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BAYESIAN SPECTRAL DECOMPOSITION METHOD FOR OPERATIONAL MODAL IDENTIFICATION IN WIRELESS SENSOR NETWORK

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Abstract. *Structural health monitoring (SHM) employing wireless sensor networks (WSN) is becoming increasingly popular in recent years. A Bayesian spectral decomposition (BSD) method employing a distributed computing strategy is presented for structural modal identification in WSN using output-only response data. This method uses the statistical properties of the largest eigenvalue of the output spectral matrix to obtain not only the optimal values of the updated modal frequencies and damping ratios but also their associated uncertainties by calculating the posterior joint probability distribution of these parameters. Mode shapes are obtained by singular value decomposition (SVD) of the output spectral matrix at corresponding discrete frequencies closest to their optimal values. This method identifies each mode, the modal frequency and damping ratio and the mode shape separately, which takes advantage of variable separation and can distribute the computational effort to several computational units, thus becoming suitable for implementation in wireless sensor network that provides such distributed computing environment. In addition, energy is conserved through the use of a novel distributed computing strategy. The efficacy and efficiency of the proposed methodology is demonstrated using numerical simulations.*

1 INTRODUCTION

Experimental modal analysis (EMA) using vibration data has been widely used over the years because of its importance in model updating, response prediction, vibration control and health monitoring. Many modal identification algorithms, both in the time and frequency domain, have been developed in the past three decades. Particularly, much attention has been devoted to the identification of modal parameters in the case where no input but only response measurements are available. These techniques, referred to as operational modal analysis or output-only modal identification or ambient modal identification, provide an in-operation testing solution for modal parameter identification with no need for external artificial excitations and take advantage of the ambient excitations such as micro-tremors, traffic, wind, waves and earthquakes, etc. Operational modal identification is proved to be very useful in civil engineering, where it is very difficult and expensive to excite infrastructures such as buildings and bridges with actuators to obtain artificially induced vibrations.

In recent years, the emerging wireless sensor networks (WSN) for structural health monitoring (SHM) have attracted a lot of attention from both the academic and industrial communities [1]. Wireless sensor networks have the potential to improve SHM dramatically with onboard computation and wireless communication capabilities. Compared with traditional wired structural monitoring systems, wireless sensors can locally process measured data and transmit only the important information through wireless communication, allowing for the distribution of the computation burden across the network. Moreover, there is no extensive wiring between sensors and data acquisition system, which arrows for a fast and flexible implementation, and easier maintenance. Inspired by these advantages, WSN are becoming immensely popular in structural health monitoring in recent years.

Though the wireless sensor network has the potential to improve SHM dramatically, limited resources on wireless sensors preclude direct application of existing algorithms in wireless sensor networks [2]. For example, the wireless sensors have limited computation speed, limited memory space and limited energy powered by batteries. Algorithms designed to be implemented in WSN should take account of such limitations, and it is preferable to process the data in a decentralized way. On the other hand, uncertainties are abundant in civil engineering and statistical methods for modal identification based on output-only measurements have been well developed and have been attracting more attention recently. Statistical methods are very powerful because they explicitly treat uncertainties entering the mathematical models of the structure and the excitations. Bayesian statistical approaches for modal identification have been proposed by Katafygiotis and Yuen [3-4] using ambient data under a Bayesian statistical framework. As a result, they not only obtain the optimal values of the updated parameters by maximizing the posterior probability density function (PDF), but also allow for the quantification of uncertainties associated with the identified parameters of interest. However, the critical issue that always remains is the efficient determination of optimal values and their covariance matrix. Usually, the optimal values are solved by multidimensional numerical optimization and the covariance matrix is determined by finite difference. When the number of identified parameters is moderate to large, which is typical in modal identification of civil infrastructures, numerical optimization in maximizing the posterior PDF is very challenging and requires lots of computation effort to obtain the optimal values, which would preclude such algorithms from being implemented in wireless sensor networks. A method that not only has most of the advantages related to Bayesian statistical algorithms for uncertainty treatment but also reduces much of the computation effort is desirable.

