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ROBUST STRUCTURAL HEALTH MONITORING USING A POLYNOMIAL CHAOS BASED SEQUENTIAL DATA ASSIMILATION TECHNIQUE

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Abstract. *With the recent technological advances and the evolution of advanced smart systems for damage detection and signal processing, Structural Health Monitoring (SHM) emerged as a multidisciplinary field with wide applicability throughout the various branches of engineering, mathematics and physical sciences. However, significant challenges associated with modeling the physical complexity of systems comprising these structures remain. This is mainly due to the fact that numerous uncertainties associated with modeling, parametric and measurement errors could be introduced. In cases where these uncertainties are significant, standard identification and damage detection techniques are either unsuitable or inefficient. This study presents a robust data assimilation approach based on a stochastic variation of the Kalman Filter where polynomial functions of random variables are used to represent the inherent process uncertainties. The presented methodology is combined with a non-parametric modeling technique to tackle structural health monitoring of a four-story shear building. The structure is subject to a base motion specified by a time series consistent with the El-Centro earthquake and undergoes a preset damage in the first floor. The purpose of the problem is localizing the damage in both space and time, and tracking the state of the system throughout and subsequent to the damage time. The application of the introduced data assimilation technique to SHM enhances the latter's applicability to a wider range of structural problems with strongly nonlinear dynamical behavior and with uncertain and complex governing laws.*

1 INTRODUCTION

With the recent technological advances and the evolution of advanced smart systems for damage detection and signal processing, Structural Health Monitoring (SHM) emerged as a multidisciplinary field with wide applicability throughout the various branches of engineering, mathematics and physical sciences. Typically, the SHM problem can be addressed as a statistical pattern recognition paradigm with three main components:

1) A numerical model that accurately represents the governing system dynamics
2) Real-time data acquisition and management system
3) A sequential data assimilation technique that relies on a set of observational data to calibrate and update the underlying dynamic principles governing the system under observation.

In such context, numerous uncertainties associated with modeling, parametric and measurement errors could be introduced. In cases where these uncertainties are significant, standard identification and damage detection techniques are either unsuitable or inefficient. Therefore, the need rises for robust system identification algorithms that can tackle the aforementioned challenges. This has been a very active research area over the past decade [3, 4, 6, 8, 9, 11, 12].

Sequential data assimilation has been widely used for structural health monitoring and system identification problems. Many extensions of the Kalman Filter were developed as adaptations to important classes of these problems. While the Extended Kalman Filter may fail in the presence of high non-linearities, Monte Carlo based Kalman Filters usually give satisfactory results. The Ensemble Kalman Filter (EnKF) [1, 2] was recently used for damage detection in strongly nonlinear systems [4], where it is combined with non-parametric modeling techniques to tackle structural health monitoring for non-linear systems. The EnKF uses a Monte Carlo Simulation scheme for characterizing the noise in the system, and therefore allows representing non-Gaussian perturbations. Although this combination gives good results, it requires a relatively accurate representation of the non-linear system dynamics. It also requires a large ensemble size to quantify the non-Gaussian uncertainties in such systems and consequently imposes a high computational cost.

This study presents a system identification approach based on coupling robust non-parametric non-linear models with the Polynomial Chaos methodology in the context of the Kalman Filtering techniques [10]. The proposed approach uses a Polynomial Chaos expansion [7] of the nonparametric representation of the system's non-linearity to statistically characterize the system's behavior. A filtering technique that allows the propagation of a stochastic representation of the unknown variables using Polynomial Chaos is used to identify the chaos coefficients of the unknown parameters in the model. The introduced filter is a modification of the EnKF that uses the Polynomial Chaos methodology to represent uncertainties in the system. This allows the representation of non-Gaussian uncertainties in a simpler, less taxing way without the necessity of managing a large ensemble. It also allows obtaining the probability density function of the model state or parameters at any instant in time by simply simulating the Polynomial Chaos basis.

