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OPTIMIZATION OF STRUCTURES EQUIPPED WITH VISCOELASTIC DAMPERS MODELED USING THE FRACTIONAL ORDER DERIVATIVES

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Abstract. *Viscoelastic (VE) dampers are now among preferable energy dissipation devices used for passive seismic response control. It is the aim of this paper to find the optimal damper placement and to determine the optimal parameters of dampers. The dynamic behavior of VE dampers, as described by the fractional Maxwell rheological model, is written in the form of a fractional differential equation. Fractional models have an ability to correctly describe the behavior of VE materials and dampers using a small number of model parameters. The equation of motion for the structure considered is expressed in the state space. The structure is subjected to a base acceleration. For the harmonic external forces the displacement response of the structure is determined. Solving the equation of motion yields an input-output relationship with the matrix frequency response function. The objective function, which we minimize, is the weighted sum of amplitudes of the transfer functions of interstorey drifts, evaluated at the fundamental, natural frequency of the frame with the dampers. The solution is obtained using the sequential optimization method and the particle swarm optimization method (PSO). The results obtained by both methods are compared.*

1 INTRODUCTION

Viscoelastic dampers are successfully applied to reduce excessive vibrations of buildings caused by winds and earthquakes. Incorporation of the VE dampers into a structure leads to a significant reduction of undesirable vibrations, see Soong [1]. A number of applications of VE dampers in civil engineering are listed in [2]. The dampers' behavior depends mainly on the rheological properties of the VE material the dampers are made of and their geometric parameters. In the past, several rheological models were proposed to describe the dynamic behavior of VE materials and dampers. Both the classic and the so-called fractional-derivative models of dampers are available. In the classic approach, mechanical models consisting of springs and dashpots are used to describe the rheological properties of VE dampers [3]. A good description of the VE dampers requires mechanical models consisting of a set of appropriately connected springs and dashpots. In this approach, the rheological properties of VE dampers are described using the fractional calculus and the fractional mechanical models. The fractional calculus has received considerable attention and has been used in modeling the rheological behavior of VE materials [4] and dampers [5]. The fractional models have an ability to correctly describe the behavior of VE materials and dampers using a small number of model parameters. A single equation is enough to describe the VE damper dynamics, which is an important advantage of the discussed model. However, in this case, the VE damper equation of motion is the fractional differential equation. The dynamic analysis of frame or building structures with dampers is presented in many papers [6, 7], where the fractional-derivative rheological model is used to model the dampers' behavior.

In this paper, planar frame structures with VE dampers mounted on them are considered. The VE dampers are modeled using the three-parameter fractional rheological Maxwell model. The structures are treated as linear elastic systems. The equations of motion of the whole system (structure with dampers) are written in terms of both physical and state-space variables. The proposed approach to the state space formulation is new. This is the main advantage of the proposed formulation, which does not require matrices with huge dimensions. However, the resulting matrix equation of motion is a fractional differential equation.

It is aim of the present paper to find the optimal placements of the dampers and to determine their optimal parameters. The objective function, which we minimize, is the weighted sum of amplitudes of the transfer functions of interstorey drifts, evaluated at the fundamental, natural frequency of a frame with dampers. The optimality criterion is expressed by the vector consisting of the values of the above mentioned transfer functions of the interstorey drifts.

The solution to the considered optimization problem is arrived at using the sequential optimization method and the particle swarm optimization method (PSO), which is based on the study of social behavior in a self–organized population system [8, 9]. Numerical tests carried out for a multi-storey building structure modeled as a shear plane frame with VE dampers mounted on it show that the presented methods are simple and efficient.

2 THE RHEOLOGICAL MODEL OF DAMPER

In this paper, the fractional Maxwell model is used to represent the rheological properties of VE dampers. The considered model consists of a fractional dashpot with the constants: c_i, α_i ($0 < \alpha_i \le 1$) and a spring of the stiffness k_i. The equations of motion for the Maxwell model could be written using the so-called relative internal variable v_i (see Figure 1). The above-mentioned equations of motion for the damper are as follows:

$$\begin{aligned} u_i &= c_i D_t^{\alpha_i}(x_i - v_i) \\ u_i &= k_i v_i \end{aligned} \tag{1}$$

where u_i is the damper force and x_i is the relative damper displacement. Moreover, $D_t^{\alpha_i}(\bullet)$ denotes the Riemann-Liouville fractional derivative of the order α_i with respect to time, *t*.

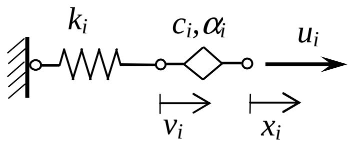

Figure 1: Rheological model of fractional Maxwell damper

The equation of motion of the classical Maxwell models could be obtained after substituting $\alpha_i = 1$ into Equations (1).

3 EQUATIONS OF MOTION

The frame with VE dampers is treated as the elastic linear system, which could be modeled as the shear frame. The mass of the system is lumped at the level of storeys.

3.1 The equations of motion expressed in physical coordinates

The equation of motion of the whole system (structure with dampers) can be written as follows:

$$\mathbf{M}_s \ddot{\mathbf{q}}_s(t) + \mathbf{C}_s \dot{\mathbf{q}}_s(t) + \mathbf{K}_s \mathbf{q}_s(t) = \mathbf{s}(t) + \mathbf{p}(t) \tag{2}$$

where the symbols $\mathbf{M}_s$, $\mathbf{C}_s$ and $\mathbf{K}_s$ denote the mass, the damping, and the stiffness ($n \times n$) matrices, respectively. Moreover, $\mathbf{q}_s(t) = col(q_{s,1},..., q_{s,j},..., q_{s,n})$ and $\mathbf{p}(t) = col(p_1,..., p_j,..., p_n)$ denote the vector of displacements of the structure and the vector of excitation forces, respectively. The $\mathbf{s}(t) = col(s_1, s_2,..., s_n)$ vector is the ($n \times 1$) vector of interaction forces between the frame and the dampers [10].

