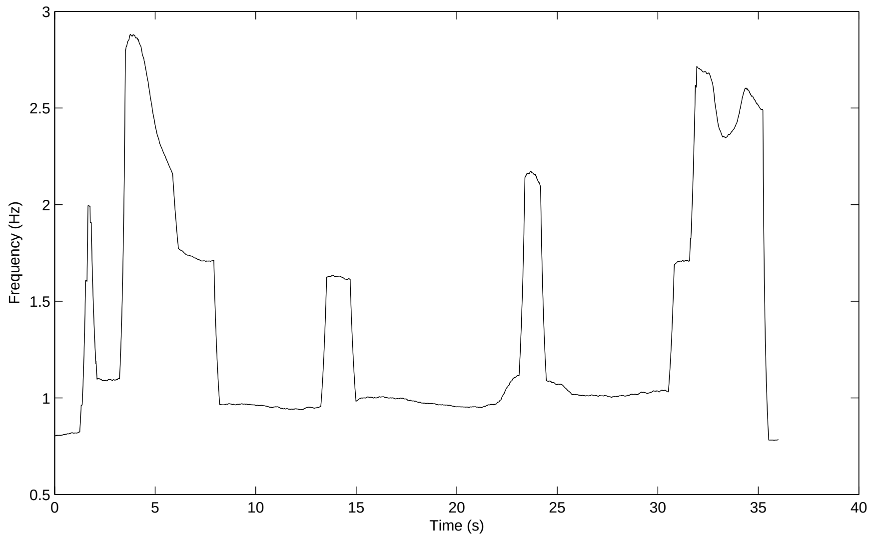

Figure 5: Instantaneous frequency of the excitation.

The primary system and mass damper's response for both TMD and SATMD cases are shown in Figures 6 to 8. The peak and RMS values of the primary system are summarized in Table 2. It can be observed that the maximum displacement response reduction in the case of SATMD compared to the TMD case is about 15%. This reduction for the maximum velocity and acceleration responses are about 13% and 4%, respectively. Furthermore, the reduction in RMS of displacement, velocity and acceleration responses is, respectively, about 39%, 39% and 36%.

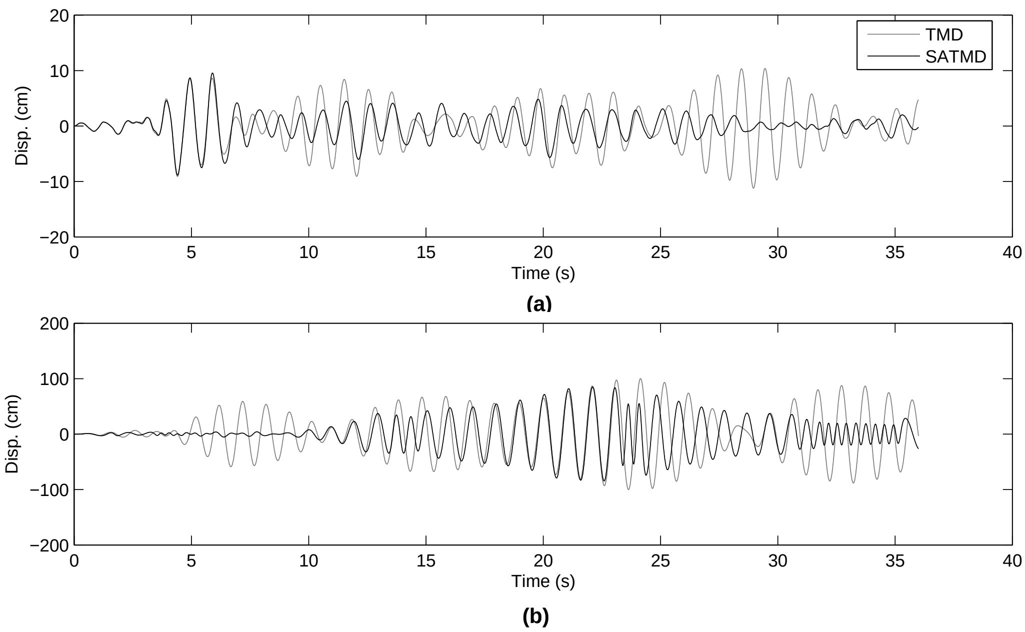

Figure 6: Displacement response of (a) primary system; (b) vibration absorber.

