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ARLEQUIN FRAMEWORK FOR STRUCTURAL TRANSIENT DYNAMICS

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Abstract. *Arlequin method is a modeling framework based on the combination of concurrent multiscale and multi mechanical models. It is a general application of overlapping domain decomposition. The choice of the coupling operators written in common areas between different models provide the main key to ensure the continuity of displacement, velocity, and equilibrium of forces. Using overlap coupling methods in the context of structural dynamics is advantageous when treating dissimilar domains, mainly when wave propagation between the different models is taken into consideration. In this paper we propose to couple and analyze different numerical time schemes using the Arlequin method. Numerical studies, based on multiscale and multimodel space coupling are used to exemplify the efficiency of our approach.*

1 INTRODUCTION

Performing numerical simulations of the dynamic response in rotating machinery is mostly based on beam structures representing the coarse model.
Taking into consideration local physical phenomena occurring at microscopic levels such as crack propagation or contact requires a fine three dimensional modeling of the structure. On the other side, carrying out calculations on very finely meshed models can be very expensive in computational terms.

Many adaptive methods [1] proposing solutions for this kind of multiscale studies have been developed in order to enhance the accuracy of the numerical approximations and also reduce CPU (Central Processing Unit) and memory problems. But in spite of their main advantage, these multi-meshes methods are considered very expensive to implement in practical computations.

An other approach is offered by multiscale methods. The main idea is to have one global domain with a coarse mesh, including a zone of interest where a very fine mesh is super-imposed. These methods are to divide into two major classes: the first one is based on coupling domains at discrete interface [2], and the second consists on using an overlapping zone [3] where the two interfaces are superposed.

Arlequin method [4] is developed in the context of multiscale overlapping methods. Using a partition of models, this approach provides a progressive passage between different models with enhanced flexibility by means of multiple parameters [5]. In static and dynamic cases, Arlequin method gives the possibility of coupling dissimilar models with concurrent scales such as 2D-1D and 3D-1D models. In dynamic studies, it presents an important ability to treat problems where aspects like wave propagation are taken into consideration. It has been shown in [6], that if the coupling parameters of this method are correctly used, wave transition is guaranteed and problems of spurious reflections are avoided while transiting between models.

Moreove, being capable of coupling different models where each one has its own time integration scheme is a very important aspect. It can be considered as a first step for introducing different time discretizations, as well as a main key for coupling multiple finite element codes. Extending the Arlequin method to such application can be very advantageous.

In this study, we will visit energy conservation aspects for continuous and discrete Arlequin formulations. Then we will analyze the energy balance of the Newmark [10] algorithm while coupling different Newmark schemes, and propose a formalism for coupling different time integration schemes in the Arlequin framework.

Based on this formalism, two numerical applications are presented. The first one considers a multiscale coupling of two bar models. The second concerns a multimodel coupling of continuous 2D and beam structures. Future industrial applications of this work will mainly be used in the domain of rotor dynamics where multimodel coupling authorizing multiscale time integration is believed to be useful for such studies.

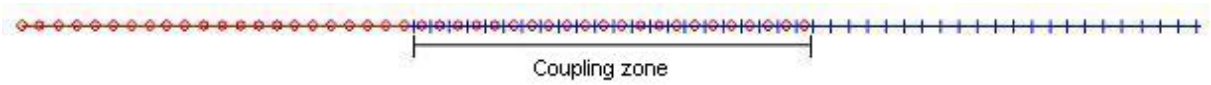

Figure 1: 1D-1D mesh coupling.

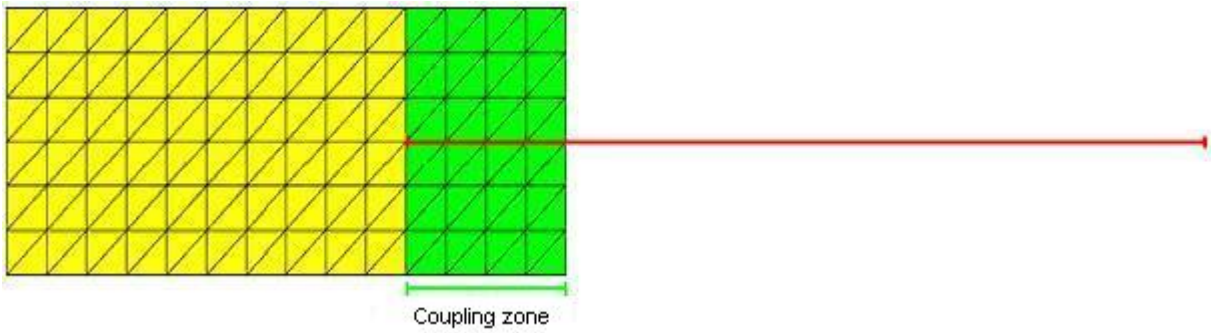

Figure 2: 2D-1D mesh coupling.

