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COUPLED DAMAGE-PLASTICITY BASED CONSTITUTIVE MODELING OF METALLIC MEMBRANE ELEMENT UNDER CYCLIC LOADING

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Abstract. *Micromechanics is employed in order to define the whole nonlinear inelastic behavior of a structure at meso/macro scale level where the presence of the inelastic phenomena (plasticity and/or damage) affects the material constitutive response depending on the loading conditions. This concept is important in the design of civil and mechanical engineering applications. This study illustrates a comprehensive theoretical formulation for a coupled damage-plasticity model and its numerical implementation under an extreme loading type such as an earthquake, which causes cyclic response and can lead to failure. Irreversible plastic deformation by plasticity, elastic response modification by the damage and the cyclic accumulation of deformation are modeled. A couple of numerical examples are presented in order to show the capability and efficiency of the proposed model for 2D membrane element, by using the operator split methodology.*

1 INTRODUCTION

Materials, which are used in the different domains of civil and mechanical engineering, can most probably be heterogeneous at a micro-scale. It may be difficult to predict the response of the whole structure to different kinds of loading due to the material characteristics.

Simo et al. [1] employed the stress-based formulation, which is the core of the numerical part of this current study for the elasto-plastic material behavior by using the classical finite element method [2]. Here, this type of formulation is constructed in order to couple both plasticity and damage [3]. The dependent Gauss point equations, which represent the evolution equations of the internal variables for both inelastic behaviors just mentioned, are solved simultaneously, so as to provide a single return mapping algorithm per element to control the equilibrium equations at macro scale. We use the operator split method to simplify the details of the numerical implementation, concerning the calculation of internal variables and equilibrium equations resulting with finite element approach at the structural state.

Study realized by Drucker and Palgen [4] stated that the material behavior under cyclic loading are much more complex than monotonic loading and cannot be modeled by isotropy alone. Therefore, the purpose of the present research is to present a plasticity model, in which both isotropic and kinematic hardening is taken into account. This will also be coupled with the damage model, which is defined in an analogous way as the plasticity model.

There are some important notifications based on researches [5, 6] for cyclic inelastic models; (i) symmetric stress and strain cycles occur with a well defined kinematic hardening, (ii) unsymmetrical stress cycles cause the ratcheting effect and (iii) unsymmetrical strain cycles cause the progressive relaxation.

This paper is organized as follows. In section 2, we describe basic concepts of inelastic behavior of the model. In section 3, the computational algorithm is presented. Section 4 is devoted to numerical examples for illustrating the proposed model. Finally, a brief conclusion is presented in section 5.

2 MODEL FORMULATION

2.1 Basic concepts of internal variables

State variables defining the inelastic behavior of the material, which are composed of the plastic strain ($\boldsymbol{\varepsilon}^p$), damage compliance ($\mathbf{D}$), the internal variables ($\xi^p, \boldsymbol{\kappa}^p$), which control isotropic and kinematic hardening of the plasticity, and the internal variable (ξ^d) of damage, are obtained by using the standard thermodynamic consideration

$$0 \le \dot{\mathcal{D}} = \sigma : \dot{\boldsymbol{\varepsilon}} - \dot{\psi} \tag{1}$$

and the principle of maximum plastic and damage dissipations, which are decomposed from total inelastic dissipation $\mathcal{D}$, with these three fundamental equations firstly defining;

- the decomposition of the total strain

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^p + \boldsymbol{\varepsilon}^d \tag{2}$$

- the total strain energy

$$\psi(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^d, \mathbf{D}, \xi^d, \boldsymbol{\varepsilon}^p, \xi^p, \boldsymbol{\kappa}^p) = \psi^e(\boldsymbol{\varepsilon}^e) + \psi^d(\boldsymbol{\varepsilon}^e, \mathbf{D}) + \Xi^p(\xi^p) + \Xi^d(\xi^d) + \Lambda^p(\boldsymbol{\kappa}^p) \tag{3}$$

- the yield criteria of plasticity and damage

$$\phi^p(\boldsymbol{\sigma}, q^p, \boldsymbol{\alpha}) = \|\mathbf{Dev}\boldsymbol{\sigma} + \boldsymbol{\alpha}\| - \sqrt{\frac{2}{3}}\left(\sigma_y - q^p\right) \leq 0$$
$$\phi^d(\boldsymbol{\sigma}, q^d) = \frac{1}{3}\mathbf{Tr}(\boldsymbol{\sigma}) - \left(\sigma_f - q^d\right) \leq 0 \tag{4}$$

where q^p, q^d are the stress-like variables describing the hardening phenomena, $\boldsymbol{\alpha}$ is the deviatoric back stress representing the kinematic hardening behavior and σ_y, σ_f are the yield and fracture stresses, respectively.

