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DESIGN OPTIMIZATION OF FOUNDATION FOR ROTATING MACHINERY AGAINST STANDING-WAVE VIBRATION IN A BUILDING

Bin Niu, Niels Olhoff

Department of Mechanical and Manufacturing Engineering, Aalborg University
Fibigerstraede 16, DK-9220 Aalborg East, Denmark
e-mail: {bni, no}@m-tech.aau.dk

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Abstract. *This paper deals with the problem of optimum design of a foundation for rotating machinery on a storey of a building with a view to minimize the level of standing-wave vibration in the building. The foundation is usually designed as a base plate for the machinery, with some resilient mounts fixed to the bottom of the base plate and supported by the floor of the storey in order to provide a suitable level of vibration isolation of the building. Due to variable service speeds and the existence of non-balanced masses, the rotating machinery may be considered a source that within a given range of excitation frequencies excites forced vibration of the foundation, and thereby the floors and walls, etc., of the building. The transmission of such vibrations through the building may result in undesirable sound emission and unsatisfactory comfort conditions for the people in dwellings and offices of the building. To remedy this, the objective of this work is to develop and implement a method of design optimization to determine optimum stiffness values of resilient mounts subject to constraints on availability of physical properties of material to be used. The design objective is chosen as minimization of the power transmitted from the machine to the floor of the building where the foundation for the rotating machinery is mounted. At the current stage of our project, this problem is only carried out for a given, quite simplified model of a building. However, for this building model, the design and performance of the optimized machinery foundation will be illustrated and discussed using several numerical examples. In the next stage of our work, a multi-material, parameterized building model will be developed with detailed dimensions and connections of components, and the current problem will be extended to encompass simultaneous design optimization of both the building and the foundation for the rotating machinery in order to minimize the level of standing-wave vibration in the building.*

1 INTRODUCTION

Rotating machinery in buildings is usually applied in central heating and ventilation systems, and larger machinery of this type including a pump is normally mounted on a foundation, which is usually designed as a base plate for the machinery with some resilient mounts fixed to the bottom of the base plate and supported by the floor of the storey. Due to variable service speeds and the existence of non-balanced masses, the rotating machinery may be considered a source that within a given range of excitation frequencies excites forced vibration of the foundation, and thereby the floors and walls, etc., of the building. The transmission of such vibrations through the building may result in undesirable sound emission and unsatisfactory comfort conditions for the people in dwellings and offices of the building. Aside from that, vibrations increase safety hazards in machinery, buildings and installations. The primary goals of vibration insulation are to restrict the detrimental effects of vibrations on people to within reasonable limits, and to protect sensitive apparatus and safety systems from excessive stresses from vibrations.

Problems of design optimization of machinery foundations against vibration have been mainly studied from two aspects:

1. Free vibration design, also termed as frequency design. This aims at keeping the operating frequency as far away as possible from the eigenfrequencies of the machinery mounting system by adjusting the mounting system in order to avoid resonance. It is usually realized by maximization of the fundamental eigenfrequency or frequency gaps between two consecutive eigenfrequencies of the machinery mounting system.
2. Forced vibration design. The machinery mounting system is assumed to be subjected to a time-varying unbalanced mechanical loading, and this system will be designed by minimizing a chosen cost function describing the level of vibration response or transmission.

The problem of forced vibration design optimization of the installation systems of machinery in buildings has been extensively researched under the assumption of a rigid supporting structure [1-3]. The design based on a rigid supporting structure model is reasonable for the installation of machinery in many real engineering situations. However, this rigid support based model may not be appropriate for the problem studied in the present paper where the machinery is to be installed on a relatively flexible floor of the storey in a lightweight building. Based on a flexible support model, Ashrafiuon [4] studied design optimization of aircraft engine-mount systems for vibration isolation, and Xie et al [5] considered optimization of the mounting system for microelectronics manufacturing equipment using the receptance matrix method. In these works, minimization of the transmitted force from the vibrating machine to the receiver is chosen as the design objective. In the work [6] it was suggested to choose the power flow as the cost function because it combines both forces and velocities in a single concept. Furthermore, the transmitted power from the unbalanced machine to the floor is closely related to the structural noise emission from the floor. Power flow is considered to be a more reasonable measure of the vibratory state in vibro-acoustic modeling. The power flow from a vibrating machine to different flexible receivers through resilient mounts is presented in the works [6-9].

When rotating machinery is to be mounted on building floors rather than directly on a soil foundation, suitable resilient mounts should be provided as vibration isolation elements under the machine with a view to reduce the transmission of vibration.

The design optimization of machinery mounting systems is studied in this paper. The system consists of a rotating, unbalanced machine as vibration source, a mounting system as isolator, and a flexible floor as receiver. By assuming a simple time harmonic excitation generated within the machine by its operating mechanisms, it is convenient to characterize the individual sub-structures by their complex mobilities evaluated at the interfaces of contiguous sub-structures. The mobility is defined as the ratio of the complex amplitudes of the velocities and forces at any interface for a given frequency [10]. The physical quantities, such as velocities, forces and moments at the interfaces, are solved in terms of the mobility matrices of sub-structures.

A generalized mathematical model of mobility power flow is developed in this paper for evaluation of the response of the total system subjected to a given external excitation. In this model, the machine is modeled as a rigid mass subjected to a harmonically time-varying force. The base plate is assumed to be rigidly connected with the machine, and included in the modeling of the machine. The mounting system of the machine is designed as some resilient supports fixed to the bottom of the machine and supported by the floor of the storey in order to provide a suitable level of vibration isolation of the floor. The flexible floor is modeled as an elastic uniform plate, and the driving point and transfer mobilities of the plate are adopted in the model. The degrees of freedom associated with flexural vibration of the supporting structure are of main interest because the flexural wave motion is usually dominating the sound radiation compared with the in-plane wave motion.

The objective of minimizing vibration transmission is realized by sizing optimization of the stiffness coefficients of the resilient mounts. The design objective is chosen as minimization of the power flow transmitted to the building floor through the resilient mounts at the excitation frequency of the machinery. The design and performance of the optimized machinery foundation will be illustrated and discussed using several numerical examples.