Au (2011) developed a fast Bayesian FFT method for ambient modal identification with separate modes [5], which allows for fast computation of the optimal values and covariance matrix. In this work, we present a Bayesian spectral decomposition (BSD) method for modal identification using ambient data which is based on the statistical properties of the largest eigenvalue of the output spectral matrix. This method identifies each mode, the modal frequency and damping ratio and the mode shape separately. As a result, the number of parameters to be identified by numerical optimization reduces to four for each mode. After the optimal modal frequencies have been obtained, mode shapes are determined by singular value decomposition of the output spectral matrix at corresponding frequencies. Moreover, a distributed computing strategy is proposed for energy conversation in WSN. A numerical example is presented to demonstrate its procedures and features.

2 THEORETICAL ASPECTS

Some of the theoretical aspects related to the presented work are briefly discussed in the following sections.

2.1 Bayesian spectral density method

Let the acceleration time history measured at N_s DOFs of a structure be $\mathbf{Y}_N=\{\mathbf{y}(m)\in\mathbb{R}^{N_s}$, $m=1,\ldots,N\}$, where N is the number of samples per channel. Herein, the measured acceleration is modeled as $\mathbf{y}$(m)=$\mathbf{x}$(m)+$\mathbf{n}$(m), where $\mathbf{x}$(m) is the acceleration response of the structural model defined by a set of model parameters $\boldsymbol{\theta}$, the parameters to be identified; $\mathbf{n}$(m) is the prediction error which accounts for the difference between the model response and measured data, due to measurement noise and modeling error. Based on $\mathbf{Y}_N$ we introduce the following discrete estimator of the spectral density matrix

$$\mathbf{S}_{y,N}(\omega_k)=\mathcal{Y}_N(\omega_k)\mathcal{Y}_N(\omega_k)^H \tag{1}$$

where the superscript H denotes conjugate transpose, $\mathcal{Y}_N(\omega_k)$ denotes the (scaled) Fourier transform of the vector process $\mathbf{y}(t)$ at frequency ω_k, as follows:

$$\mathcal{Y}_N(\omega_k)=\sqrt{\frac{\Delta t}{2\pi N}}\sum_{m=0}^{N-1}\mathbf{y}(m)e^{-j\omega_k m\Delta t} \tag{2}$$

where $j^2=-1$, Δt is the sampling interval, $\omega_k=k\Delta\omega$, $k=0,\ldots,N_1-1$ with $N_1=INT(N/2)$, $\Delta\omega=2\pi/T$, and $T=N\Delta t$. The scaling factor of the FFT in Eq(2) is defined such that the spectral density is two-sided with respect to the circular frequency in rad/s.

For a liner classically damped structure subjected to white noise excitation and independent and identical distributed (i.i.d.) Gaussian prediction error, Katafygiotis and Yuen (2001) derived the PDF for the spectral density matrix and applied it to Bayesian modal identification [3]. Consider a set of independent, identically distributed, time histories $\mathbf{Y}_N^{(1)},\ldots\mathbf{Y}_N^{(M)}$. Assuming that $N\to\infty$, the corresponding Fourier transforms $\mathcal{Y}_N^{(m)}(\omega_k), m=1,\ldots,M$ are independent and follow an identical complex N_s-variate normal distribution with zero mean, and the average spectral density estimate

$$\mathbf{S}_{y,N}^M(\omega_k)=\frac{1}{M}\sum_{m=1}^{M}\mathbf{S}_{y,N}^{(m)}(\omega_k)=\frac{1}{M}\sum_{m=1}^{M}\mathcal{Y}_{y,N}^{(m)}(\omega_k)\mathcal{Y}_{y,N}^{(m)}(\omega_k)^H \tag{3}$$

follows a central complex Wishart distribution of dimension N_s with M degrees of freedom. The PDF of this distribution is given by:

$$p(\mathbf{S}_{y,N}^{M}(\omega_k)) = \frac{\pi^{-N_s(N_s-1)/2} M^{N_s(M-N_s)} \left|\mathbf{S}_{y,N}^{M}(\omega_k)\right|^{M-N_s}}{(\prod_{p=1}^{N_s}(M-p)!) \left|E[\mathbf{S}_{y,N}(\omega_k)]\right|^{M}} \exp(-M\mathrm{tr}[E[\mathbf{S}_{y,N}(\omega_k)]^{-1}\mathbf{S}_{y,N}^{M}(\omega_k)]) \quad (4)$$

where $|A|$, tr[A] and E[A] denote the determinant, the trace and the expectation, respectively, of a matrix A. Note that this PDF exists only when $M \geq N_s$. Furthermore, it can be shown that in the limit when $N \to \infty$ the matrices $\mathbf{S}_{y,N}^{M}(\omega_k)$ and $\mathbf{S}_{y,N}^{M}(\omega_l)$ are independently Wishart distributed for $k \neq l$.