The principle of maximum inelastic (plastic and damage) dissipation states that among all the admissible values of dual variables $(\boldsymbol{\sigma}, q^p, \boldsymbol{\alpha}, q^d)$, those, which maximize the plastic and damage dissipation, must be selected. This maximization problem is presented as a minimization problem by introducing Lagrange multiplier. We obtain the evolution equations of internal variables for plasticity and damage phenomenon separately by using the Kuhn-Tucker optimality conditions. Once the suitable values of internal variables are obtained, we can go through the finite element calculation.

2.2 Variational formulation

We use the mixed variational formulation of the Hellinger-Reissner type in order to couple both plastic and damage behaviors for fixed given values of internal variables.

$$\Pi(\boldsymbol{\sigma}, \mathbf{u}) = \int_V \left(\psi^e(\boldsymbol{\varepsilon}^e) + \psi^d(\boldsymbol{\varepsilon}^d, \mathbf{D})\right) dV - \int_{\partial V} \bar{\mathbf{t}}.\mathbf{u} dS \tag{5}$$

where we can rewrite the same functional by using the complementary energies instead of the strain energies.

$$\Pi(\boldsymbol{\sigma}, \mathbf{u}) = \int_V \left(\sigma : \boldsymbol{\varepsilon}^e - \chi^e(\boldsymbol{\sigma}) + \boldsymbol{\sigma} : \boldsymbol{\varepsilon}^d - \chi^d(\boldsymbol{\sigma}, \mathbf{D})\right) dV - \int_{\partial V} \bar{\mathbf{t}}.\mathbf{u} dS \tag{6}$$

The stationary conditions of the functional for each independent displacement and stress fields are taken into account as follows.

$$G_u(\mathbf{u}, \boldsymbol{\sigma}, \boldsymbol{\delta}\mathbf{u}) = \int_\Omega \nabla^s \boldsymbol{\delta}\mathbf{u}\boldsymbol{\sigma} dV - \int_{\Gamma_\sigma} \boldsymbol{\delta}\mathbf{u}\bar{\mathbf{t}} dS$$
$$G_\sigma(\mathbf{u}, \boldsymbol{\sigma}, \boldsymbol{\delta\sigma}) = \int_\Omega \boldsymbol{\delta\sigma}(\nabla^s \mathbf{u} - \boldsymbol{\varepsilon}^p - \frac{\partial \chi^e}{\partial \boldsymbol{\sigma}} - \frac{\partial \chi^d}{\partial \boldsymbol{\sigma}}) dV \tag{7}$$

The first equation is the weak form the local equilibrium equation based on Euler-Lagrange equation and the second one is the weak form of the additive decomposition field.

The discretized model is constructed from the weak formulations (7) by using the interpolation functions for the stress (**S)** and the displacement (**N)** fields.

$$\mathbf{u} = \mathbf{N}\mathbf{U}$$
$$\boldsymbol{\sigma} = \mathbf{S}\boldsymbol{\beta} \tag{8}$$

Next, we implement into the equation (7) the internal variables considering with the time integration and then define the residuals, which is commonly used for the finite element method.

3 COMPUTATIONAL ALGORITHM

We define three levels of computations; (i) local level computation at each Gauss quadrature point for plastic and damage internal variables, for which the implicit backward Euler time integration is used, (ii) element level computation, which is characterized by the stress field, (iii) global level computation of the set of equilibrium equations from which we obtain the displacement values. We verify the convergence of the result at a given level in the spirit of operator-split approach before going through the subsequent level. Newton method is used in order to solve the nonlinear equations, for which 3 to 10 iterations are sufficient at each level. This computational model is implemented into FEAP [7] for numerical examples.

