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SEISMIC STRUCTURAL ANALYSIS USING INTERVAL RESPONSE SPECTRUM

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Abstract. *Seismic analysis is an essential procedure to design a structure subjected to a system of suddenly applied loads due to earthquake excitations. However, throughout conventional seismic analyses, the structure is subjected to a limited number of recorded earthquake excitations. Moreover, the presence of variations and uncertainties in the recorded excitations from different earthquakes is not considered in current seismic analysis procedures. One of the methods to quantify the impreciseness and uncertainty is the interval or unknown-but-bounded representation.*

In this work, a new computationally feasible method for seismic structural analysis with interval uncertainty in the response spectrum is developed, which is capable of obtaining the bounds on the structure's dynamic response. Using this method, first, the response spectra from various recorded earthquakes are combined as an interval function and the interval earthquake response spectrum is constructed. Then, using the developed interval function, the response spectrum analysis is performed and the structure's dynamic response is obtained. This method shows that calculating the bounds on the dynamic response does not require a Monte-Carlo simulation procedure. An example problem that illustrates the developed algorithm with comparison to the different solutions is presented.

1 INTRODUCTION

In structural engineering, design of an engineered system requires that the performance of the system is guaranteed over its lifetime. In the case where the structure is subjected to loads induced by earthquake excitations, seismic analysis is a crucial procedure for reliable design. Using conventional seismic analysis procedures, the structure is subjected to a limited number of recorded earthquake excitations. However, the conventional procedures do not simultaneously consider the presence of variations and uncertainties in the recorded excitations in the analysis.

Treating uncertainty in the seismic analysis of a structure requires two major considerations: first, quantification of variations and uncertainties in the earthquake response spectra; and second, development of schemes that are capable of considering the presence of uncertainty throughout the solution process. Those developed schemes must be consistent with the structure's physical behavior and must also be computationally feasible.

The set-theoretic (unknown-but-bounded), or interval representation of vagueness is one possible method to quantify the uncertainty. The interval representation of uncertainty in the parametric space has been motivated by the lack of detailed probabilistic information on possible distributions of parameters and computational issues in obtaining solutions.

In this work, a new method for seismic structural analysis with interval uncertainty in the response spectrum is developed which is capable of bounding the structure's dynamic response. Using this method, first, the response spectra from various recorded earthquakes are combined as an interval function (with respect to frequency or period) and the interval earthquake response spectrum is constructed. Then, response spectrum analysis is performed using the constructed interval function to obtain the structure's upper-bounds of dynamic response.

This work represents the synthesis of two historically independent fields, seismic structural analysis and interval analysis. In order to represent the background for this work, a review of development of both fields is presented. First, a background and the analytical procedure for seismic structural analysis are presented. Next, a background and fundamentals of interval uncertainty analysis are presented. Following that, the new method for seismic analysis with response spectrum is introduced. Next, the upper-bounds on dynamic response of the structure are determined. Finally, exemplar and numerical results are presented that are followed by observations and conclusions.

2 DETERMINISTIC SEISMIC STRUCTURAL ANALYSIS

2.1 Historical background

Modern theories of structural dynamics were introduced mostly in mid 20^{th} century. Biot (1932) [1] introduced the concept of earthquake response spectra and Housner (1941) [2] was instrumental in the widespread acceptance of this concept as a practical means of characterizing ground motions and their effects on structures. Newmark (1952) [3] introduced computational methods for structural dynamics and earthquake engineering. Anderson (1952) [4] developed methods for considering the effects of lateral forces on structures induced by earthquake and wind. Also, Hudson (1956) [5] developed techniques for response spectrum analysis in engineering seismology. Veletsos (1957) [6] determined natural frequencies of continuous flexural members. Rosenblueth (1959) [7] introduced methods for combining modal responses and characterizing earthquake analysis. Biggs (1964) [8] developed dynamic analyses for structures subjected to blast loads. Moreover, Penzien and Clough (1993) [9] further developed numerical methods for dynamics of structures and modal analysis.