For the Maxwell model of dampers, the vector of interactive forces $\mathbf{s}(t)$ is treated as a sum of two vectors, i.e., $\mathbf{s}(t) = \mathbf{s}_1(t) + \mathbf{s}_2(t)$. The vector $\mathbf{s}_1(t)$ contains interactive forces which are reactions of the elastic part of the Maxwell dampers to the frame, while the vector $\mathbf{s}_2(t)$ contains interactive forces which are reactions of the dashpot part of the dampers. It is assumed that the dashpot part of the Maxwell model is joined with the upper storey while the elastic part is joined with the lower storey. Moreover, the brace stiffness could be taken into account in the stiffness parameter of the Maxwell model. If a structure with only one damper, denoted as the damper number *i*, mounted between two successive storeys, *j* and *j+1*, is considered then the vectors $\mathbf{s}_1(t)$ and $\mathbf{s}_2(t)$ could be written in the following form:

$$\begin{aligned} \mathbf{s}_1(t) &\equiv \mathbf{s}_{1i}(t) = col(0,..., s_j = u_i,.....,0) = \tilde{\mathbf{e}}_i u_i(t) \\ \mathbf{s}_2(t) &\equiv \mathbf{s}_{2i}(t) = col(0,..., s_{j+1} = -u_i,.....,0) = \hat{\mathbf{e}}_i u_i(t) \end{aligned} \tag{3}$$

where $\tilde{\mathbf{e}}_i = col(0,..., \tilde{e}_j = 1, \tilde{e}_{j+1} = 0,...,0)$, $\hat{\mathbf{e}}_i = col(0,..., \hat{e}_j = 0, \hat{e}_{j+1} = -1,...,0)$.

Taking into account that $q_{s,j}(t)) = \tilde{\mathbf{e}}_i^T \mathbf{q}_s(t)$ and $q_{s,j+1}(t) = -\hat{\mathbf{e}}_i^T \mathbf{q}_s(t)$, the damping force $u_i(t)$ of the Maxwell damper could be shown in two equivalent forms:

$$
\begin{aligned}
u_i(t) &= k_i(v_i(t) - q_{s,j}(t)) = k_i v_i(t) - k_i \tilde{\mathbf{e}}_i^T \mathbf{q}_s(t) \\
u_i(t) &= c_i(D_t^{\alpha_i} q_{s,j+1}(t) - D_t^{\alpha_i} v_i(t)) = -c_i D_t^{\alpha_i} v_i(t) - c_i \hat{\mathbf{e}}_i^T D_t^{\alpha_i} \mathbf{q}_s(t)
\end{aligned} \tag{4}
$$

and the interaction force vectors $\mathbf{s}_{1,i}(t)$ and $\mathbf{s}_{2,i}(t)$ are given by:

$$
\begin{aligned}
\mathbf{s}_1^{(i)}(t) &= \tilde{\mathbf{e}}_i k_i v_i(t) - \tilde{\mathbf{e}}_i k_i \tilde{\mathbf{e}}_i^T \mathbf{q}_s(t) = \tilde{\mathbf{e}}_i k_i \mathbf{h}_i^T \mathbf{q}_r(t) - \tilde{\mathbf{e}}_i k_i \tilde{\mathbf{e}}_i^T \mathbf{q}_s(t) \\
\mathbf{s}_2^{(i)}(t) &= -\hat{\mathbf{e}}_i c_i D_t^{\alpha_i} v_i(t) - \hat{\mathbf{e}}_i c_i \hat{\mathbf{e}}_i^T D_t^{\alpha_i} \mathbf{q}_s(t) = -\hat{\mathbf{e}}_i c_i \mathbf{h}_i^T D_t^{\alpha_i} \mathbf{q}_r(t) - \hat{\mathbf{e}}_i c_i \hat{\mathbf{e}}_i^T D_t^{\alpha_i} \mathbf{q}_s(t)
\end{aligned} \tag{5}
$$

where the vector of internal variables $\mathbf{q}_r(t) = col(v_1(t),...,v_i(t),...,v_m(t))$ and the vector $\mathbf{h}_i = col(0,...,h_i = 1,...,0)$ have the dimension ($m \times 1$). Moreover, from the equilibrium condition of the internal node of the Maxwell model of damper we obtain:

$$
-c_i D_t^{\alpha_i} q_{s,j+1}(t) + c_i D_t^{\alpha_i} v_i(t) - k_i q_{s,j}(t) + k_i v_i(t) = 0 \tag{6}
$$

In the matrix notation, Equation (6) for $i = 1,2,...m$, may be rewritten in the form:

$$
c_i \hat{\mathbf{e}}_i^T D_t^{\alpha_i} \mathbf{q}_s(t) + c_i \mathbf{h}_i^T D_t^{\alpha_i} \mathbf{q}_r(t) - k_i \tilde{\mathbf{e}}_i^T \mathbf{q}_s(t) + k_i \mathbf{h}_i^T \mathbf{q}_r(t) = 0 \tag{7}
$$