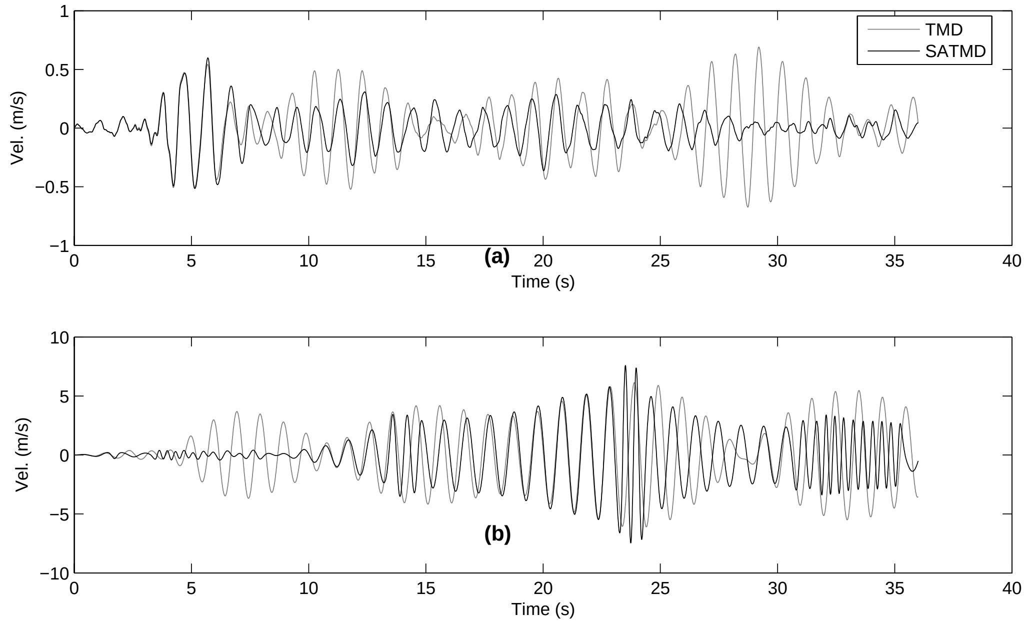

Figure 7: Velocity response of (a) primary system; (b) vibration absorber.

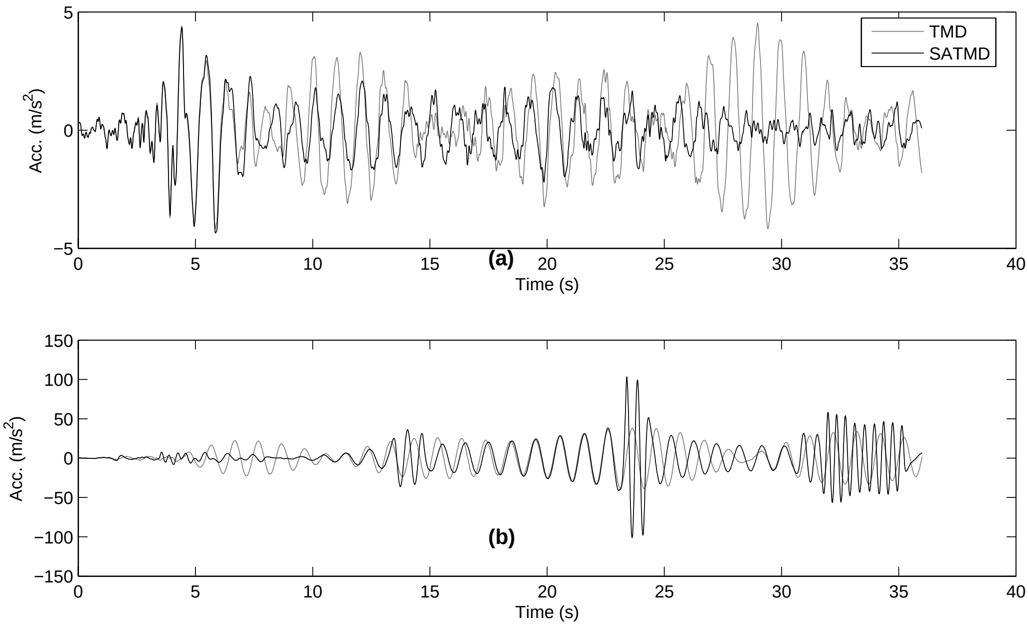

Figure 8: Acceleration response of (a) primary system; (b) vibration absorber.

Table 2: max and RMS values of the primary system response.