2.2 Review of response spectrum analysis

The method of response spectrum analysis for computing the dynamic response of a multiple degree-of-freedom structure to a system of dynamic loads can be sequenced as following:

1. Define the structural properties.
   - Determine the stiffness matrix $[K]$ and mass matrix $[M]$.
   - Assume the modal damping ratio ζ_n.
2. Perform a generalized eigenvalue problem between the stiffness and mass matrices.
   - Determine natural circular frequencies (ω_n).
   - Determine mode shapes $\{\varphi_n\}$.
3. Compute the maximum modal response.
   - Determine the maximum modal coordinate $D_{n,\max}$ using the excitation response spectrum for the corresponding natural circular frequency and modal damping ratio.
   - Determine the modal participation factor Γ_n.
   - Compute the maximum modal response as a product of maximum modal coordinate, modal participation factor and mode shape.
4. Combine the contributions of all maximum modal responses to determine the maximum total response using square root of sum of squares (SRSS) or other combination methods.

3 INTERVAL ANALYSIS

3.1 Historical background

The concept of representation of an imprecise real number by its bounds is quite old. In fact, Archimedes (287-212 B.C.) [10] defined the irrational number (π) by an interval, $(3+10/71<\pi<3+1/7)$ which he found by approximating the circle with the inscribed and circumscribed 96-side regular polygons. The introduction of digital computers in the 1950's provided impetus for further interval analysis as discrete representations of real numbers with associated truncation error. Interval mathematics was further developed by Sunaga (1958) [11] who introduced the theory of *interval algebra* and its applications in numerical analysis. Also, Moore (1966) [12] introduced *interval analysis*, interval vectors and interval matrices as a set of techniques that provides error analyses for computational results.

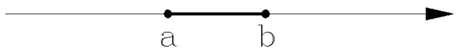
Interval analysis provides a powerful set of tools with direct applicability to important problems in scientific computing. Alefeld and Herzberger (1983) [13] presented an extensive treatment of interval linear and non-linear algebraic equations and interval methods for systems of equations. Moreover, Neumaier (1990) [14] investigated the methods for solution of interval systems of equations.

The concept of interval systems has been further developed in analysis of structures with interval uncertainty. Muhanna and Mullen (1999) [15] developed fuzzy finite-element methods for solid mechanics problems. For the solution of interval finite element method (IFEM) problems, Muhanna and Mullen (2001) [16] introduced an Element-by-Element interval finite element formulation, in which a guaranteed enclosure for the solution of interval linear systems of equations was achieved. The research in interval eigenvalue problem began to emerge as its wide applicability in science and engineering was realized. Modares and Mullen (2004) [17] introduced a method for the solution of the parametric interval eigenvalue problem resulting from semi-discretization of structural dynamics which determines the exact bounds of the natural frequencies of a structure with uncertainties in their mechanical characteristics.

3.2 Interval (convex) variable

A real interval is a closed set defined by extreme values as (Figure 1):

$$\tilde{Z} = [z^l, z^u] = \{z \in \Re \mid z^l \leq z \leq z^u\} \tag{1}$$



$$\tilde{x} = [a, b]$$

Figure 1: An interval quantity.

One interpretation of an interval number is a random variable whose probability density function is unknown but non-zero only in the range of interval. Another interpretation of an interval number includes intervals of confidence for α -cuts of fuzzy sets.

This interval representation transforms the point values in the deterministic system to inclusive set values in the system with bounded uncertainty. Interval arithmetic is a computational tool that can be used to represent uncertainty as 1) a set of probability density functions, 2) In Dempster-Shafer models for epistemic probability and, 3) α - cuts in fuzzy sets, etc. In this work, the symbol (~) represents an interval quantity.

3.3 Interval arithmetic operations

Considering $\tilde{X} = [a, b]$ and $\tilde{Y} = [c, d]$ as two interval numbers, the basic interval arithmetic operations are:

Addition:
$$\tilde{X} + \tilde{Y} = [a + c, b + d] \tag{2}$$

Subtraction:
$$\tilde{X} - \tilde{Y} = [a - d, b - c] \tag{3}$$

Multiplication:
$$\tilde{X} \times \tilde{Y} = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)] \tag{4}$$

Division:
$$\frac{\tilde{X}}{\tilde{Y}} = [a, b] \times [\frac{1}{d}, \frac{1}{c}], (0 \notin [c, d]) \tag{5}$$

Interval Vector (2-D):
$$\tilde{V} = \begin{Bmatrix} \tilde{X} \\ \tilde{Y} \end{Bmatrix} = \begin{Bmatrix} [a, b] \\ [c, d] \end{Bmatrix} \tag{6}$$

which represents a “box” in 2-D space as the enclosure (Figure 2).