When m dampers are present in the frame, and all the fractional parameters are equal, (i.e., $\alpha_i = \alpha = const.$) then the interaction force vectors are:

$$
\begin{aligned}
\mathbf{s}_1(t) &= \sum_{i=1}^{m} \tilde{\mathbf{e}}_i k_i \mathbf{h}_i^T \mathbf{q}_r(t) - \sum_{i=1}^{m} \tilde{\mathbf{e}}_i k_i \tilde{\mathbf{e}}_i^T \mathbf{q}_s(t) \\
\mathbf{s}_2(\mathrm{t}) &= -\sum_{i=1}^{m} \hat{\mathbf{e}}_i c_i \mathbf{h}_i^T D_t^{\alpha} \mathbf{q}_r(t) - \sum_{i=1}^{m} \hat{\mathbf{e}}_i c_i \hat{\mathbf{e}}_i^T D_t^{\alpha} \mathbf{q}_s(t)
\end{aligned} \tag{8}
$$

After pre-multiplying Equation (7) by $\mathbf{h}_i$ and summing up all equations with respect to *i*, we have:

$$
\sum_{i=1}^{m} \mathbf{h}_i c_i \hat{\mathbf{e}}_i^T D_t^{\alpha} \mathbf{q}_s(t) + \sum_{i=1}^{m} \mathbf{h}_i c_i \mathbf{h}_i^T D_t^{\alpha} \mathbf{q}_r(t) - \sum_{i=1}^{m} \mathbf{h}_i k_i \tilde{\mathbf{e}}_i^T \mathbf{q}_s(t) + \sum_{i=1}^{m} \mathbf{h}_i k_i \mathbf{h}_i^T \mathbf{q}_r(t) = \mathbf{0} \tag{9}
$$

Taking into account that $\mathbf{s}(t) = \mathbf{s}_1(t) + \mathbf{s}_2(t)$ and substituting Equations (8) into (2) we obtain the following equation of motion for the frame with the Maxwell dampers:

$$
\begin{aligned}
\mathbf{M}_s D_t^2 \mathbf{q}_s(t) + \mathbf{C}_s D_t^1 \mathbf{q}_s(t) + \mathbf{C}_{ss}^d D_t^{\alpha} \mathbf{q}_s(t) + (\mathbf{K}_s + \mathbf{K}_{ss}^d)\, \mathbf{q}_s(t) + \\
+ \mathbf{C}_{sr}^d D_t^{\alpha} \mathbf{q}_r(t) - \mathbf{K}_{sr}^d \mathbf{q}_r(t) = \mathbf{p}(t)
\end{aligned} \tag{10}
$$

Equation (9) could be rewritten in the form:

$$
\mathbf{C}_{rs}^d D_t^{\alpha} \mathbf{q}_s(t) + \mathbf{C}_{rr}^d D_t^{\alpha} \mathbf{q}_r(t) - \mathbf{K}_{rs}^d \mathbf{q}_s(t) + \mathbf{K}_{rr}^d \mathbf{q}_r(t) = \mathbf{0} \tag{11}
$$

The following symbols

$$
\mathbf{C}_{ss}^d = \sum_{i=1}^{m} \hat{\mathbf{e}}_i c_i \hat{\mathbf{e}}_i^T ,\; \mathbf{C}_{sr}^d = \sum_{i=1}^{m} \hat{\mathbf{e}}_i c_i \mathbf{h}_i^T ,\; \mathbf{C}_{rs}^d = \sum_{i=1}^{m} \mathbf{h}_i c_i \hat{\mathbf{e}}_i^T = (\mathbf{C}_{sr}^d)^T ,\; \mathbf{C}_{rr}^d = \sum_{i=1}^{m} \mathbf{h}_i c_i \mathbf{h}_i^T ,
$$

$$\mathbf{K}_{rs}^{d}=\sum_{i=1}^{m}\mathbf{h}_i k_i \tilde{\mathbf{e}}_i^T=(\mathbf{K}_{sr}^{d})^T\,,\ \mathbf{K}_{rr}^{d}=\sum_{i=1}^{m}\mathbf{h}_i k_i \mathbf{h}_i^T\,,\ \mathbf{K}_{sr}^{d}=\sum_{i=1}^{m}\tilde{\mathbf{e}}_i k_i \mathbf{h}_i^T\,,\ \mathbf{K}_{ss}^{d}=\sum_{i=1}^{m}\tilde{\mathbf{e}}_i k_i \tilde{\mathbf{e}}_i^T\,,$$

were introduced in Equations (10) and (11). The system of fractional differential Equations (10) and (11) constitute a set of equations from which the dynamic response of the structure with the Maxwell dampers can be determined.

3.2 The equations of motion expressed in the state space

In many cases it is very convenient to use the equation of motion expressed in the state space. In the case of frames with the Maxwell dampers the vector of state variables and the vectors of state variables' derivatives are defined as:

$$\begin{aligned}
&\mathbf{z}(t)=col(\mathbf{q}_r(t),\ \mathbf{q}_s(t),\ D_t^1\mathbf{q}_s(t))\\
&D_t^1\mathbf{z}(t)=col(D_t^1\mathbf{q}_r(t),\ D_t^1\mathbf{q}_s(t),\ D_t^2\mathbf{q}_s(t))\\
&D_t^{\alpha}\mathbf{z}(t)=col(D_t^{\alpha}\mathbf{q}_r(t),\ D_t^{\alpha}\mathbf{q}_s(t),\ D_t^{\alpha+1}\mathbf{q}_s(t))
\end{aligned}\tag{12}$$