On the other hand, for a high sampling rate and long duration of data, the term $E[\mathbf{S}_{y,N}(\omega_k)]$ in Eq(4) can be expressed as

$$E[\mathbf{S}_{y,N}(\omega_k)] = \mathbf{\Phi}\mathbf{H}_k\mathbf{\Phi}^T + \mathbf{S}_{n0} \quad (5)$$

where $\mathbf{\Phi} \in \mathbb{R}^{N_s \times N_m}$ is the mode shape matrix confined to the measured DOFs (the i-th column corresponds to the i-th mode shape); $\mathbf{S}_{n0}$ is the spectral density matrix (constant) of the prediction error; $\mathbf{H}_k$ is the spectral density matrix of the modal response with (i,j) element given by

$$H_k(i,j) = \frac{S_{ij}\omega_k^4}{[(\omega_i^2-\omega_k^2)+2j\omega_k\omega_i\zeta_i][(\omega_j^2-\omega_k^2)-2j\omega_k\omega_j\zeta_j]} \quad (6)$$

It should be noted that this is only valid for acceleration response, and the exponential of ω_k in the numerator will be the values 0 or 2 when the response corresponds to displacement or velocity, respectively.

In the context of modal identification the set of modal parameters $\boldsymbol{\theta}$ consists of the natural frequencies, damping ratios, mode shapes, spectral density matrix of modal excitations and spectral density of the prediction error. Assuming a non-informative prior distribution, the posterior PDF of $\boldsymbol{\theta}$ given the spectral density data is proportional to the likelihood function

$$p(\boldsymbol{\theta}|\mathbf{S}_{y,N}^{M,k_1,k_2}) \propto p(\mathbf{S}_{y,N}^{M,k_1,k_2}|\boldsymbol{\theta}) \simeq c_1 \prod_{k=k_1}^{k_2} \frac{\left|\mathbf{S}_{y,N}^{M}(\omega_k)\right|^{M-N_s}}{\left|E[\mathbf{S}_{y,N}(\omega_k)]\right|^{M}} \exp(-Mtr[E[\mathbf{S}_{y,N}(\omega_k)]^{-1}\mathbf{S}_{y,N}^{M}(\omega_k)]) \quad (7)$$

where c_1 is a normalizing constant.

The most probable parameters $\hat{\boldsymbol{\theta}}$ are obtained by minimizing the log-likelihood function $L(\boldsymbol{\theta})$

$$p(\boldsymbol{\theta}|\mathbf{S}_{y,N}^{M,k_1,k_2}) \propto \exp[-L(\boldsymbol{\theta})] \quad (8)$$

where

$$L(\boldsymbol{\theta}) = M\sum_{k=k_1}^{k_2}\left[\ln\left|E[\mathbf{S}_{y,N}(\omega_k)]\right| + tr[E[\mathbf{S}_{y,N}(\omega_k)]^{-1}\mathbf{S}_{y,N}^{M}(\omega_k)]\right] \quad (9)$$

Furthermore, with a sufficient large amount of data, the posterior PDF of the parameters $\boldsymbol{\theta}$ can be well-approximated by a Gaussian distribution $N(\hat{\boldsymbol{\theta}}, \mathbf{H}^{-1}(\hat{\boldsymbol{\theta}}))$ with mean $\hat{\boldsymbol{\theta}}$ and covariance matrix $\mathbf{H}^{-1}(\hat{\boldsymbol{\theta}})$, where $\mathbf{H}(\hat{\boldsymbol{\theta}})$ denotes the Hessian of $L(\boldsymbol{\theta})$ calculated at $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$ .Consider a second order expansion for the log-likelihood function

$$L(\boldsymbol{\theta}) \simeq L(\hat{\boldsymbol{\theta}}) + \mathbf{J}(\hat{\boldsymbol{\theta}})(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}) + \frac{1}{2}(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})^T \mathbf{H}(\hat{\boldsymbol{\theta}})(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}) \quad (10)$$