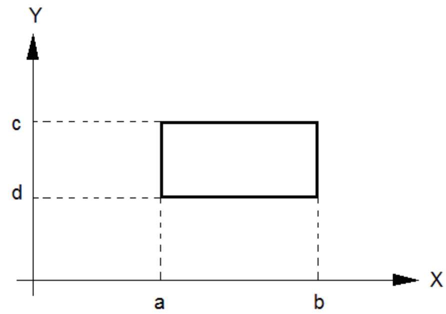


Figure 2: An interval vector

3.4 Functional dependency and sharpness considerations of interval operations

Considering $\tilde{X} = [-2,2]$ and $\tilde{Y} = [-2,2]$ as two independent interval numbers, the functional dependent interval multiplication results in:

$$\tilde{X} \times \tilde{X} = [0,4]$$

In contrast, the functional independent interval multiplication results in:

$$\tilde{X} \times \tilde{Y} = [-4,4]$$

In interval operations, the functional dependency of intervals must be considered in order to attain sharper results. In fact, the issue of sharpness and overestimation in interval bounds is the key limitation in the application of interval methods. Naïve implementation of interval arithmetic algorithms (substituting interval operations for their scalar equivalence) will yield bounds that are not useful for engineering design. Therefore, there is a need to develop algorithms to calculate sharp or nearly sharp bounds to the underlying set theoretic interval problems.

4 SEISMIC ANALYSIS USING INTERVAL RESPONSE SPECTRUM

The developed method for seismic analysis of structures using interval response spectrum enhances the conventional dynamic response spectrum analysis by: a) introducing an interval response spectrum constructed from a cluster of recorded excitation and, b) obtaining the bounds on the structure's dynamic response. The method is composed of following steps:

1. Define the physical or geometrical characteristics.
   - Determine the stiffness matrix $[K]$ and mass matrix $[M]$.
   - Assume the modal damping ratio ζ_n.

2. Perform an interval eigenvalue problem between the stiffness and mass matrices.
   - Determine the natural circular frequencies ω_n.
   - Determine the mode shapes $\{\varphi_n\}$.

3. Construct interval response spectrum.
   - Create a cluster of response spectra obtained from various recorded earthquakes.
   - Determine interval response spectrum as an interval function of excitation frequency.

4. Compute the interval modal response.
   - Determine the interval modal coordinate $\tilde{D}_n$ using the interval response spectrum for the corresponding natural circular frequency and assumed modal damping ratio.
   - Determine the modal participation factor Γ_n.
   - Compute the interval modal response as the product of interval modal coordinate, modal participation factor and mode shape.

5. Combine and maximize the contributions of all interval modal responses to determine the maximum total response using SRSS or other combination methods.

4.1 Interval Response Spectrum

In order to construct interval response spectrum, first, a cluster of response spectra obtained from various recorded earthquake excitations is combined. Then, by enclosing all the response spectra, an interval response spectrum function is determined which is an interval function of frequency or period. Moreover, additional uncertainties in response spectra can also be considered. Figure 3 depicts an example for an interval response spectrum.