2 THE POLYNOMIAL CHAOS KALMAN FILTER (PCKF)

The Kalman Filter is an optimal sequential data assimilation method for linear dynamics and measurement processes with Gaussian error statistics. The PCKF builds on the mathematics of the original Kalman Filter to allow the propagation of a stochastic representation of the unknown variables using Polynomial Chaos. In the PCKF, the model state is given by,

$$x = \sum_{i=0}^{P} x_i \psi_i(\xi), \tag{1}$$

where, $P+1$, is the number of terms in the Polynomial expansion of the state vector, $\{\psi_i\}$ is set of Hermite polynomials function of the Gaussian random variable, ξ. Consequently, the covariance matrix of the model state is defined around the mean, the zero order term, of the stochastic representation,

$$\begin{aligned} P &\approx \left\langle \left(\sum_{i=0}^{P} x_i \psi_i - x_0 \right) \left(\sum_{i=0}^{P} x_i \psi_i - x_0 \right)^T \right\rangle \\ &\approx \left\langle \left(\sum_{i=1}^{P} x_i \psi_i \right) \left(\sum_{i=1}^{P} x_i \psi_i \right)^T \right\rangle \\ &\approx \sum_{i=1}^{P} x_i x_i^T \left\langle \psi_i^2 \right\rangle, \end{aligned} \tag{2}$$

where, P is the covariance matrix, and $\langle\ \rangle$ denotes the mathematical expectation. The Polynomial Chaos representation depicts all the information available through the complete probability density function, and therefore allows the propagation of all the statistical moments of the unknown parameters and variables.

The observations are also treated as random variables represented via a Polynomial Chaos expansion with a mean equal to the first-guess observations. Since the model and measurement errors are assumed to be independent, the latter is represented as a Markov process

2.1 Analysis Scheme

For computational efficiency, the dimensionality and order of the Polynomial Chaos expansion are homogenized through out the solution. These parameters are initially defined based on the uncertainty within the problem at hand and are assumed to be constant thereafter. Since the model state and measurement vectors are assumed independent, the Polynomial Chaos representation of these variables has a sparse structure.

Let $\boldsymbol{A}$ be the matrix holding the chaos coefficients of the state vector $y \in R^n$,

$$\mathbf{A} = (x_0, x_1, \ldots, x_P) \in R^{n \times (P+1)}, \tag{3}$$

where $P+1$ is the total number of terms in the Polynomial Chaos representation of x and n is the size of the model state vector. The mean of x is stored in the first column of $\boldsymbol{A}$ and is denoted by x_0. The state perturbations are given by the higher order terms stored in the remaining columns. Consequently, the state error covariance matrix $\boldsymbol{P} \in R^{n \times n}$is defined as:

$$\mathbf{P} = \sum_{i=1}^{P} x_i x_i^T \left\langle \psi_i^2 \right\rangle \tag{4}$$

Given a vector of measurements $d \in R^m$, with m being the number of measurements at each occurrence, a Polynomial chaos representation of the measurements is defined as

$$d = \sum_{j=0}^{P} d_j \psi_j(\xi), \tag{5}$$

where the mean d_0 is given by the actual measurement vector, and the higher order terms represent the measurement uncertainties. The representation d can be stored in matrix form as:

$$\mathbf{B} = (d_0, d_1, ..., d_P) \in R^{m\times(P+1)}. \tag{6}$$

Based on Eq. 5, the measurement error covariance matrix, $\boldsymbol{R}$, is defined as:

$$\mathbf{R} = \sum_{i=1}^{P} d_i d_i^T \left\langle \psi_i^2 \right\rangle \in R^{m\times m} \tag{7}$$

The Kalman Filter forecast step is carried out by employing a stochastic Galerkin scheme, and the assimilation step consists if the traditional Kalman Filter correction step applied on the Polynomial Chaos expansion of the model state vector,

$$\sum_{i=0}^{P} x_i^a \psi_i = \sum_{i=0}^{P} x_i^f \psi_i + \mathbf{PH}^T (\mathbf{HPH}^T + \mathbf{R})^{-1} \left(\sum_{i=0}^{P} d_i \psi_i - \mathbf{H} \sum_{i=0}^{P} x_i^f \psi_i \right) \tag{8}$$

where, $\mathbf{H}$ is the observation matrix, and the superscripts f and a represent the forecast and analysis states respectively. Projecting the above equation on an approximating space spanned by the Polynomial Chaos $\{\psi_i\}_{i=0}^{P}$ yields,

$$x_i^a = x_i^f + \mathbf{PH}^T (\mathbf{HPH}^T + \mathbf{R})^{-1} \left(d_i - \mathbf{H} x_i^f \right) \qquad for \quad i = 0, 1, ..., P. \tag{9}$$