The rest of this paper is organized as follows. First, a generalized mathematical model of mobility power flow is developed in this paper. The formulation of minimization of transmitted power flow is presented in Section 3. Section 4 presents a simplified parametric example only considering the vertical flexural motion, and a generalized optimization example with more degrees of freedom. In Section 5, a shape optimization problem of a pad that is placed on a flexible floor to support a resilient mount, is presented. Finally, observations and conclusions are drawn based on the optimization results.

2 A GENERAL MOBILITY FORMULATION OF THE MACHINERY MOUNTING SYSTEM

A general model with a rotating, unbalanced machine as vibration source, a mounting system as isolator, and a flexible floor as receiver, is developed for analysis and optimization of vibration transmission, see the model in Figure 1. The machine is modeled as a rigid body subjected to a harmonically time-varying force. The mounting system of the machine is designed as some resilient supports fixed to the bottom of the machine and supported by the floor of the storey in order to provide a suitable level of vibration isolation of the floor. The flexible floor is modeled as an elastic uniform plate, and the driving point and transfer mobilities [11] of the plate are adopted in the model. Figure 1 gives a representation of a vibratory rigid body resiliently mounted on a four-edge simply supported plate via multiple resilient mounts. The forces and velocities at the interface of the contiguous sub-systems are shown in Figure 2, where the arrows define the positive directions of the forces and velocities.

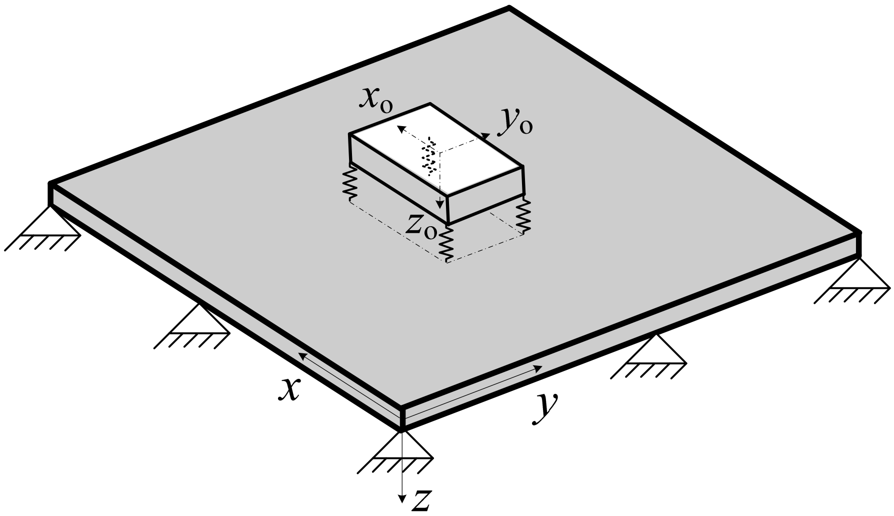


Figure 1: A machine mounted to a flexible floor via multiple resilient mounts

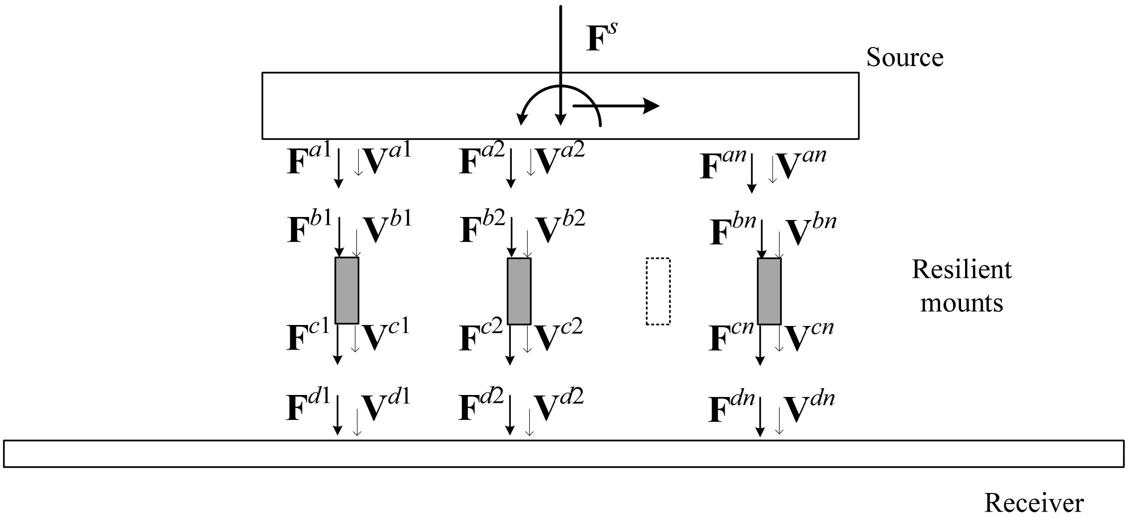


Figure 2: Description of forces and velocities between three sub-systems: source, resilient mounts and receiver

2.1 Rigid body

A local coordinate system $x_o y_o z_o$, cf. Figure 1, is adopted with origin in the center of gravity of the machine. It is assumed that the machine is excited by a generalized concentrated force vector $\mathbf{F}^s$ with three force and three moment components acting at the center of gravity. The corresponding generalized velocity vector at the center of gravity is denoted as $\mathbf{V}^s$.

$$\mathbf{F}^s = \left\{F_1^s, F_2^s, F_3^s, F_4^s, F_5^s, F_6^s\right\}^T \tag{1}$$

$$\mathbf{V}^s = \left\{V_1^s, V_2^s, V_3^s, V_4^s, V_5^s, V_6^s\right\}^T \tag{2}$$

where the superscript T represents the transpose of a matrix or vector.

A number of resilient mounts are assumed to be attached to the bottom of the machine, and the generalized output force and velocity vectors from the rigid body to the resilient mounts at the n junctions are assembled in the vectors

$$\mathbf{F}^a = \left\{ \mathbf{F}^{a1} \quad \mathbf{F}^{a2} \quad \dots \quad \mathbf{F}^{an} \right\}^T, \ \mathbf{V}^a = \left\{ \mathbf{V}^{a1} \quad \mathbf{V}^{a2} \quad \dots \quad \mathbf{V}^{an} \right\}^T \tag{3}$$

Correspondingly, the generalized force and velocity vectors acting at the *j*-th resilient mount by the machine are $\mathbf{F}^{bj}$ and $\mathbf{V}^{bj}$, and each of them includes six components, respectively.