4 NUMERICAL EXAMPLES

In this part, three numerical simulations are presented for a quadrilateral element, which is fixed at one side with displacements or forces imposed at the other side. It is shown that different kinds of cyclic response can be obtained due to the loading types. A comparison is made between the plasticity versus coupled plasticity-damage phenomenon for the same material. The characteristic of the material, which is stainless steel 304, is given in Table 1.

Elasticity Modulus E (MPa)	Poisson ratio ν	Yield stress σ_y (MPa)	Saturation stress σ_∞ (MPa)	Fracture stress σ_f (MPa)
$1,93*10^5$	0,29	241	579	300

Table 1: Characteristics of the stainless steel 304

4.1 Cyclic response for symmetric imposed displacement

In this example displacement is imposed at the right side of the membrane, which value is between 0.05 and -0.05 in the direction of x-axis for a time interval 0 to 25 seconds, defining as “loading condition 1” in Fig.1.

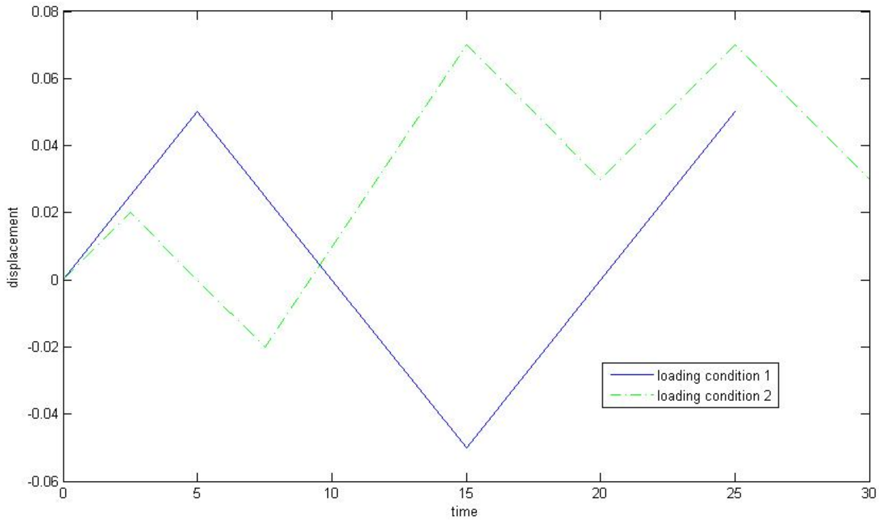

Figure 1: Comparison of the plasticity and coupled plasticity-damage behavior

The diagram obtained from this type of loading shows the coupled plasticity-damage effect over the plasticity phenomenon. It can be seen that the stress values decrease in the same imposed displacement by taking into account damage and plasticity together. Besides, the slope, which gives an idea about the tangent modulus of the material, is reduced for the coupled phenomenon.

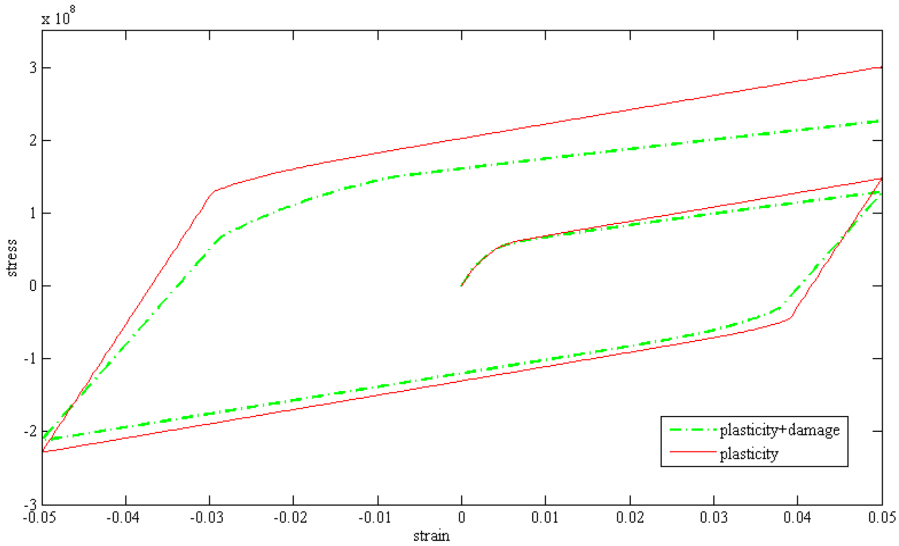

Figure 2: Comparison of the plasticity and coupled plasticity-damage behavior

4.2 Progressive relaxation effect

This response of the material is caused by the strain cycling between any two fixed values. We should define the cyclic loading condition in two parts. At the beginning, it takes the values between [-0.02, 0.02] in the time interval T [0, 7.5s] and then jumps to the value of 0.07. This type of imposed displacement is presented with the name “loading condition 2” in Fig.1. We can see from Fig.3 that stress attained by the element increase with these increasing levels of such imposed displacement.