Moreover, when the following additional matrix equation:

$$\mathbf{M}_s D_t^1\mathbf{q}_s(t)-\mathbf{M}_s D_t^1\mathbf{q}_s(t)=\mathbf{0}\tag{13}$$

is appended to the Equation (10) and (11) a set of equations is obtained which could be rewritten using the state variables defined above. The resulting matrix equation is in the form:

$$\mathbf{A}\,D_t^1\mathbf{z}(t)+\mathbf{A}_1 D_t^{\alpha}\mathbf{z}(t)+\mathbf{B}\mathbf{z}(t)=\tilde{\mathbf{p}}(t)\tag{14}$$

where

$$\mathbf{A}=\begin{bmatrix}\mathbf{0}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{C}_s&\mathbf{M}_s\\ \mathbf{0}&\mathbf{M}_s&\mathbf{0}\end{bmatrix},\ \mathbf{A}_1=\begin{bmatrix}\mathbf{C}_{rr}^d&\mathbf{C}_{rs}^d&\mathbf{0}\\ \mathbf{C}_{sr}^d&\mathbf{C}_{ss}^d&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}\end{bmatrix},\ \mathbf{B}=\begin{bmatrix}\mathbf{K}_{rr}^d&-\mathbf{K}_{rs}^d&\mathbf{0}\\ -\mathbf{K}_{sr}^d&\mathbf{K}_s+\mathbf{K}_{ss}^d&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&-\mathbf{M}_s\end{bmatrix},\ \tilde{\mathbf{p}}(t)=\begin{Bmatrix}\mathbf{0}\\ \mathbf{p}(t)\\ \mathbf{0}\end{Bmatrix}$$

The above approach to the state space formulation is new. In comparison with previous ones, such as those given in [11, 12], matrices with huge dimensions were not required which is the main advantage of the proposed formula. Moreover, all of the matrices appearing in Equation (14) are symmetrical.

4 DYNAMIC ANALYSIS

4.1 Dynamic characteristics of structures

Applying the Laplace transform, taking into account that $\tilde{\mathbf{p}}(t)=\mathbf{0}$:

$$\mathcal{L}[\mathbf{z}(t)]=\mathbf{Z}\,,\ \mathcal{L}\left[D_t^{\alpha}\,\mathbf{z}(t)\right]=s^{\alpha}\,\mathbf{Z}\,,\ \mathcal{L}\left[D_t^1\mathbf{z}(t)\right]=s\mathbf{Z}\,,\tag{15}$$

the equation of motion (14) can be written as

$$\left(s\mathbf{A}+s^{\alpha}\mathbf{A}_1+\mathbf{B}\right)\mathbf{Z}=\mathbf{0}\,,\tag{16}$$

Equation (16) constitutes a non-linear eigenproblem which can be solved using the continuation method. The solution to the considered non-linear equation could be shown as a curve in the configuration space, i.e., the s, $\mathbf{Z}$ space. The first point in this curve is obtained for $\alpha=1$, in this case Equation (16) expresses the linear eigenvalue problem. Next, the solution to the eigenproblem (16) for the chosen value of $\alpha \in (0,1)$ is investigated. The incremental-iteration method, presented in detail in [10], is used. Usually, one incremental step and three or four iterations are enough to reach the solution for the final value of the fractional parameter. The continuation method enables the eigenvalues s_i to be determined.

The dynamic behavior of a frame with viscoelastic dampers is characterized by the natural frequency ω_i and the non-dimensional damping parameter γ_i. Similarly to viscous damping, the above-mentioned properties are defined as follows:

$$\omega_i^2 = \mu_i^2 + \eta_i^2 \,, \; \gamma_i = -\mu_i / \omega_i \,, \tag{17}$$

where $\mu_i = \mathrm{Re}(s_i)$, $\eta_i = \mathrm{Im}(s_i)$.

4.2 Frequency response functions

In this section we investigate the steady state harmonic responses of structures governed by Equations (14). For the harmonic external forces described by:

$$\mathbf{p}(t) = \mathbf{P} \exp(i\lambda t) \,, \tag{18}$$

where $i = \sqrt{-1}$, λ is the frequency of excitation, the displacement response of the structure can be expressed as:

$$\mathbf{q}_s(t) = \mathbf{Q}_s(\lambda) \exp(i\lambda t) \,. \tag{19}$$

If relationships (18) and (19) are substituted into the equation of motion (14), written in the state space, the following equation is obtained:

$$\mathbf{Q}_s(\lambda) = \tilde{\mathbf{H}}(\lambda)\tilde{\mathbf{P}} \,, \tag{20}$$

where:

$$\tilde{\mathbf{H}}(\lambda) = \left[i\lambda\, \mathbf{A} + (i\lambda)^{\alpha} \mathbf{A}_1 + \mathbf{B} \right]^{-1} . \tag{21}$$

When the structure is subjected to a base acceleration $\ddot{u}_g(t)$, the excitation vector is written as $\mathbf{p}(t) = -\mathbf{M}\, \mathbf{r}\, \ddot{u}_g(t)$, where $\mathbf{r} = col\{1,1,.....,1\}$. For the harmonic external forces, $\ddot{u}_g(t) = \ddot{U}_g \exp(i\lambda t)$. The displacement response of the structure is given by relationship (19) and $\mathbf{Q}_s(\lambda)$ is determined from:

$$\mathbf{Q}_s(\lambda) = \mathbf{H}(\lambda)\ddot{U}_g \,. \tag{22}$$

where the vector $\mathbf{H}(\lambda) = -\tilde{\mathbf{H}}(\lambda)\mathbf{Mr}$ will be called the vector of frequency transfer functions of displacements.