The dynamics governing equation of the rigid body [12] can be written in terms of the mobility matrices,

$$\begin{Bmatrix} \mathbf{V}^s \\ \mathbf{V}^a \end{Bmatrix} = \begin{bmatrix} \mathbf{M}_{11}^s & \mathbf{M}_{12}^s \\ \mathbf{M}_{21}^s & \mathbf{M}_{22}^s \end{bmatrix} \begin{Bmatrix} \mathbf{F}^s \\ \mathbf{F}^a \end{Bmatrix} \tag{4}$$

where $\mathbf{M}_{11}^s = \dfrac{1}{i\omega}\mathbf{J}^{-1}$, $\mathbf{M}_{12}^s = \dfrac{1}{i\omega}\mathbf{J}^{-1}\mathbf{R}$, $\mathbf{M}_{21}^s = \dfrac{1}{i\omega}\mathbf{R}^T\mathbf{J}^{-1}$, $\mathbf{M}_{22}^s = \dfrac{1}{i\omega}\mathbf{R}^T\mathbf{J}^{-1}\mathbf{R}$. The symbol ω is the excitation frequency, i represents the imaginary unit, $i = \sqrt{-1}$, $\mathbf{R}$ represents the location matrix of the resilient mounting junctions with respect to the center of gravity of the machine, and $\mathbf{J}$ is the general mass matrix. The time dependent term $\exp(i\omega t)$ is omitted in the remainder.

2.2 Resilient mounts

The resilient mounts are used as vibration isolators for minimizing vibration transmission from the machine to the building floor. At the mounting junctions on the plate, the generalized force and velocity vectors at the bottom ends of the resilient mounts, see Figure 2, are assembled in the vectors

$$\mathbf{F}^c = \left\{ \mathbf{F}^{c1} \quad \mathbf{F}^{c2} \quad \dots \quad \mathbf{F}^{cn} \right\}^T, \ \mathbf{V}^c = \left\{ \mathbf{V}^{c1} \quad \mathbf{V}^{c2} \quad \dots \quad \mathbf{V}^{cn} \right\}^T \tag{5}$$

The relation between the velocity vectors and force vectors at the two ends of the *j*-th resilient mount is given by using the four-pole equation [13, 14],

$$\begin{Bmatrix} \mathbf{F}^{bj} \\ \mathbf{V}^{bj} \end{Bmatrix} = \begin{bmatrix} \mathbf{diag}\left(\mathbf{T}_{11}^j\right) & \mathbf{diag}\left(\mathbf{T}_{12}^j\right) \\ \mathbf{diag}\left(\mathbf{T}_{21}^j\right) & \mathbf{diag}\left(\mathbf{T}_{22}^j\right) \end{bmatrix} \begin{Bmatrix} \mathbf{F}^{cj} \\ \mathbf{V}^{cj} \end{Bmatrix} \tag{6}$$

where $\mathbf{diag}\left(\mathbf{T}_{pq}^j\right)$, $p, q = 1, 2$, means the diagonal transmission sub-matrix of the *j*-th resilient mount.

For generality, in this study, each resilient mount is modelled as six lumped stiffness components with negligible mass, where three components with translational stiffness and three with rotational stiffness are assumed without coupling between the different stiffness components. For example, the *p*-th component of the *j*-th resilient mount is described by the following four-pole equation

$$\begin{Bmatrix} F_p^{bj} \\ V_p^{bj} \end{Bmatrix} = \begin{bmatrix} -1 & 0 \\ -\dfrac{i\omega}{\mu_p^j} & 1 \end{bmatrix} \begin{Bmatrix} F_p^{cj} \\ V_p^{cj} \end{Bmatrix} \tag{7}$$

where F_p^{bj} denotes the *p*-th (p = 1, …, 6) component of the generalized force at the *j*-th resilient mount, and μ_p^j denotes the stiffness coefficient of the *p*-th stiffness component in the *j*-th resilient mount.

For all resilient mounts, the transmission matrix equation is obtained as

$$\begin{Bmatrix} \mathbf{F}^b \\ \mathbf{V}^b \end{Bmatrix} = \begin{bmatrix} \mathbf{T}^{11} & \mathbf{T}^{12} \\ \mathbf{T}^{21} & \mathbf{T}^{22} \end{bmatrix} \begin{Bmatrix} \mathbf{F}^c \\ \mathbf{V}^c \end{Bmatrix} \tag{8}$$

The conditions for force equilibrium and motion compatibility at the junctions can be written as

$$\mathbf{F}^a = -\mathbf{F}^b,\ \mathbf{V}^a = \mathbf{V}^b,\ \mathbf{F}^c = -\mathbf{F}^d,\ \mathbf{V}^c = \mathbf{V}^d \tag{9}$$

2.3 The supporting floor

The supporting floor is modeled as a thin, elastic uniform plate. When there are no significant reflections from the boundaries or from discontinuities within the receiver, an infinite, uniform plate model may be assumed. Otherwise, a finite plate with suitable boundary conditions should be considered. The supporting plate floor is excited by the force and moment components at the bottom of each resilient mount. The force and velocity vectors at the plate mounting points, see Figure 2, are assembled in the vectors

$$\mathbf{F}^d = \begin{Bmatrix} \mathbf{F}^{d1} & \mathbf{F}^{d2} & \dots & \mathbf{F}^{dn} \end{Bmatrix}^T,\ \mathbf{V}^d = \begin{Bmatrix} \mathbf{V}^{d1} & \mathbf{V}^{d2} & \dots & \mathbf{V}^{dn} \end{Bmatrix}^T \tag{10}$$

The force vector at each resilient mount junction j = 1, …, n includes six components,

$$\mathbf{F}^{dj} = \left\{ F_1^{dj}, F_2^{dj}, F_3^{dj}, F_4^{dj}, F_5^{dj}, F_6^{dj} \right\}^T \tag{11}$$

Accordingly, the velocity vector at each resilient mount junction j = 1, …, n includes three translation components and three rotational components, i.e.,

$$\mathbf{V}^{dj} = \left\{ V_1^{dj}, V_2^{dj}, V_3^{dj}, V_4^{dj}, V_5^{dj}, V_6^{dj} \right\}^T \tag{12}$$