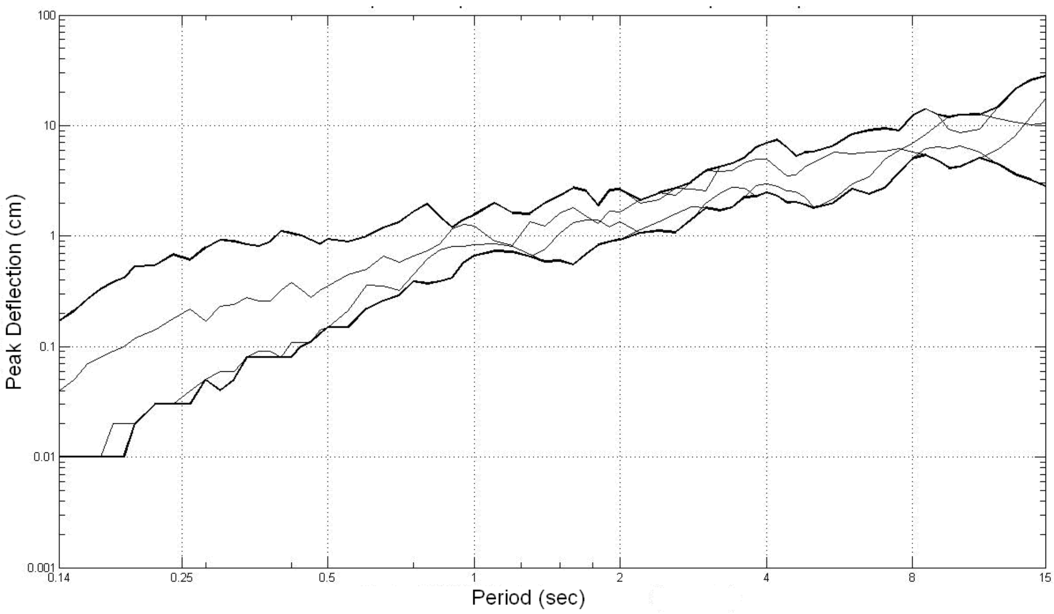


Figure 3: An interval response spectrum

4.2 Bounding Dynamic Response

The interval modal coordinate $\tilde{D}_n$ is determined using the interval response spectrum evaluated for the corresponding natural circular frequency ω_n and assumed modal damping ratio (Figure 4).

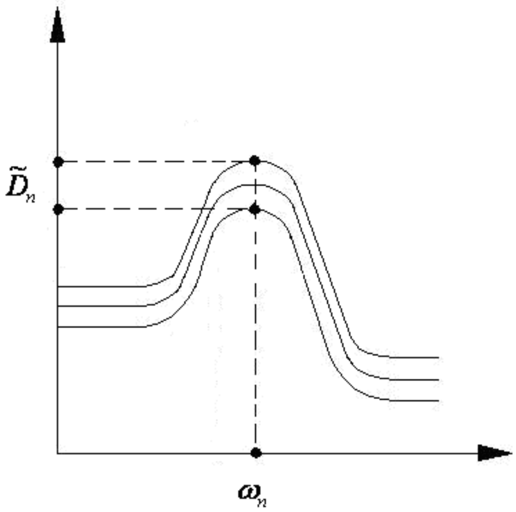


Figure 4: Determination of $\tilde{D}_n$ corresponding to a ω_n from an interval response spectrum.

If excitation is proportional, the modal participation factor is obtained as:

$$\Gamma_n = \frac{\{\varphi_n\}^T [M]\{r\}}{\{\varphi_n\}^T [M]\{\varphi_n\}}$$
$$\{r\} = \{1 \quad 1 \quad \dots \quad 1\}^T \tag{7}$$

The interval modal response is determined as the product of interval modal coordinate, modal participation factor and mode shape as:

$$\{\widetilde{U}_n\} = (\widetilde{D}_n)(\Gamma_n)\{\varphi_n\} \tag{8}$$

Finally, the contributions of all interval modal responses are combined and maximized to determine the maximum total response using SRSS or other combination methods as:

$$\{U_{\max}\} = \max\left(\sqrt{\sum_{n=1}^{N}\{\widetilde{U}_n^2\}}\right) \tag{9}$$

in which the upper-bounds of response due to presence of variations and uncertainty in the response spectra is obtained.

5 NUMERICAL EXAMPLE

In this example, the bounds of the dynamic response of a 2-D truss structure subjected to multiple ground-motions are obtained using the developed method (Figure 5.)

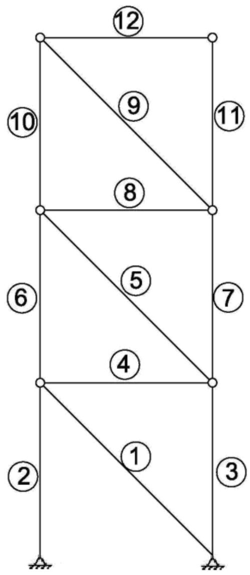


Figure 5: The structure of 2-D truss

The Young's moduli for all elements is $E = 200\ GPa$. The load on first floor, second floor and roof are 10,000 *kN*, 10,000 *kN* and 8,000 *kN*, respectively. The length of all horizontal and vertical members is 4*m*. The areas of the members are: $A_1 = A_6 = A_7 = A_8 = A_{12} = 65\ cm^2$, $A_2 = A_3 = 100\ cm^2$, $A_4 = 75\ cm^2, A_{10} = A_{11} = 35\ cm^2$.