Note that the first term is a constant, and the second term vanishes since the gradient vector $\mathbf{J}(\hat{\boldsymbol{\theta}})$ equals to zero due to the minimization nature. Substituting Eq.(10) into Eq.(8), the posterior PDF can be approximated by a Gaussian distribution

$$p(\boldsymbol{\theta}|\mathbf{S}_{y,N}^{M,k_1,k_2}) \propto \exp[-\frac{1}{2}(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})^T \mathbf{C}(\hat{\boldsymbol{\theta}})^{-1}(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})] \quad (11)$$

where $\mathbf{C}(\hat{\boldsymbol{\theta}}) = \mathbf{H}(\hat{\boldsymbol{\theta}})^{-1}$ is the posterior covariance matrix, inverse of the Hessian matrix.

The critical issue that remains is the efficient determination of the most probable parameters and their covariance. In the original method, the optimal values are solved by numerical optimization and the Hessian is determined by finite difference. In particular, the objective function is a nontrivial nonlinear function of the modal parameters, so the determination of most probable parameters is computationally very demanding. In addition, the inverse of $E[\mathbf{S}_{y,N}(\omega_k)$ is ill-conditioned in a resonance frequency band. If all the most probable parameters were to be found by numerical optimization then the computational effort grows with the number of measured DOFs N_s and the number of contributed modes N_m. By noting the symmetry of the spectral density matrix of modal excitations and the normalization characteristic of mode shapes, there are $2N_m + N_m(N_s-1) + N_m(N_m+1)/2 + N_s = O(N_m^2 + N_mN_s)$ parameters to be identified. The growth of the dimension with N_s is a major issue because it can be moderate or large in typical civil engineering applications (e.g., $N_s>10$). The growth of the dimension with N_m, although quadratic, is not significant because one can focus on a resonance frequency band dominated by a small number of modes. For well-separate modes one can identify each mode separately and $N_m=1$ in this case.

In the next section we shall analyze the statistical property of the largest eigenvalue of the output spectral matrix and utilize it to effectively determine the optimal values of modal frequency and damping ratio and their covariance matrix when the structure has separated modes.

2.2 Modal identification with single mode

Assume that in a resonance frequency band the response is dominated by a single mode and only the spectral density data in this band are used for modal identification. For simplification, the spectral density of the prediction error is assumed to be the same for all measured DOFs. In this case, the parameters $\boldsymbol{\theta}$ consist of the natural frequency ω, damping ratio ζ, mode shape $\Phi \in \mathbb{R}^{N_s}$, spectral density S_f of the modal excitation and the prediction error S_n. Here the modal index is omitted for simplification. It is assumed that the mode shape is normalized to have unit norm, i.e., $\|\Phi\|^2 = 1$.

We start by examming the mathematical structure of $E[\mathbf{S}_{y,N}(\omega_k)]$ in Eq.(5). In this case,

$$E[\mathbf{S}_{y,N}(\omega_k)] = \alpha_k \Phi\Phi^T + S_n \mathbf{I}_{N_s} \quad (12)$$

where

$$\alpha_k = \frac{S_f}{(\beta_k^2-1)^2 + (2\zeta\beta_k)^2} = S_f A_k \quad (13)$$

where $\beta_k = \omega/\omega_k$, and $A_k = 1/[(\beta_k^2 - 1)^2 + (2\zeta\beta_k)^2]$ is the modal dynamic amplification. To facilitate deriving the analytical form of largest eigenvalue, we shall express $E[\mathbf{S}_{y,N}(\omega_k)]$ in a more suitable form. The key is to express $E[\mathbf{S}_{y,N}(\omega_k)]$ via a suitable eigendecomposition. Define a set of orthonormal bases E = $\{e_j \in \mathbb{R}^{N_s}$: j = 1, …, $N_s\}$, where $e_1 = \Phi \in \mathbb{R}^{N_s}$ and $\{e_2, e_3, \ldots, e_{N_s}\}$ form an orthonormal basis in the complement subspace. Using this basis, the identity matrix can be represented as $\mathbf{I}_{N_s} = \sum_{j=1}^{N_s} e_j e_j^T$, so the Eq.(12) can be rewritten as

$$E[\mathbf{S}_{y,N}(\omega_k)] = \alpha_k e_1 e_1^T + S_n \sum_{j=1}^{N_s} e_j e_j^T = (\alpha_k + S_n) e_1 e_1^T + S_n \sum_{j=2}^{N_s} e_j e_j^T \tag{14}$$