The mobility equation of the plate can be now written as follows [11, 15-18], where $[\mathbf{Y}]$ is the mobility matrix,

$$\begin{Bmatrix} \mathbf{V}^{d1} \\ \mathbf{V}^{d2} \\ \vdots \\ \mathbf{V}^{dn} \end{Bmatrix} = \begin{bmatrix} \mathbf{y}^{d11} & \mathbf{y}^{d12} & \dots & \mathbf{y}^{d1n} \\ \mathbf{y}^{d21} & \mathbf{y}^{d22} & \dots & \mathbf{y}^{d2n} \\ \dots & & & \vdots \\ \mathbf{y}^{dn1} & \mathbf{y}^{dn2} & \dots & \mathbf{y}^{dnn} \end{bmatrix} \begin{Bmatrix} \mathbf{F}^{d1} \\ \mathbf{F}^{d2} \\ \vdots \\ \mathbf{F}^{dn} \end{Bmatrix} = [\mathbf{Y}] \begin{Bmatrix} \mathbf{F}^{d1} \\ \mathbf{F}^{d2} \\ \vdots \\ \mathbf{F}^{dn} \end{Bmatrix} \tag{13}$$

Since the flexural vibration dominates the noise emission of the building floor, only the out-of-plane flexural wave will be taken into account. Thus, the in-plane shear and longitudinal motions induced by in-plane forces, and drilling motion of the plate induced by twisting moment are neglected. The out-of-plane flexural waves are induced by the out-of-plane force and the in-plane moment components. For example, the mobility sub-matrix $\mathbf{y}^{d21}$ can be written as

$$\mathbf{y}^{d21}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & y_{wF_z}^{21} & y_{wM_x}^{21} & y_{wM_y}^{21} & 0 \\ 0 & 0 & y_{\theta_x F_z}^{21} & y_{\theta_x M_x}^{21} & y_{\theta_x M_y}^{21} & 0 \\ 0 & 0 & y_{\theta_y F_z}^{21} & y_{\theta_y M_x}^{21} & y_{\theta_y M_y}^{21} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{14}$$

where the mobility term $y_{wF_z}^{21}$ relates an out-of-plane translational velocity V_z^{d2} at the junction no.2 to an out-of-plane force excitation F_z^{d1} at junction no.1, where the latter junction is the excitation point, and the former junction is the response point.

The mobility representation is a useful tool to describe the dynamic properties of the supporting floor. For a realistic complicated floor, force and moment mobilities at the mounting junctions can be measured by experiments. For some simple infinite and finite structures, the mobility matrices can be derived analytically. The mobility formulations for an elastic uniform plate with simply supported edges and an infinite plate can be found in, e.g., Refs. [10, 11, 15].

2.4 Power flow

The time averaged power flow transmitted from the machine to the building floor can be expressed as

$$\Pi=\frac{1}{2}\mathrm{Re}\left[\left(\mathbf{F}^d\right)^*\mathbf{V}^d\right]=\frac{1}{2}\mathrm{Re}\left[\left(\mathbf{F}^d\right)^*\mathbf{Y}\mathbf{F}^d\right] \tag{15}$$

where the symbol $(\ )^*$ represents conjugate transpose of a matrix or vector.

The transmitted force $\mathbf{F}^d$ can be solved from Eqs. (4), (8) and (13) by applying the conditions of force equilibrium and motion compatibility in Eq. (9). For brevity, the somewhat lengthy derivation is omitted here, as the same result can be found in Ref. [14]. Thus one gets the following expression for the transmitted force $\mathbf{F}^d$,

$$\mathbf{F}^d=\left(-\mathbf{T}^{21}-\mathbf{M}_{22}^s\mathbf{T}^{11}+\mathbf{T}^{22}\mathbf{Y}+\mathbf{M}_{22}^s\mathbf{T}^{12}\mathbf{Y}\right)^{-1}\mathbf{M}_{21}^s\mathbf{F}^s \tag{16}$$

In order to avoid excessive vibration of the machine, a suitable constraint on the velocity $\mathbf{V}^s$ of the rigid body may be applied. Thus, the velocity $\mathbf{V}^s$ is sometimes of interest in design optimization of machinery mounting system. The expression for the velocity $\mathbf{V}^s$ given in Eq. (17) is obtained by solving Eqs. (4), (8) and (13) with the conditions of force equilibrium and motion compatibility in Eq. (9),

$$\mathbf{V}^s=\mathbf{M}_{11}^s\mathbf{F}^s+\mathbf{M}_{12}^s\left(\mathbf{T}^{11}-\mathbf{T}^{12}\mathbf{Y}\right)\mathbf{F}^d \tag{17}$$

3 OPTIMIZATION FORMULATION FOR MINIMIZATION OF POWER TRANSMISSION

3.1 Optimization formulation

The problem of design optimization of the rotating machinery mounting system with the objective of minimizing the total power flow transmitted from the machine to the building floor via several resilient mounts, can be formulated as

$$\begin{aligned} &\min_{\mu} \left\{ \Pi = \frac{1}{2} \left(\mathbf{F}^d \right)^* \mathrm{Re}\left\{ \mathbf{Y} \right\} \mathbf{F}^d \right\} \\ &\text{s.t. Given constraints} \\ &\quad \mu_{\min} \le \mu_i \le \mu_{\max},\, i = 1, \dots, n^{dv} \end{aligned} \tag{18}$$

The present work aims to realize this objective by optimizing stiffness coefficients μ_i of resilient mounts in a given range between $\mu_{\min}$ and $\mu_{\max}$. The lower and upper limits of the stiffness coefficients are usually determined by physical properties of the resilient mounts. A reasonable lower limit is important for satisfying requirements on the static displacement of the machine, or the motion during starting and stopping stages. It is well-known that the stiffness of resilient mounts is normally frequency dependent. However, for simplicity the stiffness will be assumed to be independent of frequency in the following.

In the expression for the total transmitted power Π in (18), $\mathbf{F}^d$ denotes the vector of amplitudes of the loading vector acting on the plate with the excitation frequency ω. The given constraints are specified by physical and geometrical requirements on mounting systems. The symbol n^{dv} denotes the number of design variables.