In matrix form, the assimilation step is expressed as:

$$\mathbf{A}^a = \mathbf{A}^f + \mathbf{PH}^T \left(\mathbf{HPH}^T + \mathbf{R} \right)^{-1} \left(\mathbf{B} - \mathbf{HA}^f \right) \tag{10}$$

3 NUMERICAL EXAMPLE

The efficiency of the presented method is assessed by applying it to the structural health monitoring of the four-story shear building shown in Figure 1. This model has a constant stiffness on each floor and a 5% damping ratio in all modes. All structural elements of this frame are assumed to involve hysteretic behavior, and it is supposed that a change in the hysteretic loop of the first floor element occurs at some point. It is of utmost importance to localize that point in time and track the state of the system throughout and subsequent to that point.

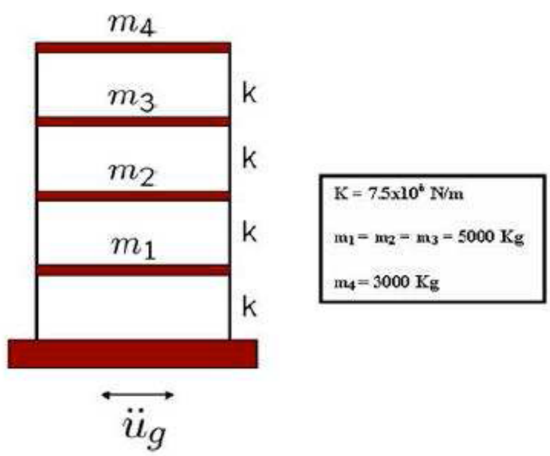


Figure 1: Shear Building Under Analysis

A synthetically generated dataset representing measurements of the displacements and velocities at each floor is obtained by representing the hysteretic restoring force by the Bouc-Wen model, which is therefore considered as the exact hysteretic behavior of the system. Thus, the equation of motion of the system is given by,

$$M\ddot{u}(t)+C\dot{u}(t)+K_{el}u(t)+(1-\alpha)K_{in}z(x,t)=-M\tau\ddot{u}_g(t) \tag{11}$$

where, M, C, K_{el}, and K_{in} are the mass, damping, elastic and inelastic stiffness matrices respectively; α is the ratio of the post yielding stiffness to the elastic stiffness, τ is the influence vector, u is the displacement vector, x is the inter-story drift vector, and z is an n-dimensional evolutionary hysteretic vector whose i^{th} component is give by the Bouc-Wen model as,

$$\dot{z}_i = A_i\dot{x}_i - \beta|\dot{x}_i||z_i|^{n_i-1} - \gamma_i\dot{x}_i|z_i|^{n_i}, \qquad i=1,\ldots,n \tag{12}$$

A, β, and ϒ are the Bouc-Wen model parameters. The adopted values for these parameters are shown in Table 1.

Model Coef.	Pre-Change	Post-Change
α	0.15	0.15
β	0.1	10
n	1	1
γ	0.1	10
A	1	1

Table 1: Bouc-Wen Model Coefficients

The structure is subject to a base motion specified by a time series consistent with the El-Centro earthquake shown in Figure 2, and a change of the first floor hysteric behavior is assumed to take place five seconds after the excitation. A monitoring scenario where it is assumed that measurements are available every 5 time steps is adopted. A nonparametric representation of the system nonlinearity is adopted, and the filtering technique is used to characterize the latter representation in order to capture any ambiguous behavior of the structure examined.