5 OPTIMIZATION PROBLEM

It is the aim of this paper to find the optimal dampers' placements and to determine the optimal parameters of the dampers k_{di} and c_{di}. The objective function, which is minimized, is the weighted sum of amplitudes of the transfer functions of interstorey drifts, $h_i(\lambda)$, evaluated at the fundamental, natural frequency ($\lambda = \omega_1$) of the frame with the dampers. The optimality criteria may by described as follows:

$$F = \mathbf{w}^T \mathbf{h}(\omega_1)\,. \tag{23}$$

where the vector $\mathbf{h}(\omega_1) = col(h_1(\omega_1), h_2(\omega_1), ..., h_n(\omega_1))$ consists of the values of the above mentioned amplitudes of transfer functions of interstorey drifts, $\mathbf{w} = col(w_1, w_2, ..., w_n)$ is the vector of weight coefficients, and n stands for the number of the structure's degrees of freedom.

The considered optimization problem is subjected to some constraints. We assume that the sum of damping coefficients and the sum of stiffness parameters are known and constant. Moreover, the values of the parameters of damping, $c_{d,i}$, and stiffness, $k_{d,i}$, for every damper must be non-negative. The above constraints are written as:

$$\sum_{i=1}^{m} c_{d,i} = C_d\,, \quad \sum_{i=1}^{m} k_{d,i} = K_d\,, \quad c_{d,i} \geq c_{\min}\,, \quad k_{d,i} \geq k_{\min}\,. \tag{24}$$

where $c_{\min}, k_{\min}$ represent the assumed low-value positive numbers ($c_{\min} = 1.0 N \sec^{\alpha}/m$ was chosen in our example).

The vector $\mathbf{H}_d(\lambda)$ of the frequency transfer functions of interstorey drifts can be calculated from the following formula:

$$\mathbf{H}_d(\lambda) = \mathbf{T}\mathbf{H}(\lambda)\,, \tag{25}$$

where the transformation matrix is:

$$\mathbf{T} = \begin{bmatrix} 1 \; 0 \, 0 ... 0 \;\; 0 \;\; 0 \\ -1 \; 1 \; 0 ... 0 \;\; 0 \;\; 0 \\ .... \\ 0 \;\; 0 \;\; 0 ... 0 -1 \;\; 1 \end{bmatrix}. \tag{26}$$

The solution is obtained using the sequential optimization method and the particle swarm optimization method (PSO). In the first case, for every possible location of one damper the values of the objective function are calculated. The right fixed location of the damper is the position for which the minimum value of the objective function is obtained. When the first damper's location is determined the procedure is repeated until all locations are found for the dampers. However, there is no proof for the solution's convergence although many examples show that this method is simple and efficient in many engineering applications (for instance [13]).

The PSO algorithm which is based on the study of social behavior in a self-organized population system (i.e., ant colonies, fish schools), searches a space by adjusting the trajectories of so-called particles. In the considered optimization problem, the vector of the *i*-th particle's position $\mathbf{p}_i$ consists of damping coefficients of dampers currently mounted on the structure, i.e., $\mathbf{p}_i = col(c_{d,1}^{(i)}, c_{d,2}^{(i)}, ..., c_{d,n}^{(i)})$. The dimension of the vector $\mathbf{p}_i$ is equal to the number of building storeys. Moreover, to reduce the number of elements of the particle position vector and because of technological requirements for the damper, the stiffness parameter of the damper is calculated assuming that the ratio $c_{d,i}/k_{d,i}$ is given and will not change during iteration.

A population of particles is initialized with random positions and velocities [9]. Taking into account the best positions of the particles at subsequent iteration *k*+1, the algorithm adjusts the behavior of the particles by the following rules:

$$\mathbf{v}_i(k+1) = w(k)\mathbf{v}_i(k) + \frac{c_1}{\Delta t}\mathbf{R}_1(k)\big(\mathbf{b}_i(k) - \mathbf{p}_i(k)\big) + \frac{c_2}{\Delta t}\mathbf{R}_2(k)\big(\mathbf{g}_i(k) - \mathbf{p}_i(k)\big)$$
$$\mathbf{p}_i(k+1) = \mathbf{p}_i(k) + \mathbf{v}_i(k+1)\Delta t \tag{27}$$

where $\Delta t = 1$, $\mathbf{p}_i(k)$ is the position of i-th particle at k-th iteration, $\mathbf{v}_i(k)$ is the corresponding velocity vector; $\mathbf{b}_i(k)$ and $\mathbf{g}_i(k)$ stand for the best position found by the particle i and the best position in the particle's neighbourhood achieved so far, respectively; $\mathbf{R}_1(k)$, $\mathbf{R}_2(k)$ are the diagonal matrices of independent random numbers uniformly distributed in the range $(0, \ 1)$; $w(k)$ is the inertia factor providing balance between exploration and exploitation, c_1 is the individuality constant, and c_2 is the sociality constant. To speed up convergence, the inertia weight could be linearly reduced. A new velocity, which moves the particle in the direction of a potentially better solution, is calculated based on its previous value, and the particle location at which the best fitness so far has been achieved.
The initial values of the elements $v_{i,j}(0)$ of the velocity vector $\mathbf{v}_i(0)$ are calculated from the following formula:

$$v_{i,j} = r_3\, C_d\, \varepsilon_0\,, \tag{28}$$

where r_3 is the random number taken from the range (0, 1) and ε_0 is a constant ($\varepsilon_0 = 0.05$ is assumed). The initial values of the elements of the vector $\mathbf{p}_i(0)$ are determined from the following relationship:

$$c_{d,i}(0) = \frac{\tilde{r}_i C_d}{\sum_{j=1}^{m} \tilde{r}_j}\,, \tag{29}$$

where $\tilde{r}_i$ is a random number taken from the range (0, 1). The above choices assure that all assumed initial approximations of dampers parameters, i.e., vectors $\mathbf{p}_i(0)$ and $\mathbf{v}_i(0)$ fulfill the optimization constraints (24).