5.1 Construction of interval response spectrum

For this example, ten recorded seismic response spectra are used which are obtained from five recent major earthquakes (two spectra per each earthquake measured in perpendicular directions). Table 1 summarizes the earthquakes' information. Figure 6 depicts the ten historical response spectra, and the bounding interval response spectrum.

Earthquake Location	Earthquake Date	Recording Location	Distance from Epicenter
Chile	Feb. 27, 2010	Concepcion, Chile	109 km
Samoa	Nov. 29, 2009	Afimalu, Samoa	179 km
Sumatra	Nov. 9, 2007	Sikuai Island, West Sumatra	392 km
Sumatra (aftershock)	Nov. 12, 2007	Sikuai Island, West Sumatra	165 km
Haiti	Jan. 12, 2010	Presa de Sabaneta, Dominican Republic	144 km

Table 1: Information for the earthquakes used in the example problem.

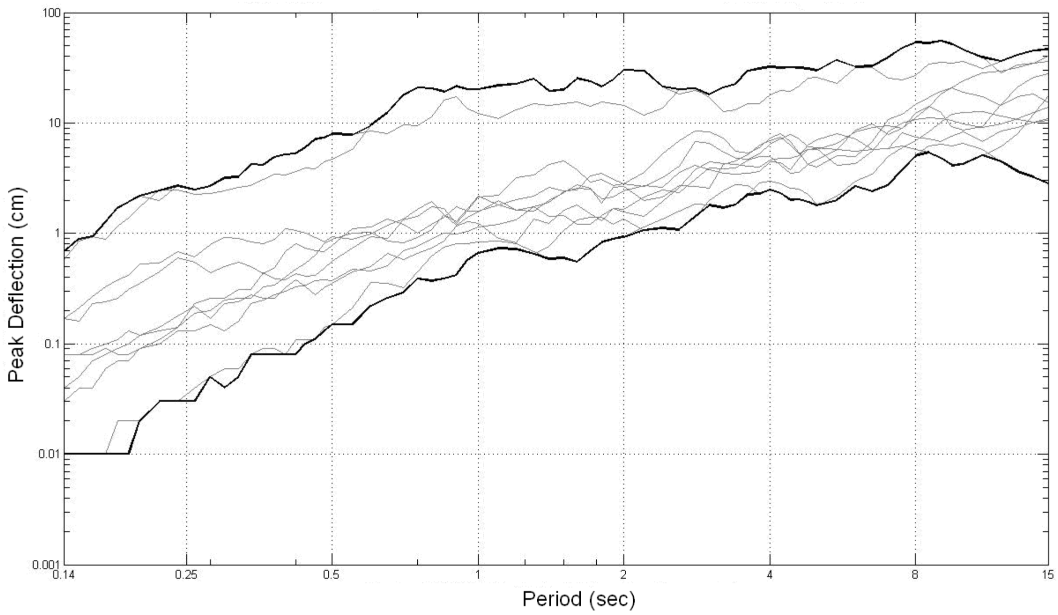


Figure 6: Constructed interval response spectrum for the example problem.

5.2 Solution

The problem is solved using the present method and the results are compared with solutions for 10^4 Monte Carlo Simulations using uniformly distributed random variables. The structure's degrees-of-freedom are statically condensed to only three lateral degrees-of-freedom (one per floor).

Using the obtained interval response spectrum, interval modal coordinates $\tilde{D}_n$ for structure's natural periods are determined. The results are shown in Figure 7 and summarized in Table 2.