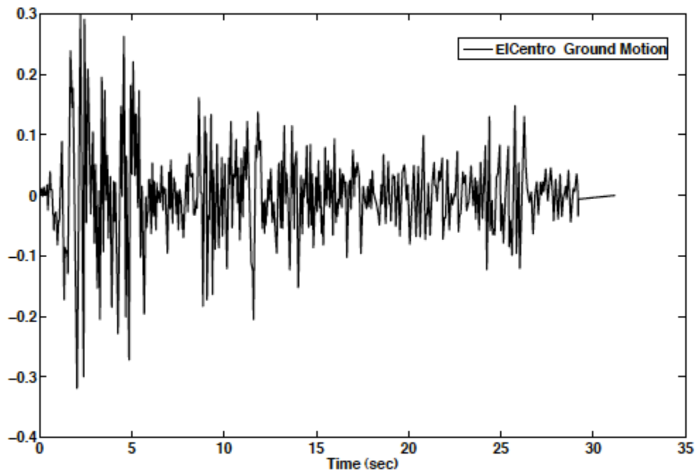


Figure 2: The Elcentro Excitation Applied to the Structure

4 NON-PARAMETRIC REPRESENTATION OF THE NON-LINEARITY

The proposed filtering methodology is combined with a non-parametric modeling technique to tackle structural health monitoring of non-linear systems but instead of adopting a deterministic nonparametric representation of the non-linearity, a stochastic representation via Polynomial Chaos is used. The basic idea behind the non-parametric identification technique used is to determine an approximating analytical function $\hat{F}$ that approximates the actual system non-linearities, with the form of $\hat{F}$ including suitable basis functions that are adapted to the problem at hand [8]. For general non-linear systems, a suitable choice of basis would be the list of terms in the power series expansion in the doubly indexed series,

$$S = \sum_{i=0}^{i_{\max}} \sum_{j=0}^{j_{\max}} u^i \dot{u}^j, \tag{13}$$

where u and $\dot{u}$ are used to represent the system's displacement and velocity respectively. Therefore, if $i_{max} = 3$ and $j_{max} = 3$, the basis functions become:

$$basis = \left\{1, \dot{u}, \dot{u}^2, \dot{u}^3, u, u\dot{u}, u\dot{u}^2, u\dot{u}^3, u^2, u^2\dot{u}, u^2\dot{u}^2, u^2\dot{u}^3, u^3, u^3\dot{u}, u^3\dot{u}^2, u^3\dot{u}^3\right\} \tag{14}$$

In the proposed method the displacements and velocities are stochastic processes represented by their Polynomial Chaos expansion. Thus, the approximating function is also expressed as a stochastic process via a Polynomial Chaos representation. The model adopted within the Kalman Filter is hence given by

$$M\ddot{u}(t) + F(u,\dot{u}) = -M\tau\ddot{u}_g(t) \tag{15}$$

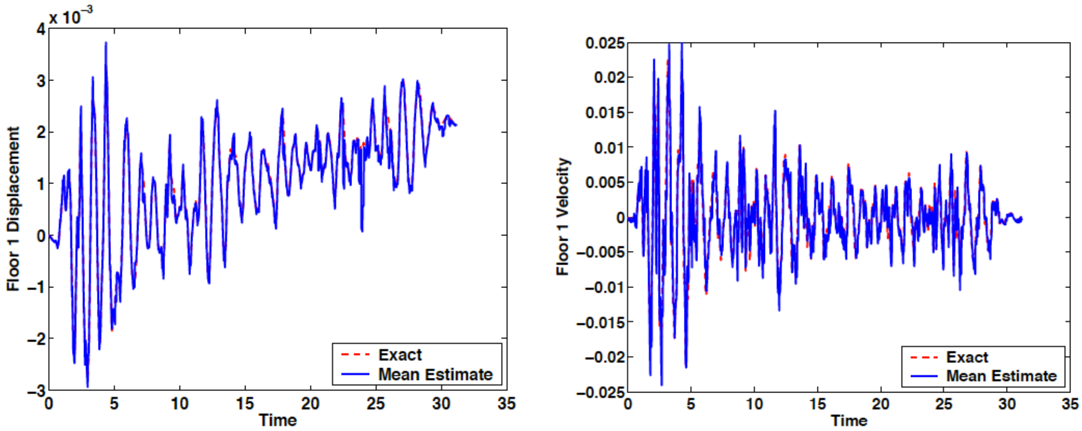
where, F is the non-parametric representation of the non-linearity whose i^{th} floor component is given by