The eigenvalues of $E[\mathbf{S}_{y,N}(\omega_k)]$ are $\alpha_k + S_n$, S_n, …, S_n, with corresponding eigenvector $e_1, e_2, \ldots, e_{N_s}$, and the largest eigenvalue d_k of $E[\mathbf{S}_{y,N}(\omega_k)]$ is

$$d_k = \alpha_k + S_n = \frac{S_f}{(\beta_k^2 - 1)^2 + (2\zeta\beta_k)^2} + S_n \tag{15}$$

A considerable amount of mathematical effort has been devoted on finding distributions for the eigenvalues of a Wishart matrix. Let s_k be the largest eigenvalue of $\mathbf{S}_{x,N}^M(\omega_k)$. According to Ref. [6], the largest eigenvalue s_k are asymptotically independently normally distributed with

$$E(s_k) = d_k, \quad \mathrm{var}(s_k) = \frac{2d_k^2}{M} \tag{16}$$

In the context of modal identification, assuming a non-informative prior distribution, the posterior PDF of $\boldsymbol{\theta}$ given the eigenvalue data is proportional to the likelihood function

$$p(\boldsymbol{\theta} | \mathbf{s}_k^{k_1,k_2}) \propto p(\mathbf{s}_k^{k_1,k_2} | \boldsymbol{\theta}) \simeq \prod_{k=k_1}^{k=k_2} \frac{\sqrt{M}}{\sqrt{4\pi} d_k} \exp\left(-\frac{M(s_k - d_k)^2}{4d_k^2} \right) \tag{17}$$

In the context of wireless sensor network, in order to utilize the distributed computing capacity, the whole network may be divided into n clusters of sensors. In this case, each cluster can determine a posterior PDF based on its own data, and n sets of posterior PDF will be obtained. The fusion of all the information obtained by various clusters can be conducted in Bayesian manner. Take one cluster as a priori, continuously include the data in other clusters, then the final posterior PDF $p(\boldsymbol{\theta} | \mathbf{s}_k^{n,k_1,k_2})$ will be a product of the n posterior PDFs corresponding to all clusters,

$$p(\boldsymbol{\theta} | \mathbf{s}_k^{n,k_1,k_2}) \propto p(\mathbf{s}_k^{n,k_1,k_2} | \boldsymbol{\theta}) \simeq \prod_n \prod_{k=k_1}^{k=k_2} \frac{\sqrt{M}}{\sqrt{4\pi} d_k} \exp\left(-\frac{M(s_k - d_k)^2}{4d_k^2} \right) \tag{18}$$

The most probable parameters $\hat{\boldsymbol{\theta}}$ are obtained by maximizing the posterior PDF $p(\boldsymbol{\theta} | \mathbf{s}_k^{n,k_1,k_2})$. This is equivalent to minimizing $L(\boldsymbol{\theta}) = -\ln[p(s_k^{n,k_1,k_2} | \boldsymbol{\theta})]$. Various optimization algorithms can be employed to minimize $L(\boldsymbol{\theta})$ and obtain the optimal parameters $\hat{\boldsymbol{\theta}}$, and then central difference or analytical formulas can be used to calculate the Hessian matrix $\mathbf{H}(\hat{\boldsymbol{\theta}})$.

Furthermore, it is found that the updated PDF of the parameters $\boldsymbol{\theta}$ can be well approximated by a Gaussian distribution $N(\hat{\boldsymbol{\theta}}, \mathbf{H}^{-1}(\hat{\boldsymbol{\theta}}))$ with mean $\hat{\boldsymbol{\theta}}$ and covariance matrix $\mathbf{H}^{-1}(\hat{\boldsymbol{\theta}})$.