An important part of the PSO algorithm is the way of handling the constraints introduced in the optimization problem. Here, the following very simple procedure is used to fulfill the constraints (24):

- if non-admissible values $c_{d,i}(k+1) < 0$ result from the relationship (27) then $c_{d,i}(k+1) = c_{\min}$ is artificially introduced,
- in order to fulfill the constraint (27.1), elements of the vector $\mathbf{p}_i(k+1)$ are normalized in such a way that:

$$\tilde{c}_{d,i} = \frac{c_{d,i}}{\sum_{j=1}^{m} c_{d,j}} C_d \tag{30}$$

The PSO procedure is ceased if the change of the objective function is sufficiently small, i.e., when:

$$\left|F(k+1) - F(k)\right| \le \varepsilon_1 F(k+1) \tag{31}$$

where ε_1 is an assumed low-value number.

6 NUMERICAL TEST

In the numerical example, a ten-storey building structure modeled as a shear plane frame with VE dampers mounted on it is investigated. The bending rigidity of columns varies in sequence, for every two storeys: $k_1 = k_2 = 68710.0\ kN/m$, $k_3 = k_4 = 54010.0\ kN/m$, $k_5 = k_6 = 42170.0\ kN/m$, $k_7 = k_8 = 28660.0\ kN/m$, $k_9 = k_{10} = 16450.0\ kN/m$, but the mass value is the same for every floor: $m_s = 2.07 Mg$. The structure's damping ratios corresponding to the stiffness of the storeys are: $c_1 = c_2 = 4.76\ kN \sec/m$, $c_3 = c_4 = 3.73\ kN \sec/m$, $c_5 = c_6 = 2.91\ kN \sec/m$, $c_7 = c_8 = 1.98\ kN \sec/m$, $c_9 = c_{10} = 1.44\ kN \sec/m$ (data taken from [13]).

Firstly, the calculations were carried out for a frame without dampers (see Figure 2a), only the damping properties of structure were taken into account.

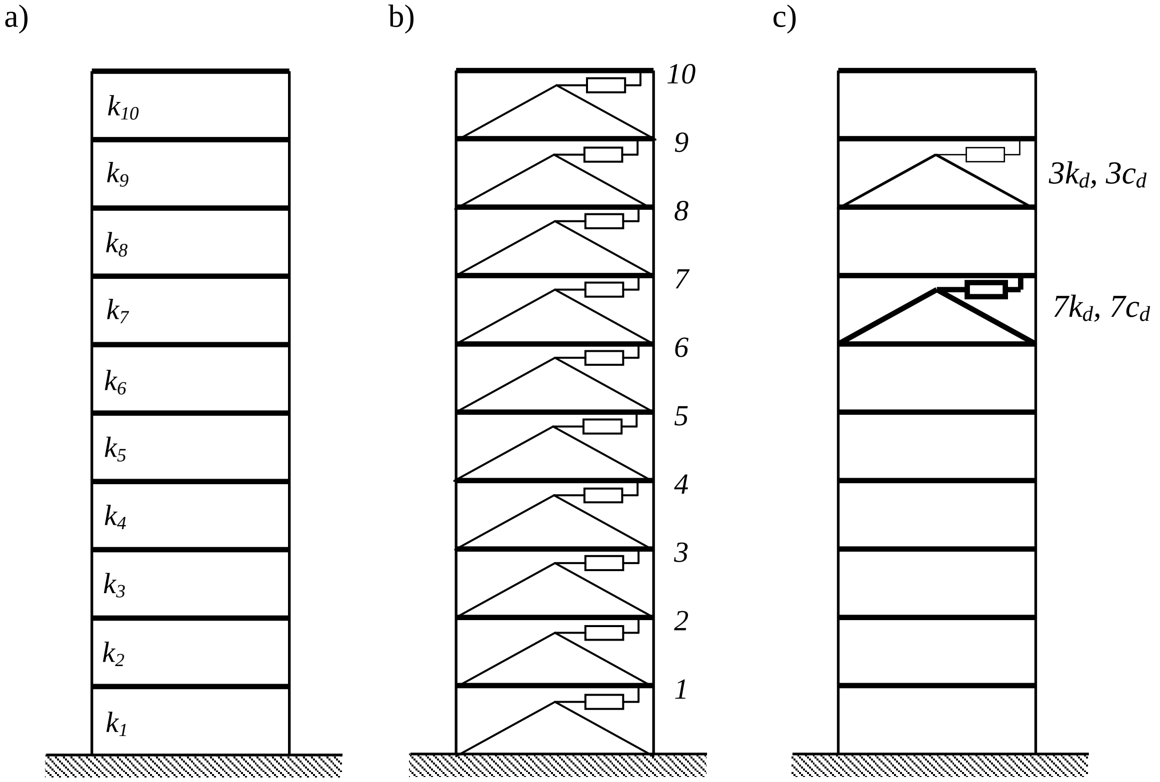

Figure 2: Diagram of a 10-storey frame with different distributions of dampers

The solution to Equation (16), where $\mathbf{A}_1 = \mathbf{0}$ and $\mathbf{K}^d_{rr} = \mathbf{K}^d_{sr} = \mathbf{K}^d_{rs} = \mathbf{K}^d_{ss} = 0$, leads to the eigenvalues s_i which enable determination of the dynamic properties of the structure described by Equation (17). The results, the natural frequencies of the structure and the values of non-dimensional damping factor are presented in Table 1.