2 ARLEQUIN CONTINUOUS FORMULATION IN DYNAMIC REGIMES

An Arlequin-based mechanical study consists in dividing a global model Ω into two or more sub-domains Ω_i. These sub-domains have a common overlap area S.
Considering two domains Ω_1 and Ω_2, the overlap is equivalent to their intersection $S = S_1 \cap S_2$.

In general terms, the Arlequin formulations are obtained by three main steps:

- super-imposing a local model Ω_2 to a global one Ω_1 in a zone of interest S.
- distributing the mechanical energies between the overlapping sub-systems by means of partition of unity functions.
- gluing the models in a part S_g of S.

We note that for the numerical applications presented in this paper, the gluing area S_g is equal to the overlapping zone S (external junction).
We consider a time dependent problem, where Ω_1 and Ω_2 are subjected to a field of volume density of forces f_i $(i = 1, 2)$ and clamped in a non zero measured part Γ_1 of its boundary $d\Omega_1$.
We denote W_1 and W_2 as the spaces of kinematical admissible fields related to Ω_1 and Ω_2, and M the Lagrange multiplier space called the mediator space (for more details on these definitions we refer to [7]):

$$W_1 = \left\{ v_1 \in H^1(\Omega_1);\ v_1 = 0\, on\, \Gamma_1 \right\} \tag{1}$$

$$W_2 = H^1(\Omega_2) \tag{2}$$

Based on the Virtual Work Principle (VWP), the classical continuous Arlequin formulation consists in finding the admissible displacement fields u_1 and u_2, as well as the Lagrange multipliers fields λ.
It can be written as follows:

$$Find\ (u_1,\ u_2,\ \lambda) \in W_1 \times W_2 \times M; \tag{3}$$

$$\forall(w_1,\ w_2,\ \mu) \in W_1 \times W_2 \times M, \tag{4}$$

$$G_{dyn}(u_1, w_1, u_2, w_2, \alpha) + G_{int}(u_1, w_1, u_2, w_2, \beta) + G_{arl}(\lambda, w_1, w_2) = G_{ext}(w_1, w_2, \varphi) \tag{5}$$

$$G_{arl}(\mu, u_1, u_2) = 0 \tag{6}$$

Weight parameters α, β, and $\varphi \in [0\ 1]$ used to distribute energy between different models are defined by two main properties:

- $\alpha = \beta = \varphi = 1$ in $\Omega | S_g$
- $0 < \alpha, \beta, \varphi < 1$ in S_g

We note:

- $\alpha_2 = 1 - \alpha_1$
- $\beta_2 = 1 - \beta_1$
- $\varphi_2 = 1 - \varphi_1$

G_{dyn} , G_{int} , G_{arl} , G_{ext} represent respectively the work of inertial forces, the work of the internal forces, the work of the Arlequin forces, and finally the work of the external forces applied on the different models. They are defined by:

$$G_{dyn}(u_1, w_1, u_2, w_2, \alpha) = \int_{\Omega_1} \alpha_1 \rho_1 \ddot{u}_1 w_1 d\Omega_1 + \int_{\Omega_2} \alpha_2 \rho_2 \ddot{u}_2 w_2 d\Omega_2 \tag{7}$$

$$G_{int}(u_1, w_1, u_2, w_2, \beta) = \int_{\Omega_1} \beta_1 \sigma(u_1) : \varepsilon(w_1) d\Omega_1 + \int_{\Omega_2} \beta_2 \sigma(u_2) : \varepsilon(w_2) d\Omega_2 \tag{8}$$

$$G_{ext}(\varphi, w_1, w_2) = f_1(w_1) + f_2(w_2) = \int_{\Omega_1} \varphi_1 f.w_1 d\Omega_1 + \int_{\Omega_2} \varphi_2 f.w_2 d\Omega_2 \tag{9}$$

$$G_{arl}(\lambda, w_1, w_2) =< \lambda, w_1 - w_2 > where\ \ <,> represent\ a\ coupling\ operator \tag{10}$$

Where $\sigma(u)$ and $\varepsilon(u)$ respectively denote the strain and stress tensors associated to the displacement field u.