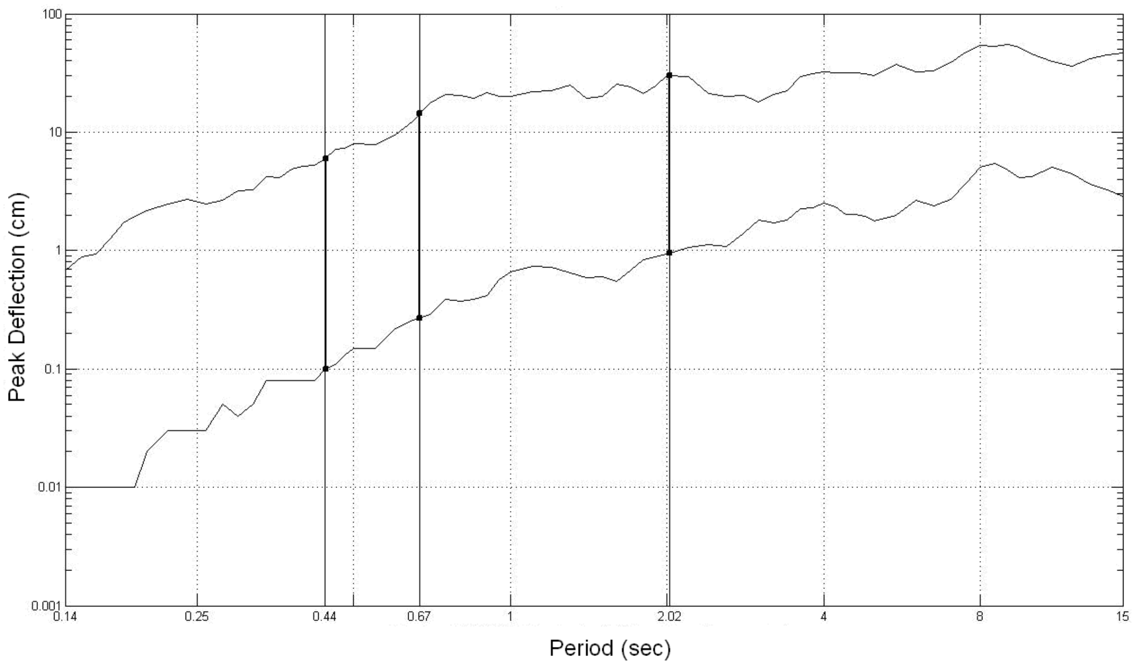


Figure 7: Determination of interval modal coordinates from interval response spectrum

Mode	Natural period T_n (sec)	Interval modal coordinate $\tilde{D}_n$ (cm)
1	0.44	[0.10 , 5.97]
2	0.67	[0.27 , 14.52]
3	2.02	[0.95 , 30.14]

Table 2: Interval modal coordinates

Next, the upper-bounds of modal responses are determined. Then, the dynamic response for each floor is obtained by combining modal responses using SRSS method. The analysis procedure is performed using both present method and Monte-Carlo simulations and the results are compared. Table 3 summarizes the results.

Mode	Maximum Response $U_{\max}$ (cm) (*present method*)	Maximum Response $U_{\max}$ (cm) (*simulation*)
1	10.64	10.50
2	25.53	25.42
3	40.56	40.48

Table 3: The structure's upper-bounds of dynamic responses

Figure 8 depict the structure's response obtained from both present method and Monte-Carlo simulation solution geometrically.

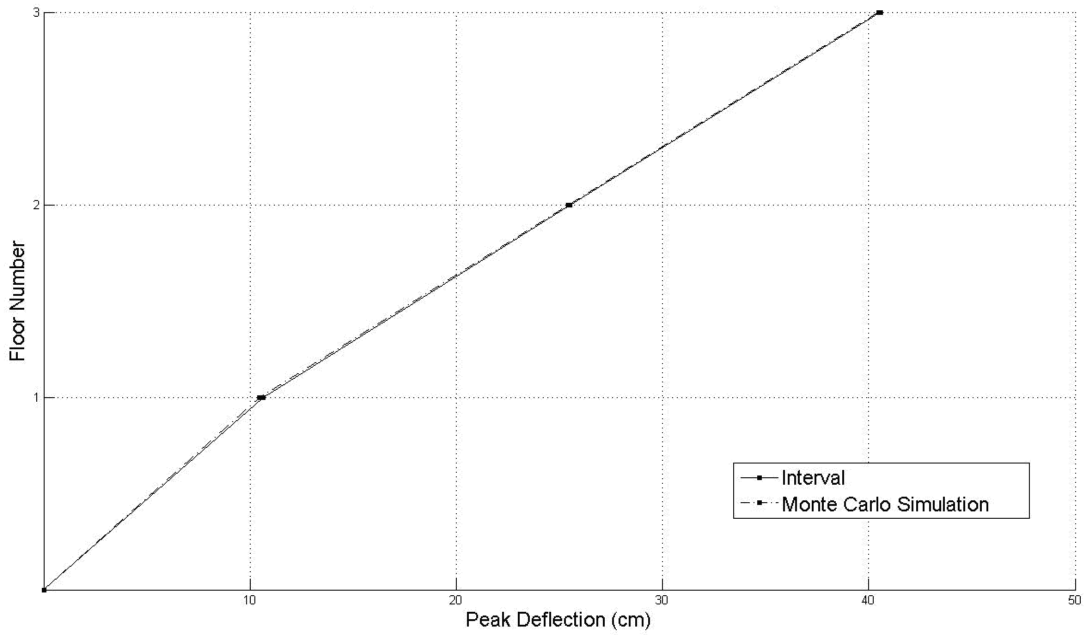


Figure 8: Geometric depiction of the structure's upper-bounds of dynamic responses