$$\begin{aligned}
F^i &\approx \sum_j F^i_j(u,\dot{u})\psi_j \\
F^i &\approx \sum_j a^i_j \psi_j \left(\sum_k (u_k - u^{i-1}_k)\psi_k \right) + \sum_j a^{i+1}_j \psi_j \left(\sum_k (u^i_k - u^{i+1}_k)\psi_k \right) \\
&+ \sum_j b^i_j \psi_j \left(\sum_k (u^i_k - u^{i-1}_k)\psi_k \right)^2 + \sum_j b^{i+1}_j \psi_j \left(\sum_k (u^i_k - u^{i+1}_k)\psi_k \right)^2 \\
&+ \sum_j c^i_j \psi_j \left(\sum_k (\dot{u}^i_k - \dot{u}^{i-1}_k)\psi_k \right) + \sum_j c^{i+1}_j \psi_j \left(\sum_k (\dot{u}^i_k - \dot{u}^{i+1}_k)\psi_k \right) \\
&+ \sum_j d^i_j \psi_j \left(\sum_k (u^i_k - u^{i-1}_k)\psi_k \right)\left(\sum_l (\dot{u}^i_l - \dot{u}^{i-1}_l)\psi_l \right) \\
&+ \sum_j d^{i+1}_j \psi_j \left(\sum_k (u^i_k - u^{i+1}_k)\psi_k \right)\left(\sum_l (\dot{u}^i_l - \dot{u}^{i+1}_l)\psi_l \right)
\end{aligned} \tag{16}$$

In the above equation, $\{a_j\}, \{b_j\}, \{c_j\}$, and $\{d_j\}$ represent the chaos coefficients of the unknown parameters to be identified. The fourth order Runge-Kutta method is used for the time stepping and a stochastic Galerkin approach is employed to solve the system at each time step.

5 RESULTS

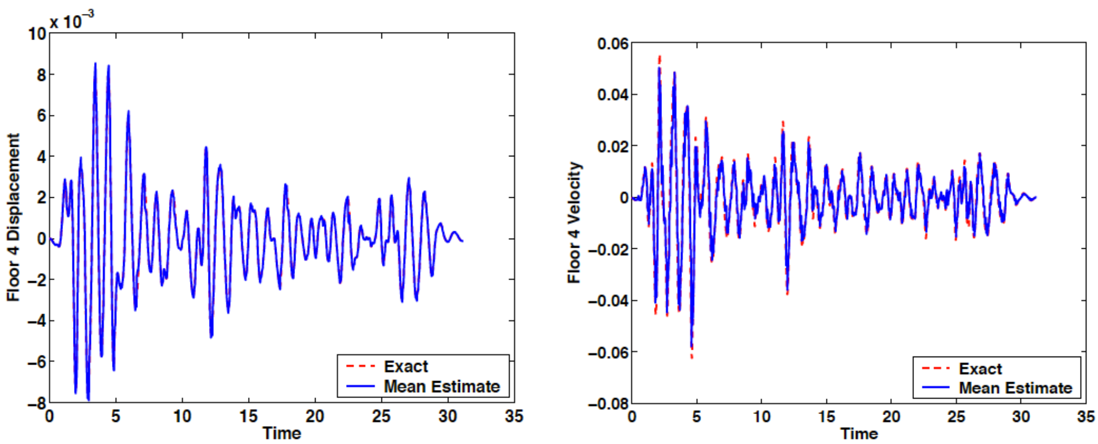
In the numerical example, it is assumed that observations of displacements and velocities from all floors are available. The noise signals perturbing both the model and measurements are modeled as first order, one dimensional, independent, Polynomial Chaos expansions having zero-mean and an RMS of 0.05 and 0.001 respectively. The parametric uncertainties on the other hand, are modeled as second order, one dimensional, Polynomial Chaos expansions whose coefficients are to be determined in accordance with the available observations. This is done to incorporate the possibility that the unknown parameters may deviate from Gaussianity. Furthermore, it is assumed that the first floor undergoes a change in its hysteretic behavior 5 seconds after the ground excitation. The purpose of the application is to detect this behavioral change.



(a) (b)

Figure 3: Estimate of the first floor parameters, (a) displacement, (b) velocity

Figure 3 and Figure 4 describe the tracking of the displacement and velocity for the first and fourth floor respectively. Excellent match between the results estimated using the Polynomial Chaos based Kalman Filter and the true state is observed.