2.3 Mode shape identification and assembling

The mode shapes can be identified as follows: since the modal frequency ω is identified in the previous section, the nearest discrete frequency $\omega_k \to \omega$ can be obtained. From Eq. (14), we know that the corresponding singular vector $\mathbf{u}_k$ is an estimate of the r^{th} mode shape Φ with unitary normalization. The principle for mode shape identification is the same as that of FDD method [9].

The mode shapes extracted from a particular cluster of sensors are referred to as local mode shapes. The local mode shapes have to be rescaled and assembled to global mode shapes. Consider the global mode shape Φ^{Ω} for the r^{th} mode, along with local mode shapes Φ^{Ω_1}, $\Phi^{\Omega_2}, \ldots \Phi^{\Omega_n}$ associated with respective clusters of sensors $\Omega_1, \Omega_2, \ldots \Omega_n$. The local mode shapes Φ^{Ω_i} and Φ^{Ω_j} associated with two neighboring clusters with overlapped sensors can be expressed as

$$\Phi^{\Omega_i} = \begin{pmatrix} \phi_{i1} \\ \phi_{i2} \\ \vdots \\ \phi_{ip} \\ \phi_{i1} \\ \vdots \\ \phi_{io} \end{pmatrix} \text{ and } \Phi^{\Omega_j} = \begin{pmatrix} \phi_{j1} \\ \phi_{j2} \\ \vdots \\ \phi_{jq} \\ \phi_{j1} \\ \vdots \\ \phi_{jo} \end{pmatrix} \tag{19}$$

where o is the number of overlapping nodes, and p and q are the number of non-overlapping nodes in the i^{th} and j^{th} clusters, respectively. To allow assembly, the mode shapes in Equation (19) should be rescaled to have the same values at the overlapping nodes, i.e.

$$\begin{pmatrix} \phi_{i1} \\ \vdots \\ \phi_{io} \end{pmatrix} = R_j \begin{pmatrix} \phi_{j1} \\ \vdots \\ \phi_{jo} \end{pmatrix} \tag{20}$$

where R_j is a rescaling factor for the mode shape Φ^{Ω_j}. The global mode shape is the union of the local mode shapes as

$$\Phi^{\Omega} = \bigcup_{i=1}^{n} R_i \Phi^{\Omega_i} \tag{21}$$

In the presence of noise, the rescaling factors to Equation (20) for any $o>1$ does not exist in general. Therefore, the rescaling factors R_i (i=1,2,…n) must be approximately determined, for example as a solution in the least-square sense [7-8]. Using the rescaling factors, the local mode shapes are scaled and assembled to obtain the global mode shape. At the overlapping nodes, the local mode shapes are averaged to obtain the associated values of the global mode shape.

3 APPLICATION IN WIRELESS SENSOR NETWORKS

In this section, a distributed computing strategy is adopted to implement this algorithm. This distributed computing strategy can reduce the data amount of wireless transmission and take advantage of decentralized computing capacity of the wireless sensor networks (WSN). Because wireless communication often consumes more energy than other parts, algorithms which require transmission of long time history records should be avoided. Pre-processing data locally will not only make it possible to transmit smaller amount of important information but also take advantage of the autonomous computing capacity of smart sensors. A two-level hierarchical architecture is proposed for the application of WSN for Bayesian modal identification. There are three types of sensors classified by their functions in the network: gateway node, cluster head nodes and leaf nodes. The schematic network architecture is shown in Figure1.