2.1 COUPLING OPERATORS

Constructing the coupling matrix is a main step in the application of the Arlequin method especially when a multimodel coupling is taken into consideration (2D or 3D models coupled with 1D models). The gluing volume of displacement field is treated in a natural way by introducing the Lagrange multiplier field.
Several operators are proposed for coupling models in the context of the Arlequin framework [4], [5], [7], [8]. We define u as the general displacement vector formed of u_1 and u_2. The three main operators are presented as follows:

$$L^2\ coupling\ G_{arl} = \int_{S_g} \lambda.u\, dS_g \tag{11}$$

$$H^1\ coupling\ G_{arl} = \int_{S_g} \left[\lambda.u + l^2 \varepsilon(\lambda) : \varepsilon(u)\right] dS_g \tag{12}$$

where l is a strictly positive parameter homogeneous to a length.

$$Energy\,coupling\ G_{arl} = \int_{S_g} \sigma(\lambda) : \varepsilon(u)\, dS_g \tag{13}$$

Depending on the application, L^2 coupling may introduce ill conditioning of the global stiffness matrix. This can be treated in an artificial manner by introducing an homogenous parameter (H^1 coupling), or in a more natural way by using the energy coupling.
The three operators listed above are tested in the context of our applications (2D-1D an 1D-1D coupling). In general terms, they all present similar results.

3 ENERGY CONSERVATION: CONTINUOUS ARLEQUIN FORMULATION

In this section we will write the energy conservation in a transient dynamic regime while using the Arlequin continuous framework.

Considering the same problem presented in the section 2 but with no external loads, the expression of total mechanical energy (E_{tot}) of the system is given by:

$$E_{tot} = E_{kinetic} + E_{potential} \tag{14}$$

where kinetic and potential energies are respectively defined as:

$$E_{kinetic} = \int_{\Omega_1} \frac{1}{2}\alpha_1\rho_1(\dot{u}_1)^2 d\Omega_1 + \int_{\Omega_2} \frac{1}{2}\alpha_2\rho_2(\dot{u}_2)^2 d\Omega_2 \tag{15}$$

$$E_{potential} = \int_{\Omega_1} \frac{1}{2}\rho_1\sigma(u_1) : \varepsilon(u_1) d\Omega_1 + \int_{\Omega_2} \frac{1}{2}\rho_2\sigma(u_2) : \varepsilon(u_2) d\Omega_2 \tag{16}$$

We assume that the constitutive material follows a hooke's law:

$$\sigma_i = \mathbf{D}\varepsilon(u_i) \tag{17}$$

with $\mathbf{D}$ the elastic constant matrix.
The derivative of the total energy formula with respect to time gives:

$$\frac{dE_{tot}}{dt} = \int_{\Omega_1} \alpha_1\rho_1\dot{u}_1\ddot{u}_1 d\Omega_1 + \int_{\Omega_2} \alpha_2\rho_2\dot{u}_2\ddot{u}_2 d\Omega_2 + \int_{\Omega_1} \rho_1\sigma(u_1) : \varepsilon(\dot{u}_1) d\Omega_1 + \int_{\Omega_2} \rho_2\sigma(u_2) : \varepsilon(\dot{u}_2) d\Omega_2 \tag{18}$$

We assume that virtual fields are equivalent to velocity fields $w_1 = \dot{u}_1$ and $w_2 = \dot{u}_2$.
Comparing (18) and (5) enables us to write:

$$\frac{dE_{tot}}{dt} = - < \lambda, \dot{u}_1 - \dot{u}_2 > \tag{19}$$

For $\lambda = \mu$, the derivative of (6) gives:

$$\frac{dG_{arl}}{dt} = < \lambda, \dot{u}_1 - \dot{u}_2 > + < \frac{d\lambda}{dt}, u_1 - u_2 > \tag{20}$$

Assuming λ as a time independent field, the second term of 20 is reduced to zero, and we are capable to write the energy conservation as:

$$\frac{dE_{tot}}{dt} =< \lambda, \dot{u}_1 - \dot{u}_2 >= 0 \tag{21}$$

Based on (21), we proved that in a general case, when the Lagrange multiplier field is time independent, the Arlequin framework provides an energy conservation of the global problem.