3.2 Design sensitivity analysis

The sensitivity of the objective function Π in Eq. (18) with respect to the design variable μ_k can be derived as

$$\frac{\partial \Pi}{\partial \mu_k} = \frac{1}{2} \frac{\partial \left(\mathrm{Re}\left\{ \left[\mathbf{F}^d \right]^* \left[\mathbf{Y} \right] \left[\mathbf{F}^d \right] \right\} \right)}{\partial \mu_k} = \mathrm{Re}\left\{ \left[\mathbf{F}^d \right]^* \left[\mathbf{Y} \right] \frac{\partial \left[\mathbf{F}^d \right]}{\partial \mu_k} \right\},\, k = 1,\, \dots,\, n^{dv} \tag{19}$$

where the symmetric characteristics of the mobility matrix $\left[\mathbf{Y} \right]$ have been used.

In order to derive the sensitivity of the transmitted force $\dfrac{\partial \mathbf{F}^d}{\partial \mu_k}$ with respect to the design variables, Eq. (16) is rewritten as

$$\left(-\mathbf{T}^{21} - \mathbf{M}_{22}^s \mathbf{T}^{11} + \mathbf{T}^{22} \mathbf{Y} + \mathbf{M}_{22}^s \mathbf{T}^{12} \mathbf{Y} \right) \mathbf{F}^d = \mathbf{M}_{21}^s \mathbf{F}^s \tag{20}$$

By differentiating both sides of Eq. (20) with respect to the design variable μ_k, considering the condition that only $\mathbf{T}^{21}$ is dependent on design variables among the mobility matrices of the machine, resilient mounts and the floor plate, and assuming that the generalized excitation force $\mathbf{F}^s$ generated by the rotating unbalanced machine is design independent, $\dfrac{\partial \mathbf{F}^d}{\partial \mu_k}$ can be obtained as

$$\frac{\partial \mathbf{F}^d}{\partial \mu_k} = \left(-\mathbf{T}^{21} - \mathbf{M}_{22}^s \mathbf{T}^{11} + \mathbf{T}^{22} \mathbf{Y} + \mathbf{M}_{22}^s \mathbf{T}^{12} \mathbf{Y} \right)^{-1} \frac{\partial \mathbf{T}^{21}}{\partial \mu_k} \mathbf{F}^d,\ k = 1,\, \dots,\, n^{dv} \tag{21}$$

The accuracy of analytical sensitivities has been validated by overall finite difference sensitivity calculations. With these sensitivity results, the design problem (18) may be solved by a mathematical programming method, e.g., MMA by Svanberg [19].

4 NUMERICAL EXAMPLES

4.1 A simplified parametric example

First, a simple special case of the vibratory system shown in Figure 1 is considered. Here, four identical resilient mounts are placed symmetrically with respect to the machine, and mounted on a flexible floor, which is assumed to be an infinite elastic plate of constant thickness. This simplification leads to an equivalent system of the resilient mounts as shown in Figure 3, which consists of four separate sets of a spring of stiffness μ and a mass m^s equal to $m/4$, where the symbol m represents the total mass of the machine. The dynamic governing equation is simplified correspondingly, i.e., the forces acting on all four mounting points are assumed to be the same, that is, $F^0/4$, where F^0 denotes the vertical excitation force acting on the machine. A similar simplified model using a finite four-edge simply supported plate as the receiver is studied in [20].

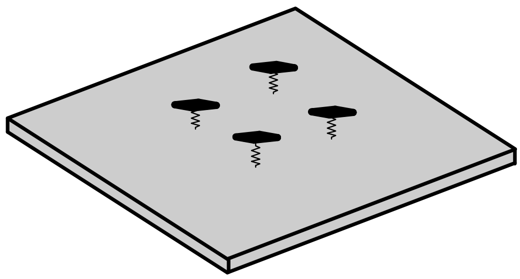


Figure 3: A simplified modeling for the machine connected to a flexible floor via four symmetrically placed mounts

From the vibration equation of a system with a single degree of freedom, see, e.g. [21], the force F_j transmitted from the mass m^s to the floor via the j-th spring is obtained from [11, 20]

$$F_j = \frac{F_0}{4} \frac{1}{1 - \dfrac{m\omega^2}{4\mu} + i\omega \dfrac{m}{4} Y_j} \tag{22}$$

where μ is the stiffness of the spring.

Since only the out-of-plane force excitation on the plate is considered, the mobility matrix of the plate in Eq. (13) can be simplified. By the concept of effective point mobility Y_j as a space averaged effective mobility over all excitation points [22], Y_j can be expressed as $Y_j = Y_{j1} + Y_{j2} + Y_{j3} + Y_{j4}$ for j=1, 2, 3, and 4. The symbol Y_{jk} is the general mobility element from the excitation point k to the response point j, where Y_{jk} with $k = j$ is the driving point mobility, while Y_{jk} with $k \neq j$ denotes the transfer mobility. The effective mobility represents the ratio of the total velocity due to all applied forces to the force acting at the point j. It can be calculated that the effective point mobility of an infinite plate Y_j (j=1, 2, 3, 4) are same, i.e., $Y_1 = Y_2 = Y_3 = Y_4$. The driving point and transfer mobilities of an infinite plate can be found in, e.g., [11, 15].

The velocity V_0 of the machine is calculated based on the same assumption

$$V_0 = \frac{F_0}{4} \frac{\frac{\mu}{i\omega} + \frac{1}{Y_1}}{\left(\frac{\mu}{i\omega} + \frac{1}{Y_1}\right)\left(\frac{\mu}{i\omega} + i\omega\frac{m}{4}\right) - \left(\frac{\mu}{i\omega}\right)^2} \tag{23}$$

Thus, from Eq. (15) the time averaged power $P = \Pi$ transmitted from the machine to the floor can be rewritten as

$$P = 2\,\mathrm{Re}\left(Y_1\right)\left|\frac{F_0}{4}\right|^2 \left|\frac{1}{1 - \frac{m\omega^2}{4\mu} + i\omega\frac{m}{4}Y_1}\right|^2 \tag{24}$$

As a reference to evaluate the effect of the spring isolator, the power flow P^{ws} from the machine to the building floor without spring isolators is given in Eq. (25) below. The modeling without isolation is obtained by removing the spring isolators from Figure 3, and we find that the power P^{ws} transmitted from the machine to the floor without spring isolators is given by

$$P^{ws} = 2\,\mathrm{Re}\left(Y_1\right)\left|\frac{F_0}{4}\right|^2 \left|\frac{1}{1 + i\omega\frac{m}{4}Y_1}\right|^2 \tag{25}$$

The velocity V_0^{ws} of the machine without spring isolators is calculated as

$$V_0^{ws} = \frac{F_0}{4} \frac{Y_1}{i\frac{m}{4}\omega Y_1 + 1} \tag{26}$$

Assuming four springs with the same stiffness coefficient, the dependence of transmitted power P in Eq. (24) on the stiffness coefficient μ will be studied for different excitation frequency and mass values of the machine. In this section, no damping is assumed for the springs and the plate.