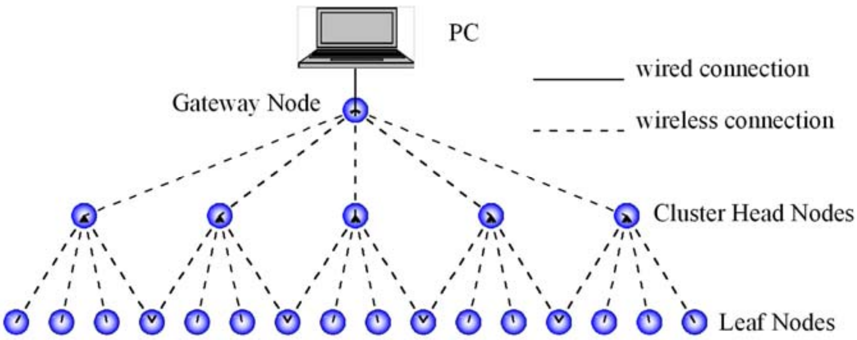

Figure 1. Two-level hierarchical architecture of WSN

Both the leaf nodes and cluster head nodes perform data acquisition, and each node calculates the FFT and then the auto-spectral density. Using Peak-Picking method, the potential modal frequencies can be obtained. Once all the local modal frequencies from leaf nodes and cluster head nodes are collected centrally in the gateway node, the first task is to pick out the true modes. The true modes should be identified obviously in the majority of clusters, while the noise modes will randomly appear. Thus, if a specific natural frequency is identified in a substantial number of clusters, it is considered as a true mode. FFT data around these modal frequencies are sent to cluster head nodes, and then cross-spectrum around these frequencies can be calculated in the cluster head node. In the cluster head node, SVD is performed in the vicinity of the modes to be identified, and then corresponding singular values (eigenvalues) are sent to the gateway node. Using Bayesian inference, modal frequencies and damping ratios together with their uncertainties are obtained in the gateway node and then the gateway node returns modal frequencies to each cluster head node. Thus, local mode shapes in each cluster can be obtained using SVD at the corresponding closest discrete frequencies. Finally, each cluster head node reports local mode shapes to the gateway node. A global mode shape is assembled in the gateway node by comparing the overlapping nodes in each cluster. After that, finalized modal parameters and their uncertainties are sent to PC via USB cable.

4 NUMERICAL EXAMPLE

In this example, we use simulated data from an 8-story shear building shown in Figure 2. It is assumed that this building has a uniformly distributed floor mass m=1×10^{5}kg and interstorey stiffness k=2.5×10^{8}N/m, and Rayleigh's damping with corresponding damping ratios $\zeta_1 = \zeta_2 = 1\%$ is assumed. The structure is assumed to be subjected to a base acceleration given by stationary Gaussian white noise with a spectral density of $0.0025\, m^2 s^{-3} rad^{-1}$. It is assumed that the accelerations at each floor were measured using a sampling interval ΔT=0.01sec. The time duration for one data set is 1000s and 10 data sets are collected. The measurement noise at different sensors is assumed to be i.i.d. Gaussian white noise with RMS=0.1 ms^{-2}. Therefore, the spectral density estimation calculated by Eq. (3) follows a Wishart distribution with M=10 degrees of freedom and their eigenvalues asymptotically follows a Gaussian distribution.

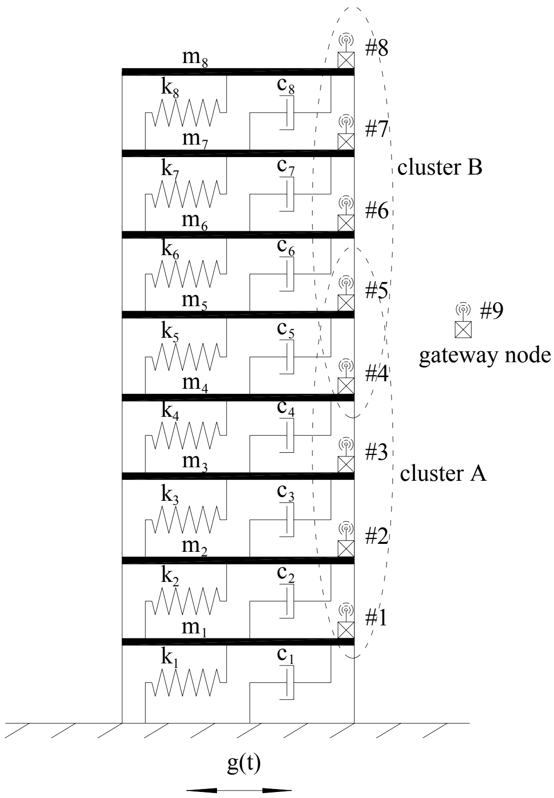

Figure 2. 8-storey shear building

The measurements can be taken by wireless sensors, which are grouped into two clusters: cluster A and cluster B. Cluster A consists of sensor #1, #2, #3, #4 and #5, one of which (excluding the overlapping nodes) is served as cluster head. Cluster B consists of sensor #4, #5, #6, #7 and #8, one of which (excluding the overlapping nodes) serves as cluster head. The overlapping nodes for these two clusters are sensor #4 and #5, which are used for global mode shape assembling. From the auto-spectrum in each sensor node, we know that there are three significant modes.