4 DISCRETIZED PROBLEM

In the following sections, we consider the energy coupling (13). This choice is based on several study results realized in the context of our 2D-1D and 1D-1D applications, and on the homogeneity between the ingredients of this operator and the quantities evaluated in (8).
We now introduce the spatio-temporal discretizations corresponding to (5) and (6). We take into consideration the effect of the damping matrix:

$$M_1\ddot{u}^1_{n+1} + C_1\dot{u}^1_{n+1} + K_1u^1_{n+1} + L^t_1\lambda_{n+1} = f^1_{n+1/ext} \tag{22}$$

$$M_2\ddot{u}^2_{n+1} + C_2\dot{u}^2_{n+1} + K_2u^2_{n+1} + L^t_2\lambda_{n+1} = f^2_{n+1/ext} \tag{23}$$

$$L_1u^1_{n+1} + L_2u^2_{n+1} = 0 \tag{24}$$

In this discretized equation system, M_i, C_i, and K_i are the mass, damping, and stiffness matrices of the system, and $f^i_{n+1/ext}$ is the external load vector. L_i matrices are the Arlequin coupling matrices (24) acting on the displacement fields in this case.

We note that the different matrices figuring in (22) and (23), with the exception of the L_i matrices, are weighted by the mean of the weight parameters defined in section 2. C_i matrix has a special importance in the rotor dynamics domain since it can contain the gyroscopic terms essential for such studies.

5 NUMERICAL INTEGRATION

For our two applications we apply the Gravouil-Combescure [14] method for coupling two sub-domains with different Newmark integration schemes and the same time scale.

5.1 Problem statement

Tests realized on the 2D-1D and 1D-1D couplings show coherent results and balanced energy Newmark algorithm when the average acceleration method is applied or when the same scheme is used for both models. However, when different Newmark schemes are involved, the energy balance is no more respected and differences caused by the work of the gluing forces appear.

5.2 Energy balance of the Newmark algorithm

Ideally, a single-step time integration algorithm should lead to a similar energy balance equation for finite increments. Using ([11]), we will establish the energy balance for multi-Newmark scheme coupling. Our main goal is to focus the contribution of the Arlequin forces in this equilibrium equation.

The work of these forces at the interface between the overlapping domains leads to undesirable energy dissipation effects. The discrete form of the energy balance equation involves the increment of the mechanical energy over the time interval from t_n to t_{n+1}. This increment can be expressed in terms of mean values and increments of the displacement and velocity as follows:

$$\sum_{i=1,2}\Big[E_c^i + E_p^i\Big]_n^{n+1} = \sum_{i=1,2}\Big[\frac{1}{2}(\dot{u}^i)^t M_i(\dot{u}_n^i) + \frac{1}{2}(u^i)^t K_i(u^i)\Big]_n^{n+1} \tag{25}$$

$$= \sum_{i=1,2}\left[(\frac{1}{2}(\dot{u}_{n+1}^i + \dot{u}_n^i)^t M_i(\dot{u}_n^i - \dot{u}_{n+1}^i) + \frac{1}{2}(u_{n+1}^i + u_n^i)^t K_i(u_n^i - u_{n+1}^i)\right] \tag{26}$$

Using different ingredient of the Newmark scheme, the energy balance equation taking into consideration the contribution of the gluing forces at the interface can be written as following:

$$\sum_{i=1,2}\Big[\frac{1}{2}(\dot{u}^i)^t M_i(\dot{u}_n^i) + \frac{1}{2}(u^i)^t K_i(u^i) + (\beta^i - \frac{1}{2}\gamma^i)\frac{1}{2}(\triangle t)^2(\ddot{u}^i)^t M_i(\ddot{u}^i)\Big]_n^{n+1} \tag{27}$$

$$= \sum_{i=1,2}(\triangle u^i)^t\left[\left(\bar{f}^i + (\gamma^i - \frac{1}{2})\triangle f^i\right)\right. \tag{28}$$

$$- \sum_{i=1,2}(\triangle u^i)^t\left(L_i^t\overline{\lambda} + (\gamma^i - \frac{1}{2})L_i^t\triangle(\lambda)\right) \tag{29}$$