For a rigid building floor, the velocity at the mounting junction on the plate is zero, $V_1 = 0$, thus the transmitted power $P = 0$. If the stiffness of the spring is reasonably selected to make the resonance frequency $\omega_0 = \sqrt{\frac{\mu}{m/4}}$ of the mass-spring system to be located far from the excitation frequency ω, the transmitted force $\left|F_i\right|$ on the floor is relatively small.

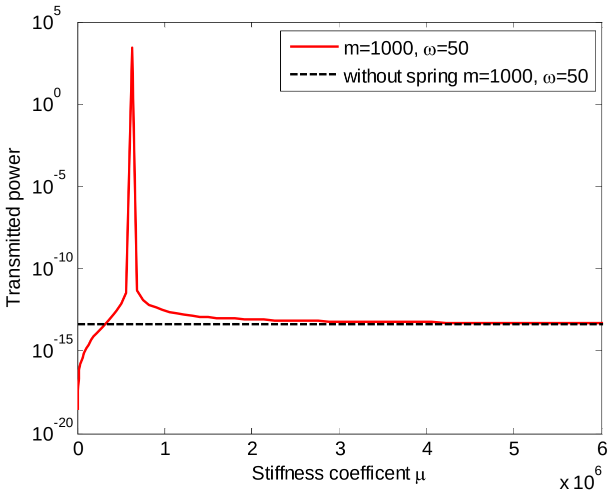
When a flexible floor is considered, the mobility of the plate must be taken in account, and the velocity of the mounting point on the plate $V_1 \neq 0$. Thus we should pay attention to the transmitted power P, the velocity V_1 and force F_1.

When the bending stiffness of the plate is quite large, the corresponding mobility Y_1 is very small. The conclusion is similar to the case of a rigid floor above. The transmitted power P, shown in Figure 4 (a), and force F_1 are very large only in the vicinity of resonance, and in other cases both of them are generally small. In this example, the given circular excitation frequency ω is 50 rad/s.

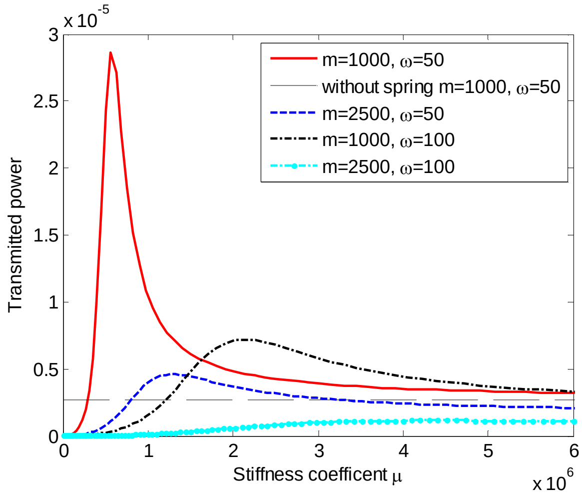
A suitably flexible plate of thickness 0.2 made of a material with Young's modulus 2×10^9, Poisson's ratio 0.3 and mass density 840, is studied here. SI units are assumed. Transmitted powers for different mass values of machine and excitation frequencies are presented in Figure 4 (b). For a specified excitation frequency and a given stiffness coefficient, it is seen that when the mass of the machine increases, then the transmitted power decreases. Actually, an inertia concrete block is sometimes placed at the bottom of machine to increase the mass of the vibrating rigid body. The extra inertia block can be deemed beneficial for reduction of transmitted power when an infinite plate is the receiver, in which case no resonant behavior can occur. Though infinite structures do not exist in reality, the assumption of infinite structures is applicable in many circumstances where there are no significant reflections from discontinuities or boundaries within the receiver [6].

The only peak of each curve in Figure 4 (b) comes from the resonance of the spring-rigid body system. Obviously, the peak will move to the right when the mass of the machine is increased, and the maximum value at the peak also decreases significantly. If the mass of the machine is the same, then the value of the transmitted power at the peak point is reduced for a higher value of the excitation frequency. This can be explained from Eq. (24) where the term $\omega m^s Y_1$ in the denominator increases with increasing excitation frequency, which can be approximately considered as increasing the effect of damping.

Compared with the power flow P^{ws} from the machine to the building floor without spring isolators, it is found that if a very soft spring is provided under the machine, the transmitted power to the floor will be considerably reduced, but it may cause significant vibration velocity of the machine, see Figure 5. If a machine is rigidly bolted to the floor in the infinite plate case, then the vibratory movement of the machine may be reduced, but the transmitted power to the floor will be relatively large. Thus, some compromise must be made between the two requirements, and motivates optimization.