5.3 Observations

The results show that using the present method, the obtained sharp solutions are upper-bounds to solutions obtained by methods that produce inner-bound results such as Monte-Carlo simulation. Moreover, the proposed method is computationally feasible because of its non-iterative process.

6 CONCLUSIONS

- In this work, a finite-element based method for seismic structural analysis with interval uncertainty in the response spectrum is developed.
- This method enhances the conventional response spectrum analysis for considering the presence of variation and uncertainty in the ground excitation.
- For a given set of recorded ground motions for various earthquakes, the response spectra are combined as an interval response spectrum used in the analysis procedure defined in the developed methodology.
- The method is capable of obtaining sharp results for the structure's dynamic response.
- This method is computationally feasible and it shows that the bounds on the dynamic response can be obtained without any iterative procedure such as Monte-Carlo simulation procedure.
- The computational efficiency of the proposed method makes it attractive to introduce uncertainty into seismic analysis of structures.

REFERENCES

[1] M. A. Biot, *Vibrations of Buildings during Earthquake*, Chapter II in Ph.D. Thesis No. 259 entitled *Transient Oscillations in Elastic System*, Aeronautics Department, Calif. Inst. of Tech., Pasadena, California, U.S.A., 1932.

[2] G. W. Housner, *Behavior of Structures during Earthquake* Proc. ASCE Vol. 85, No. EM 4 p. 109, 1959.

[3] N. M. Newmark, *A Method of Computation for Structural Dynamics*, Trans. ASCE, Vol. 127, pt.1, pp. 1406-1435 (1962).

[4] A.W. Anderson et al., *Lateral Forces on Earthquake and Wind*", Trans. ASCE, Vol. 117, p. 716, 1952.

[5] D. E. Hudson, *Response Spectrum Techniques in Engineering Seismology*, Proc. World Conference on Earthquake Eng. Earthquake Engineering Research Institute, Berkeley, California, 1956.

[6] A. S. Veletsos, N. M. Newmark, , *Natural Frequencies of Continuous Flexural Members*, Trans. ASCE, Vol. 122, p. 249, 1957.

[7] E. Rosenblueth, & Bustamente, J. I., *Distribution of Structural Response to Earthquakes*, Proc. ASCE, Vol. 88, No. EM 3 p. 75, 1962.

[8] J. M Biggs, *Introduction to Structural Dynamics*, McGraw-Hill, Inc, 1964.

[9] R.W Clough, J. Penzien, *Dynamics of Structures*, McGraw-Hill, New York, 1993.

[10] Archimedes (287-212 B.C.); by T.L. Heath, *The Works of Archimedes*, Cambridge, Cambridge University Press, 1897.

[11] T. Sunaga, *Theory of an Interval Algebra and its Application to Numerical Analysis*, RAAG Memoirs II 2, 547-565. Tokyo, Japan, 1958.

[12] Ramon E. Moore, *Interval Analysis*, Prentice Hall, Englewood, NJ, 1966.

[13] G. Alefeld, J. Herzberger, *Introduction to Linear Computation*, New York, Academic Press, 1983.

[14] A. Neumaier, *Interval Methods for Systems of Equations*, Cambridge University Press, Cambridge, 1990.

[15] R. L Muhanna, R. L. Mullen, *Formulation of Fuzzy Finite Element Methods for Mechanics Problems*, Computer-Aided Civil and Infrastructure Engineering, 14, pp. 107-117, 1999.

[16] R. L. Muhanna, R. L. Mullen, *Uncertainty in Mechanics Problems-Interval-Based Approach*, Journal of Engineering Mechanics June-2001, pp.557-566, 2001.

[17] M. Modares, R. L. Mullen, *Free Vibration of Structures with Interval Uncertainty*, 9th ASCE Specialty Conference on Probabilistic Mechanics and Structural Reliability, 2004.