$$- \sum_{i=1,2}(\gamma^i - \frac{1}{2})\left\{(\triangle u^i)^t K_i\triangle u^i + (\beta^i - \frac{1}{2}\gamma^i)(\triangle t)^2(\triangle\ddot{u}^i)^t M_i(\triangle\ddot{u}^i)\right) \tag{30}$$

$$- \sum_{i=1,2}(\triangle u^i)^t C_i\left\{\frac{1}{2}(\dot{u}_{n+1}^i + \dot{u}_n^i) + (\gamma^i - \frac{1}{2})\triangle\dot{u}^i\right\} \tag{31}$$

where
$\triangle f^i = f_{n+1}^i - f_n^i$
$\triangle\lambda = \lambda_{n+1} - \lambda_n$
$\triangle u = u_{n+1} - u_n$
$\bar{f}^i = \frac{1}{2}(f_{n+1}^i + f_n^i)$
$\overline{\lambda} = \frac{1}{2}(\lambda_{n+1} + \lambda_n)$

Terms in (27) are the total mechanical energy plus a conservative Newmark term. In (28) and (29) , we respectively have the work of the external forces and the work of gluing forces. In (30) we have a Newmark dissipative term, and in (31) the contribution of the damping matrix. We are mainly interested in (29). It represents the Arlequin contribution and it will be studied in details in the next section.

In order to show conservative terms on the left side of Newmark's energy balance, and dissipative terms on the right side, we can express (30) with quadratic terms.
If the mass matrix M_i is replaced with an equivalent mass matrix M_i^* defined by:

$$M_i^* = M_i + (\gamma - \frac{1}{2})hC \tag{32}$$

Equations (27) to (31) can be written as:

$$\sum_{i=1,2}\Big[\frac{1}{2}(\dot{u}^i)^t M_i^*(\dot{u}_n^i) + \frac{1}{2}(u^i)^t K_i(u^i) + (\beta^i - \frac{1}{2}\gamma^i)\frac{1}{2}(\triangle t)^2(\ddot{u}^i)^t M_i^*(\ddot{u}^i)\Big]_n^{n+1} \tag{33}$$

$$= \sum_{i=1,2} (\triangle u^i)^t \left[\left(\bar{f}^i + (\gamma^i - \frac{1}{2}) \triangle f^i \right) \right. \tag{34}$$

$$- \sum_{i=1,2} (\triangle u^i)^t \left(L_i^t \overline{\lambda} + (\gamma^i - \frac{1}{2}) L_i^t \triangle(\lambda) \right) \tag{35}$$

$$- \sum_{i=1,2} (\gamma^i - \frac{1}{2}) \left\{ (\triangle u^i)^t K_i \triangle u^i + (\beta^i - \frac{1}{2}\gamma^i)(\triangle t)^2 (\triangle \ddot{u}^i)^t M_i^* (\triangle \ddot{u}^i) \right) \tag{36}$$

$$\frac{1}{2} \Delta t \left\{ \Delta t^{-2} \Delta u^t C \Delta u + \frac{1}{4} (\dot{u}_{n+1} + \dot{u}_n)^t C (\dot{u}_{n+1} + \dot{u}_n) \right\} \tag{37}$$

$$- \frac{1}{2} (\beta^i - \frac{1}{2}\gamma^i) \left\{ (\gamma^i - \frac{1}{2})^2 - \frac{1}{2} (\beta^i - \frac{1}{2}\gamma^i) \right\} \Delta t^3 \Delta \ddot{u}^t C \Delta \ddot{u} \tag{38}$$

Now we have a Newmark energy balance system with conservative and dissipative terms respectively on the left and right side of the equation.

6 ARLEQUIN INTERFACE WORK

The contribution of the Arlequin gluing forces at the interface appear in the Newmark energy balance equation and it is given by:

$$W_{inter}^{arl} = \sum_{i=1,2} (\triangle u^i)^t \left(L_i^t \overline{\lambda} + (\gamma^i - \frac{1}{2}) L_i^t \triangle \lambda \right) \tag{39}$$