Table 1 shows the identified results from the output measurements. It shows the estimated optimal values $\hat{\boldsymbol{\theta}}$, the calculated standard deviations σ, coefficient of variance (COV), the value of a 'normalized distance' β. The parameter β represents the absolute value of difference between the identified optimal and actual value, normalized with respect to the corresponding calculated standard deviation. Thus, β expresses how many standard deviations away the identified value of a given parameter is from the target value. It can also be seen that the COV of damping is larger than that of natural frequency, which means it contains more uncertainty.

The mode shapes are identified using SVD of the output spectral matrix at the nearest discrete frequencies, and the corresponding singular vector $\mathbf{u}_k$ is an estimate of the r^{th} mode shape Φ with unitary normalization. The global mode shapes are assembled using least square method. The identified results are tabulated in table 2. The modal assurance criterion (MAC) values between theoretical values and identified ones are listed in the end column. The MAC values shows that the identified results closely match the theoretical ones.

	Actual θ	Optimal $\hat{\theta}$	S.D. σ	C.O.V	$\beta = \lvert\theta - \hat{\theta}\rvert / \sigma$
ω_1	6.2851	6.2831	0.0022	0.0004	0.8948
ω_2	18.6413	18.649	0.0064	0.0003	1.2129
ω_3	30.3626	30.312	0.0095	0.0003	5.3203
ζ_1	0.01	0.0102	0.0004	0.0416	0.4323
ζ_2	0.01	0.0104	0.0013	0.123	0.328
ζ_3	0.0137	0.0134	0.001	0.0763	0.3528

Table 1. Identified modal frequencies and damping ratios

mode	DOF	#1	#2	#3	#4	#5	#6	#7	#8	MAC
1st	Th*	-0.0891	-0.1752	-0.2554	-0.3268	-0.3871	-0.4342	-0.4666	-0.483	1.0000
	Id*	-0.0891	-0.1754	-0.2554	-0.3267	-0.3872	-0.4341	-0.4664	-0.4831	
2nd	Th	-0.2554	-0.4342	-0.483	-0.3871	-0.1752	0.0891	0.3268	0.4666	0.9999
	Id	-0.2568	-0.4363	-0.4862	-0.3893	-0.1781	0.0849	0.3217	0.4619	
3rd	Th	-0.3871	-0.4666	-0.1752	0.2554	0.483	0.3268	-0.0891	-0.4342	0.9977
	Id	-0.3998	-0.4739	-0.177	0.2726	0.4894	0.3283	-0.0683	-0.3976	

Note: * "Th" & "Id" denotes theoretical and identified values, respectively.

Table 2. Identified mode shapes

5 CONCLUSIONS

A Bayesian spectral decomposition (BSD) method for identifying modal parameters using output-only data is presented. This method takes advantage of the Bayesian spectral density approach (BSDA) and frequency domain decomposition (FDD) method, and avoids their limitations. Comparing with BSDA, this method reduces the dimension of identified parameters in the optimization problem to only four for each mode. It can identify each mode separately, which renders the method suitable for implementation in wireless sensor networks. Comparing with FDD, this method considers the uncertainty in a Bayesian statistical framework. It avoids the subjective peak-picking procedures for modal frequency identification and the accuracy of damping ratio is also remarkable. It obtains not only the optimal values of the modal frequencies and damping ratios but also their associated uncertainties by calculating the posterior joint probability of these parameters. The quantification of these uncertainties is very important when one plans to use modal parameters estimates for further processing.

The proposed method along with the distributed computing strategy may be suitable to be implemented in wireless sensor networks by utilizing the autonomous computing capacity of each wireless sensor. The proposed distributed computing strategy can reduce the amount of wireless communication and thus conserve the energy in an efficient manner.

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