Control System	$\max(x_1(t))$ (m)	$\max(\dot{x}_1(t))$ (m/s)	$\max(\ddot{x}_1(t))$ (m/s^2)	$\mathrm{RMS}(x_1(t))$ (m)	$\mathrm{RMS}(\dot{x}_1(t))$ (m/s)	$\mathrm{RMS}(\ddot{x}_1(t))$ (m/s^2)
TMD	0.11	0.69	4.54	0.04	0.25	1.61
SATMD	0.10	0.60	4.35	0.02	0.15	1.02

According to the cross-sectional dimensions of the beam given in Table 1, the minimum and maximum values of the moment of inertia are 117.33 mm^4 and 3549.3 mm^4, respectively. Therefore, the stiffness of SAVS device can vary between certain minimum and maximum values of 1273.1 N/m and 38513 N/m. Figure 9 shows the variation of SATMD's stiffness as a function of time. Figure 10 shows the angle of rotation of the each beams from the principal axis about which the moment of inertia is a maximum. The figure demonstrates that the angle varies between 79.94° to 23.11° to provide the required stiffness.

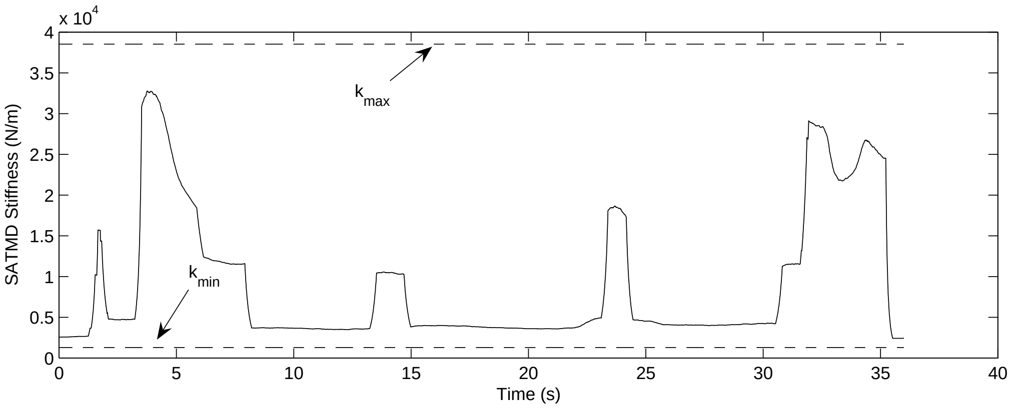

Figure 9: Variations of SATMD's stiffness versus time.

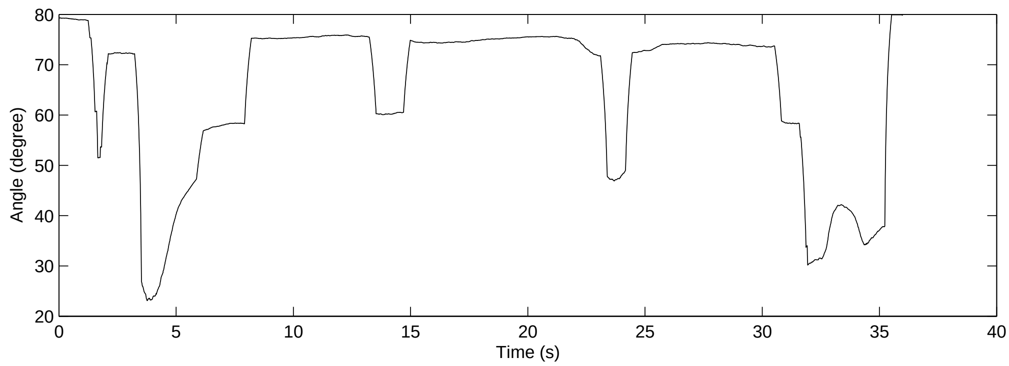

Figure 10: Angle of rotation of SAVS.

4 CONCLUSIONS

The design and formulations of a novel SAVS device is presented. The main advantages of this device can be enumerated as: simple structure, low power requirement, ease of design and manufacturing, quick adjustment of stiffness from minimum to maximum, low moving parts. A new SATMD is developed based on the SAVS device which is capable of adjusting its frequency in real-time. The effectiveness of the SATMD for suppression of a SDOF system under earthquake excitation is investigated. Wavelet transform is used to detect instantaneous frequency of the excitation and online tuning of SATMD. It was shown that the SATMD reduces the maximum and RMS displacement response of the primary system about 15% and 39%, respectively, compared to the passive TMD tuned to the operating frequency. A reduction of 13%, 4%, 39% and 36% is also observed for the maximum velocity, maximum acceleration and RMS of velocity and acceleration of the system, respectively.

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