We can re-write this term as an interpolation of λ between n and $n+1$ as follows:

$$W_{inter}^{arl} = \sum_{i=1,2} (\triangle u^i)^t \left(L_i^t \overline{\lambda} + (\gamma^i - \frac{1}{2}) L_i^t \triangle \lambda \right) = \sum_{i=1,2} (\triangle u^i)^t L_i^t \left(\gamma^i \lambda_{n+1} + (1 - \gamma^i) \lambda_n \right)$$

$$= (\triangle u^1)^t L_1^t \left(\gamma^1 \lambda_{n+1} + (1 - \gamma^1) \lambda_n \right) + (\triangle u^2)^t L_2^t \left(\gamma^2 \lambda_{n+1} + (1 - \gamma^2) \lambda_n \right) \tag{40}$$

$$= \left((\triangle u^1)^t L_1^t \gamma^1 + (\triangle u^2)^t L_2^t \gamma^2 \right) \lambda_{n+1} + \left((\triangle u^1)^t L_1^t (1 - \gamma^1) + (\triangle u^2)^t L_2^t (1 - \gamma^2) \right) \lambda_n \tag{41}$$

In order to find a solution to the problem statement (5.1) we have to be able to use the Arlequin condition (24).
In (41) we can notice that the presence of γ^1 and γ^2 makes the factorization unrealizable and the use of the Arlequin condition impossible.
Based on [13], we propose to introduce **a time independent Lagrange multiplier field $\lambda_{n+\gamma}$.** In other terms, between two successful time steps, $\lambda_{n+\gamma}$ is a constant field.
Equation (39) can now be written as:

$$W_{inter}^{arl} = \sum_{i=1,2} (\triangle u^i)^t L_i^t \lambda_{n+\gamma} \tag{42}$$

If the numerical integration conditions are well respected, between two successive time steps, Arlequin condition (24) gives:

$$\sum_{i=1,2} (L_i u_{n+1}) = 0 \; and \sum_{i=1,2} (L_i u_n) = 0 \tag{43}$$

Now we have (42) equivalent to zero.
In this section we have demonstrated that by the mean of a time independent Lagrange multiplier field, the work of the Arlequin gluing forces is reduced to zero while using two different Newmark schemes and a space recovering method.

6.1 A PRACTICAL WAY TO ENFORCE THE INTERPOLATION OF $\lambda_{n+\gamma}$

The idea proposed in the last section, is a theorical framework permitting the suppression of undesirable interface effects. But no indications concerning constant value of the time independent field $\lambda_{n+\gamma}$ are presented.
We will now suggest a method which aims to determine a general form of $\lambda_{n+\gamma}$. This form should be applicable on both models and on the gluing zone.

Based on (41),

$$\lambda_{n+\gamma} = (\gamma^i \lambda_{n+1} + (1-\gamma^i)\lambda_n) \tag{44}$$

we define $\lambda_{n+\gamma}$ as follows:

$$\lambda_{n+\gamma} = (\gamma^* \lambda_{n+1} + (1-\gamma^*)\lambda_n) \tag{45}$$

One can notice that with this definition, according a value to $\lambda_{n+\gamma}$ is equivalent to according a value to γ^*.
In order to stay coherent with the partition of unity used for spatial gluing part, we propose to weight the Newmark parameters γ^1 and γ^2 with a weight parameter ξ.

By means of this approach we will have:

- $\gamma^* = \gamma^1 \ in \ \Omega_1 | S_c$;
- $\gamma^* = \gamma^2 \ in \ \Omega_2 | S_c$;
- $\gamma^* = \xi\gamma^1 + (1-\xi)\gamma^2 \ in \ S_c$;

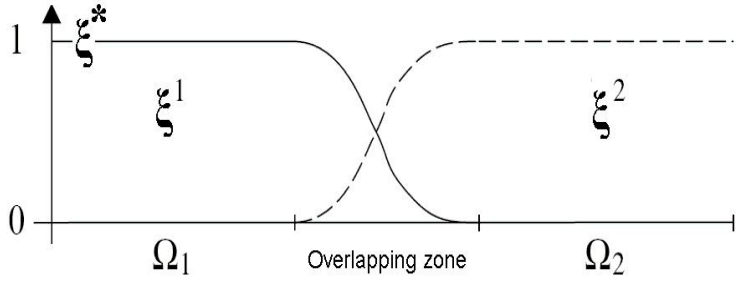

Figure 3: Wave propagation.

We are now able to write (42) as:

$$W^{arl}_{inter} = \left(L_1(\triangle u^1) + L_2(\triangle u^2)\right)\gamma^* \lambda_{n+1} + \left(L_1(\triangle u^1) + L_2(\triangle u^2)\right)(1-\gamma^*)\lambda_n \tag{46}$$

We can notice that (46) is very similar to (41). The main difference is that in 46 the Arlequin conditions can take effect and suppress the undesirable energy dissipation introduced at the interface.