(a)



(b)

Figure 4: Transmitted power for different masses of machine and excitation frequencies: (a) very stiff floor plate, (b) relatively flexible floor plate

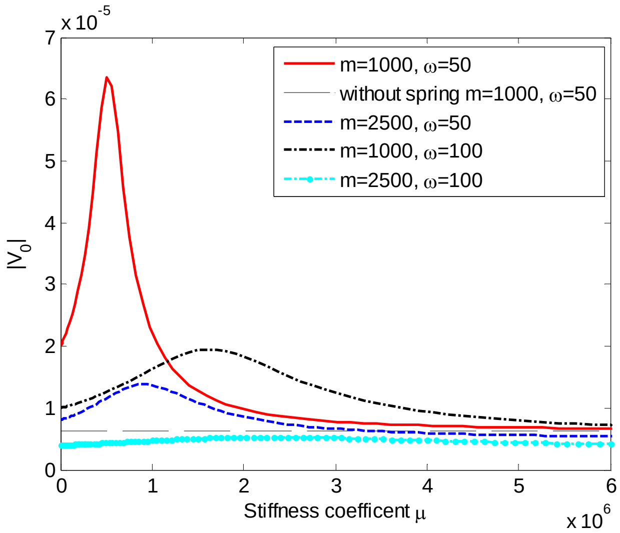


Figure 5: Velocity of machine for different masses of machine and excitation frequencies

4.2 Optimization of the mounting system of a machine with six degrees of freedom subjected to generalized excitation forces

The model shown in Figure 1 is considered for minimizing the power flow. It is now assumed that a machine with six degrees of freedom is excited by a concentrated force vector $\mathbf{F}^s$ acting at its center of gravity. The receiver is modeled as a four-edge simply supported finite plate of uniform thickness 0.1, which is made of the same material as in the previous example. A loss factor $\eta = 0.005$ is introduced for the material.

The stiffness coefficients of four resilient mounts are chosen as design variables, i.e. we have $n^{dv} = 24$ when considering the stiffness components in every direction to be design variables.

The lower and upper limits $\mu_{\min}$ and $\mu_{\max}$ of the stiffness coefficients are given as 10^2 and 10^5, respectively. The units of stiffness coefficients of translational and rotational stiffness components are N/m and Nm/rad. First, a vertical excitation force $\mathbf{F}^s = \{0, 0, 1000, 0, 0, 0\}^T$ is considered. The initial values of all design variables are taken to be 5×10^4, which provides a convenient reference for evaluation and discussion of the vibration reduction by optimization. At the same time, the power flow without resilient mounts is calculated as a reference to evaluate the effect of the isolator. When considering vertical force excitation, the design objective is independent on the stiffness components in the directions of the in-plane and twisting motions. However, the rotational stiffness with respect to the *x* and *y* directions and the vertical translational stiffness will influence the transmitted power.

Excitation frequency ω	Power flow		
	Initial design	Optimized design	Design without resilient mounts
10	1.2134	0.0997	1.3508
20	0.0712	0.0075	0.0776
50	0.2660	5.1457e-005	0.8417
100	0.0312	5.6883e-005	0.0617

Table 1: Optimized result for the case with an excitation force in the *z* direction, only

As stated in Table 1, it is found that, in comparison with the initial design and design without resilient mounts, the optimization has reduced the power flow significantly for four different excitation frequencies, ω = 10, 20, 50, 100.

Next, an excitation force $\mathbf{F}^s = 1/\sqrt{2}\{1000, 0, 1000, 0, 0, 0\}^T$ with a different direction but the same magnitude as above, is considered. When simultaneously considering the vertical and horizontal (in the *x* direction) force excitations, the design objective depends on the vertical translational stiffness, the rotational stiffnesses with respect to the *x* and *y* directions, and the horizontal stiffness with respect to the *x* direction. The same initial design is used.

Excitation frequency ω	Power flow		
	Initial design	Optimized design	Design without resilient mounts
10	0.8528	0.0499	0.7780
20	3.7194	0.0038	2.1498
50	0.1334	2.5622e-05	0.4561
100	0.0157	2.8445e-05	0.0353

Table 2: Optimized result for case with excitation forces in both the *x* and *z* directions

The results shown in Table 2 for this case also illustrate that substantial reduction of the power flow can be achieved for the excitation frequencies considered. When the excitation frequency ω is taken to be 10 or 20, the design without resilient mounts gives relatively lower values of the power flow compared with those of the initial design. This implies that an inappropriately chosen isolator may increase the vibration transmission. The important conclusion that can be drawn from the results reported in Tables 1 and 2 is that design optimization is extremely useful for the best possible selection of an isolator.

5 SHAPE OPTIMIZATION OF A SUPPORTING PAD PLACED ON A FLEXIBLE FLOOR FOR A RESILIENT MOUNT

This section deals with the shape optimization of a rotationally symmetric pad (see Figure 6) which, as illustrated in Figure 7, transfers a vertical force F from a resilient mount to a distributed pressure loading $-f(x)$ on a flexible wooden floor. One- and two-parameter analytical expressions for the distributed pressure loading are derived in Brunskog and Hammer [23] for the problem of pressing a rigid, plane indenter a small uniform distance into the plane surface of an elastic body. For simplicity and in order to apply the result from [23], we shall model the wooden floor as an infinite, isotropic, elastic plate as considered in Section 4.1. Moreover, we assume that the Young's modulus of the pad is large relative to that of the plate, and that the radius r_b of the pad is sufficiently small such that the introduction of the pad does not change the force equilibrium condition $\mathbf{F}^c = -\mathbf{F}^d$ and the motion compatibility condition $\mathbf{V}^c = \mathbf{V}^d$ in Eq. (9).

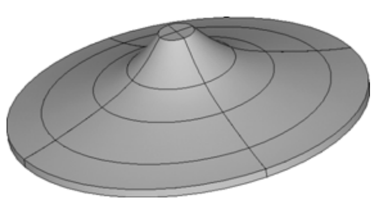


Figure 6: Schematic figure of a rotationally symmetric supporting pad for a resilient mount

The pad is assumed to be a circular truncated cone with a slant boundary subject to design. Due to rotational symmetry of the pad, the design problem can be formulated as a shape optimization problem of the curved part of the plane radial section of the pad shown in Figure 7, where the *y* axis is oriented along the center line, and *x* indicates the radial direction. The design boundary is defined by Spline curves evaluated in terms of the positions of the given points a and b and three master nodes, i.e., node 1, 2 and 3 shown in Figure 7. The coordinates of the master nodes are chosen as design variables. The design domain is meshed by a number of 4-node axisymmetric finite elements.