Next, the authors investigated a structure with one damper mounted on every storey (see Figure 2b). The assumed value of the sum of the damping coefficients and the sum of the stiffness parameters are: $C_d = 500\ kN \sec^{\alpha}/m$, and $K_d = 25000\ kNm^2$, respectively. If dampers are uniformly distributed within a structure, the data for every single damper are: $k_d = 2500\ kNm^2$, $c_d = 50\ kN\sec^{\alpha}/m$, $\tau_d = c_d / k_d = 0.02$. The values of fractional parameters for all dampers are identical, i.e., $\alpha = 0.6$. Using the suggested procedure, the dynamic properties of the considered system were computed (see Table 1).

A first solution to the optimization problem is obtained using the sequential optimization method. For every possible location of one damper, the values of fundamental frequency are calculated (see Figure 3).

Modal number	No dampers		Dampers's distribution			
			uniform		optimal	
	ω_i	γ_i	ω_i	γ_i	ω_i	γ_i
1	22.690	0.0008	22.816	0.0126	22.934	0.0162
2	56.534	0.0022	58.114	0.0246	59.421	0.0338
3	91.909	0.0035	95.255	0.0212	96.309	0.0182
4	127.472	0.0047	132.284	0.0177	129.866	0.0094
5	151.769	0.0061	159.807	0.0209	163.766	0.0204
6	182.399	0.0066	188.678	0.0151	190.317	0.0213
7	208.638	0.0073	216.278	0.0152	220.389	0.0146
8	245.147	0.0085	252.143	0.0136	261.260	0.0210
9	281.524	0.0097	288.274	0.0135	283.755	0.0122
10	324.052	0.0112	330.492	0.0139	324.065	0.0112

Table 1: Natural frequencies ω_i and non-dimensional damping factors γ_i

Next, the objective function is evaluated for the frame, taking into account every possible position of the damper. The results are presented in Figure 4. It was assumed that the values of the weight coefficients w_i in Equation (23) are equal to one.

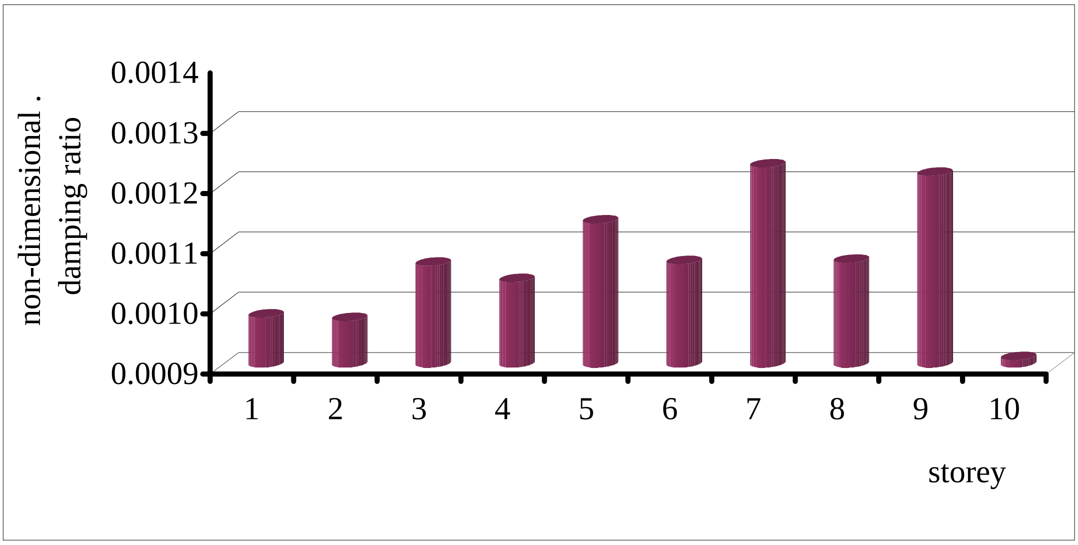

Figure 3: Non-dimensional damping factors versus first damper's position

The correct fixed location of the first damper is at the seventh storey, for which the minimum value of the objective function is obtained. When the first damper's location is determined the procedure is repeated until all locations for the dampers are found. The optimal locations of ten successive dampers are found to be: seven at the seventh storey and three at the ninth storey (see also Figure 2c). The dynamic properties of structures with optimally distributed dampers are shown in Table 1.

It can be noted that the non-dimensional damping ratio of the first mode of vibration is greater (by about 28%), compared with the same ratio for the structure with uniformly distributed dampers. Moreover, the damping ratios of the third, fourth, fifth, seventh, ninth, and tenth modes of vibration are smaller.