7 NUMERICAL APPLICATION

The proposed methodology here below, is applied for both 2D-1D and 1D-1D couplings. We used two different dissipative Newmark schemes, and a unique time scale. Results were validated by comparison with an analytic solution [9] of a free climbed bar model, undergoing a constant concentrated effort at its free edge. The same work was realized with sinusoidal loading.

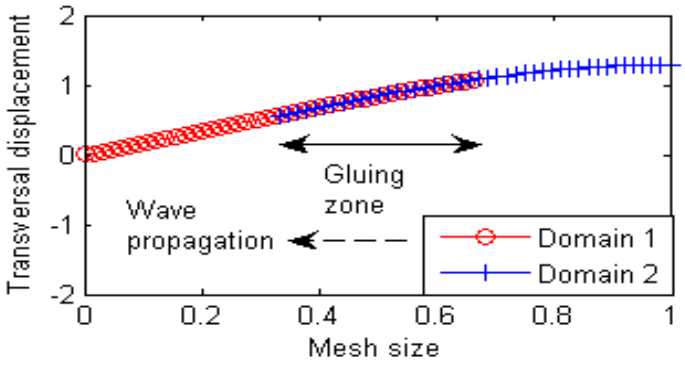

Figure 4: Wave propagation.

In figure 5, the left graph shows the work of the gluing forces at the interface while using two different Newmark schemes on the 1D-1D bar coupling. The right one represents the same work but here we use the $\lambda_{n+\gamma}$ introduced in section 5.1. We can clearly see that the numerical dissipation disappear, and the contribution of the gluing forces at the interface is reduced to zero. We can find the corresponding energy balance of the Newmark algorithm (figure7).

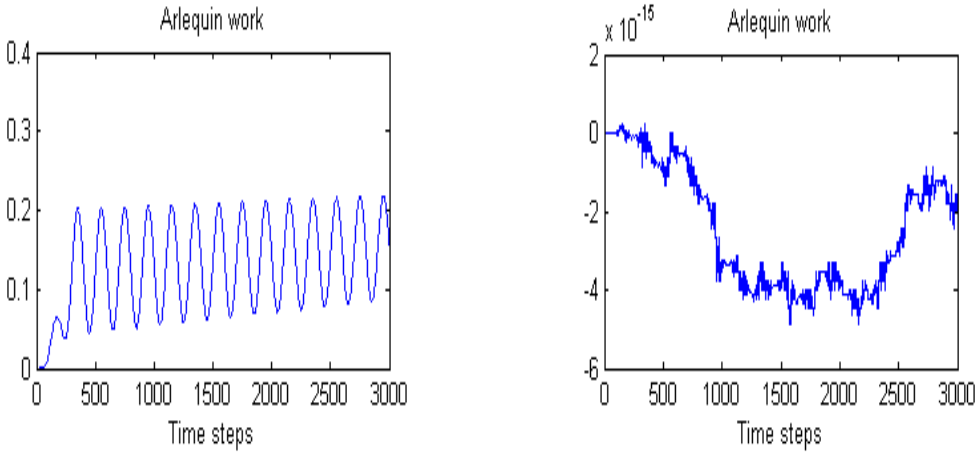

Figure 5: Arlequin interface work ; $\lambda_{n+\gamma}$ Effect.

Graphs in figure 5 are generated for compatible bar meshes. Next results (figure 6) are obtained when bar models presenting incompatible mesh are coupled. For both cases the contribution of the Arlequin forces at the interface is negligible. But we can notice a degradation of the results when incompatible mesh are used. Properly treating the numerical integration issue is the main clue to address this kind of difficulties.

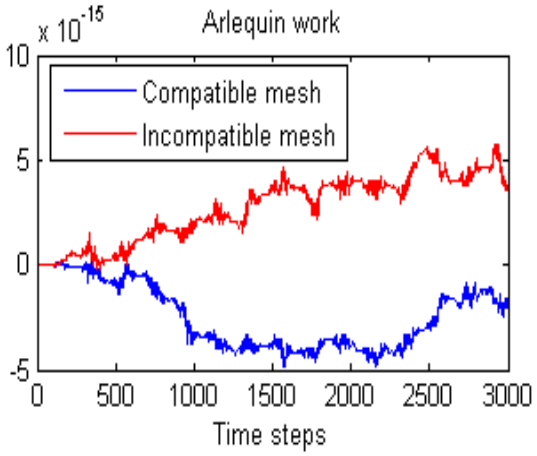

Figure 6: Arlequin interface work - Mesh compatibility effect.