(a) (b)

Figure 4: Estimate of the fourth floor parameters, (a) displacement, (b) velocity

Figure 5 presents the evolution of the mean of the unknown parameters identified by the proposed filtering technique. Error bars representing the scatter in the estimated parameters are also present in Figure 5. The different jumps within the parameters are associated with the perks in the corresponding excitation.

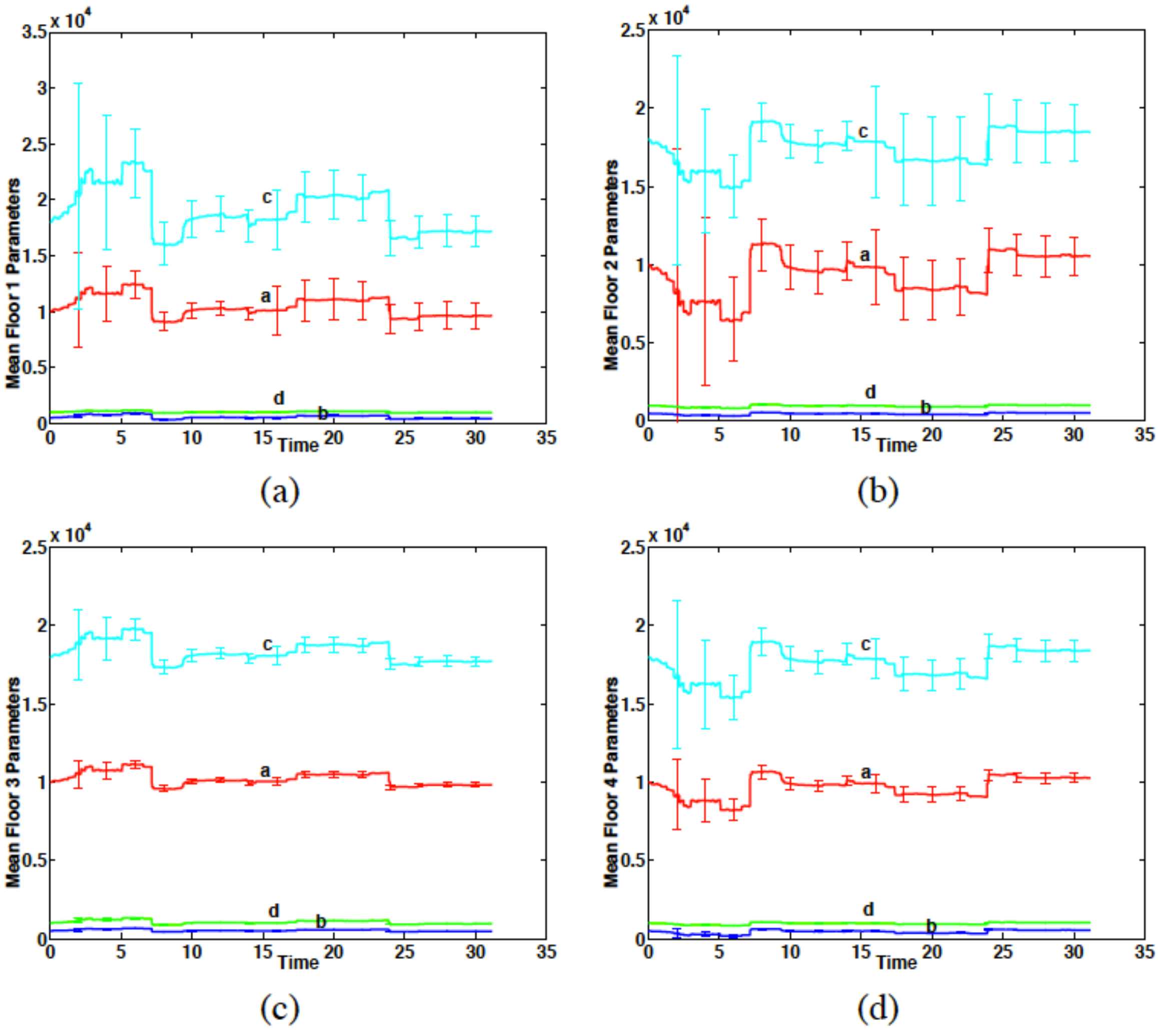


Figure 5: Estimate of the mean floor parameters

Further investigation of the parameters indicates that the main changes take place in the first floor following the 5sec time interval. Note that the parameters a and c in floors 1 and 2 undergo the greatest jumps since they are associated with inter-story drift and velocity, respectively. One of the main advantages of using the Polynomial Chaos Kalman filter is that is provides a scatter around the estimated parameters. This is represented by the probability density functions corresponding to each of the estimated parameters. Figure 6 presents the probability density functions of the estimated floor 1 parameters.