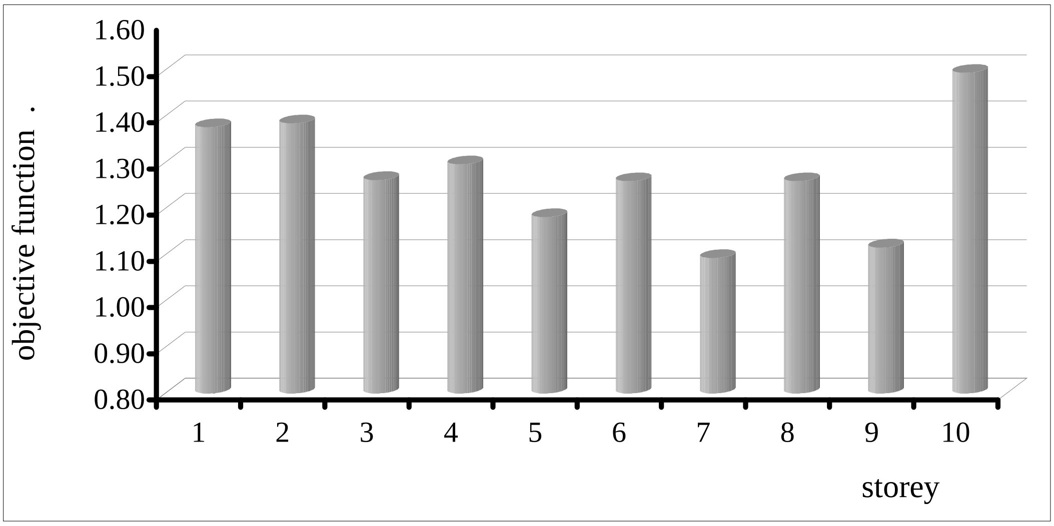

Figure 4: Objective function versus first damper's position

In the second approach, the PSO method is applied. In Equation (27), we define the values of coefficients $c_1 = c_2 = 2$ and the declining value of the inertia factor; starting with $w = 0.9$, it decreased by 0.005 at every step of iteration. A population of ten particles was initialized with random positions. The coordinates of every particle position describe the current distribution of damping properties on the frame. On every storey, the value of the damping coefficient must be non-negative and smaller than the assumed constant value $C_d = 500\, kN \sec^{\alpha} / m$ (i.e., $c_{\min} \le c_{d,i} \le C_d$). The stiffness parameters of the dampers are calculated from the value of the ratio $c_{d,i} / k_{d,i}$ which is equal to 0.02 and constant for every damper.

Changes of the best value of the objective function during the iteration process are presented in Figure 5. The solution to the optimization problem, i.e., the optimal distribution of VE dampers obtained with the help of both optimization methods is shown in Table 2.

The objective function, the weighted sum of amplitudes of the transfer functions of interstorey drifts is: $F_0 = 1.7053$, $F_U = 0.3739$, $F_S = 0.2759$ for the frame without dampers, for uniformly distributed dampers, and for the optimal solution obtained by the sequential and PSO methods, respectively.

It can be concluded that results obtained by both methods yield similar dampers' distributions on the frame. Differences between the optimal values of damping coefficients obtained as the result of optimization procedures are partially affected by an incremental way of distribution of damping coefficients in the sequential optimization method. Moreover, in the PSO method the values of the damping $c_{d,i}$ parameters of every damper must be non-negative.

Number of storey	Damping coefficient	
	Sequential method	PSO method
1	0	0.78
2	0	0.78
3	0	0.78
4	0	0.78
5	0	0.78
6	0	0.78
7	350	347.23
8	0	0.78
9	150	146.48
10	0	0.78
Total	500	499.95

Table 2: Optimal distribution of VE dampers

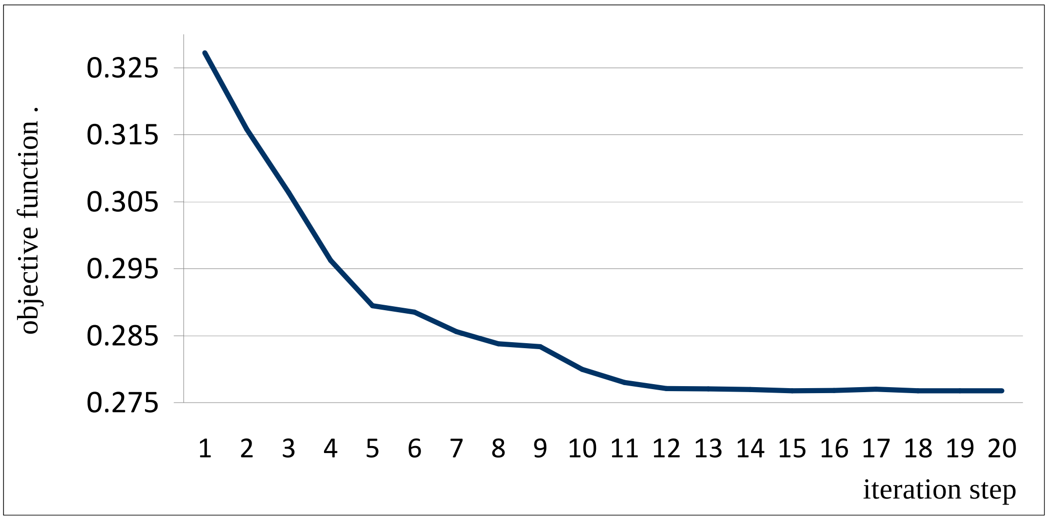

Figure 5: Convergence of objective function at PSO iteration

7 CONCLUDING REMARKS

In this paper, frame structures with viscoelastic dampers mounted on them are considered. Viscoelastic dampers are modeled using a three-parameter, fractional rheological Maxwell model which more precisely describes the VE damper's properties, compared with the classical one. The resulting matrix equation of motion is the fractional differential equation. The problem of optimal distribution of VE dampers modeled by the fractional rheological Maxwell model is solved for the first time. The considered optimization problem is solved by minimization of the objective function which is the weighted sum of amplitudes of the transfer functions of interstorey drifts. The sequential optimization method and the particle swarm optimization method are used to successfully solve the optimization problem. Examples of numerical calculations were shown. The presented results demonstrate the effectiveness and applicability of the proposed approach.

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