Energy balance equation (27) to (31) is represented in the figure 7. Both cases described in figure 5 are reviewed in the global energy balance system. (Left term holds for (27) and right term holds for (28) to (30)). The contribution of the gluing forces disappear on the right side graph. On the other side, one can notice coherent kinetic and potential energy profiles. The proposed method present satisfying results.

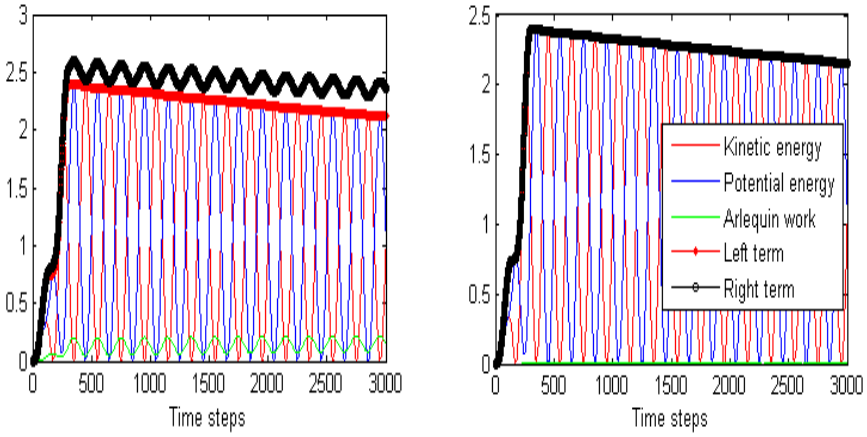

Figure 7: Newmark energy balance - 1D-1D coupling.

Other interesting aspects are currently being investigated. For instance, we are evaluating the model's response to high frequency solicitations and studying how do high frequency waves transit between the different mesh scales.

The study was reviewed in the context of a multimodel 2D-1D coupling (Figure 8), and we obtained similar results.

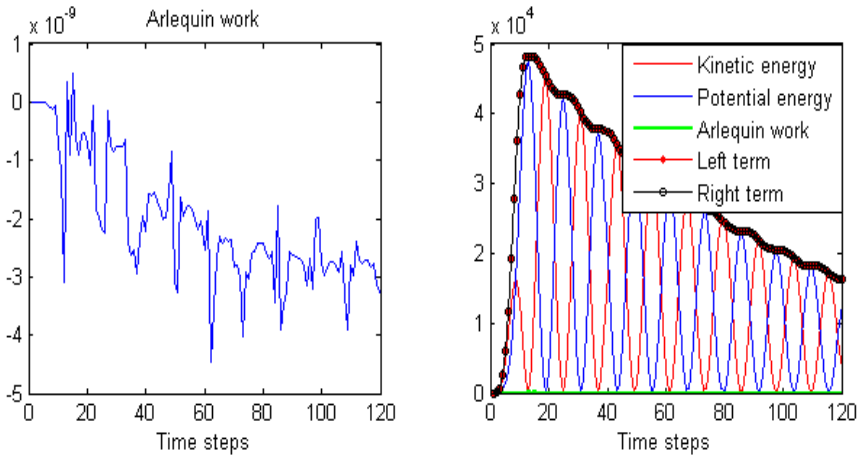

Figure 8: Newmark energy balance - 2D-1D coupling.

8 CONCLUSION

In this article we propose a general formalism to couple different time schemes without introducing undesirable numerical dissipation due to the gluing forces work effects at the interface. This formalism authorizes multiscale/multimodel coupling since it is based on the Arlequin method. Two applications (1D-1D and 2D-1D) were presented in order to exemplify the relevance of the method. An extension to a 3D-1D coupling is planned to be developed. Our main goal is to formulate a generale formalism leading to a space-time multimodel-multiscale/multischeme coupling which guaranties a global energy balance of the system during the resolution, in the context of the Arlequin method.
More elaborated applications are under progress and will be presented during the conference.

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