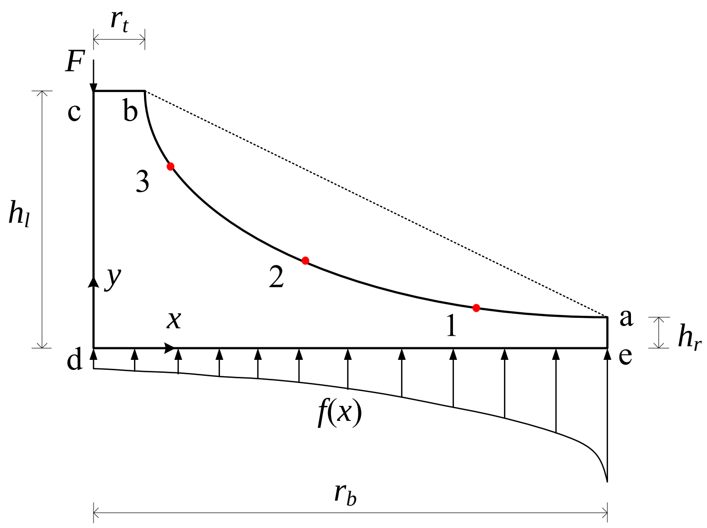


Figure 7: The axisymmetric planar design domain with three master nodes on the design boundary

All dimensions of the design domain are normalized by the radius r_b of the plane bottom surface, i.e., $r_t = \lambda_t r_b$, $h_l = \lambda_l r_b$, $h_r = \lambda_r r_b$.

The shape optimization problem is formulated as

$$\begin{aligned} &\min_{\mathbf{x}} C = \mathbf{P}^T \mathbf{U} \\ &\text{s.t. } V \le \overline{V} \\ &\quad\ \mathbf{x}_L \le \mathbf{x} \le \mathbf{x}_U \end{aligned} \tag{27}$$

where C is the static compliance of the structure. Assuming the point *c* of action of the force *F* to be fixed, the compliance C is defined as the scalar product of the nodal force vector $\mathbf{P}$ and the vector $\mathbf{U}$ of nodal displacements in the *y* direction at the bottom surface of the pad. Moreover, V denotes the volume of the axisymmetric structure, $\overline{V}$ is the given upper limit on the volume, and $\mathbf{x}_L$ and $\mathbf{x}_U$ denote allowable lower and upper limits of the design variables which are the coordinates of three master nodes on the design boundary. The upper limit of the volume is prescribed as $\overline{V}$=0.51 which corresponds to 75% of the volume of the circular truncated cone formed by 360$^\text{o}$ rotation of the area a-b-c-d-e along the *y* axis when $r_b = 1$.

The design variable vector $\mathbf{x}$ is expressed as

$$\mathbf{x} = \{x_1 \quad x_2 \quad x_3 \quad y_1 \quad y_2 \quad y_3\}^T \tag{28}$$

where subscripts 1, 2 and 3 represent node numbers of master nodes on the design boundary.

A numerical example is presented here with the dimensions of design domain taken to be $r_b = 1$, $r_t = 0.1r_b$, $h_l = 0.5r_b$, $h_r = 0.05r_b$. The pressure function is adopted from the paper [23] in the form

$$f(x) = \frac{c}{\sqrt{1 - x^2 + \varepsilon}}, \ c = 1.59 \times 10^4 \tag{29}$$

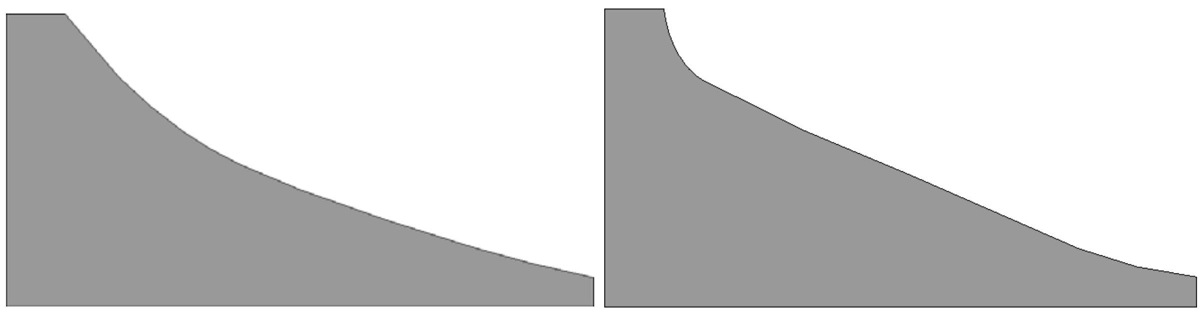
where a small value $\varepsilon = 10^{-6}$ is introduced for avoiding singularity when x equals r_b at the outer edge, and the integral of pressure over the circular bottom surface is obtained as

$$F = 2\pi \int_0^{r_b=1} f(x) x dx = 10^5 \tag{30}$$

The lower and upper limits of design variables are chosen as $0.7 \le x_1 \le 1.0$, $0.4 \le x_2 \le 0.7$, $0 \le x_3 \le 0.4$, $h_b = 0.05 \le y_1 \le 0.2$, $0.2 \le y_2 \le 0.35$, and $0.35 \le y_3 \le 0.5 = h_l$. The initial and final designs are compared in Figure 8. The values of the design variables, volume and compliance of initial and optimized designs are given in Table 3. As a result of the shape optimization, the compliance is reduced from 51.73 to 49.16.

	Initial design	Optimized design
x_1	0.80	0.80
x_2	0.50	0.51
x_3	0.20	0.17
y_1	0.10	0.10
y_2	0.20	0.22
y_3	0.40	0.38
C	51.73	49.16
V	0.50	0.51

Table 3: Comparison of initial and final designs ($\bar{V}$ =0.51)



(a) Initial design (b) Optimized design

Figure 8: Radial sections of initial and optimized pad designs

6 CONCLUSIONS

The problem of optimization of the machinery mounting system in a lightweight building is studied in this paper. A general mobility equation is developed for the system consisting of source, isolator and receiver of vibration by assembling the mobility matrices of each subsystem. The objective of minimizing vibration transmission is realized by sizing optimization of stiffness coefficients of resilient mounts. In comparison with designs without resilient mounts, designs with optimized isolators can provide significant reduction of the power flow.

Shape optimization of a supporting pad for a resilient mount is also studied. Such a pad may be used for transferring a vertical force from a resilient mount to a distributed pressure loading on a flexible floor. Artificial modeling of the resilient mounts and the floor of the building is adopted in this study. Work on more practical modeling of resilient mounts and building structures is in progress.

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