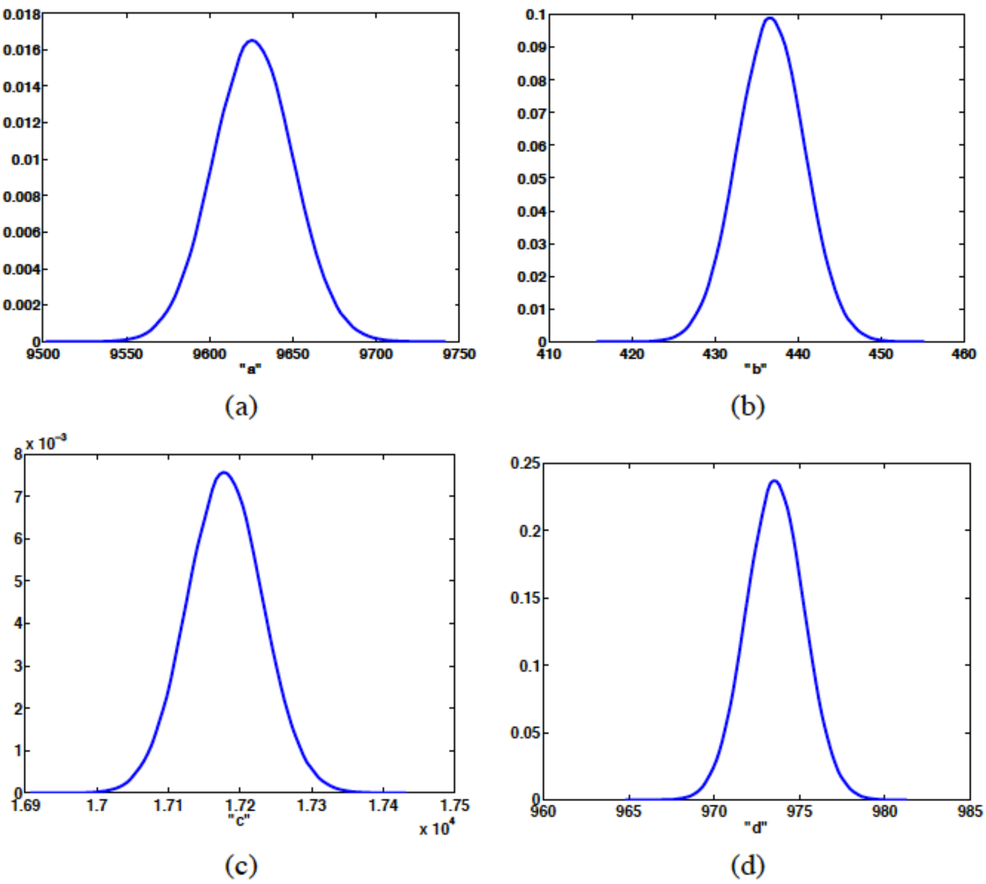


Figure 6: Probability density functions of the estimated floor 1 parameters

6 CONCLUSIONS

The combination of Polynomial Chaos with the Ensemble Kalman Filter renders an efficient data assimilation methodology that competes with other Kalman Filtering techniques while maintaining a relatively low computational cost. Although the proposed method employs traditional Kalman Filter updating schemes, it preserves all the error statistics, and hence allows the computation of the probability density function of the uncertain parameters and variables at all time steps. This is achieved by simply simulating the Polynomial Chaos representation of these parameters. Together with the non-parametric representation of the nonlinearities, the approach constitutes an effective system identification technique that accurately detects any changes in the systems behavior. The Polynomial Chaos representation of the non-parametric model for the nonlinearities is a robust innovative approach that permits damage identification and tracking the dynamical state beyond that point. Using Polynomial Chaos, the uncertainty associated with the assumed non-parametric model is inherently present and thus represents the actual nonlinearity in a more accurate way.

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