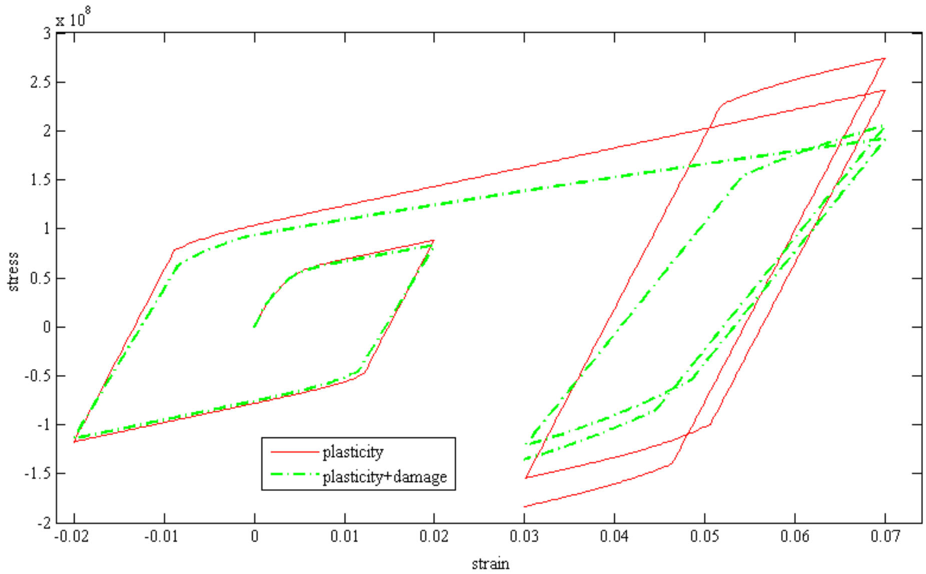

Figure 3: Strain-stress response showing the progressive relaxation phenomenon

4.3 Ratcheting effect

A cyclic loading program in tension and compression between fixed values of stress as seen in Fig.4 has been performed in order to simulate the ratcheting effect of the constitutive material behavior.

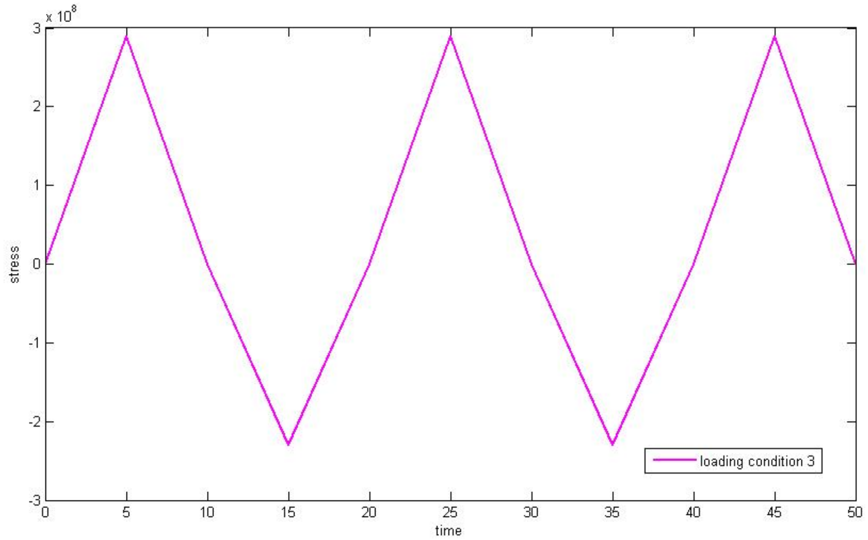

Figure 4: Stress-strain response to cyclic loading resulting in ratcheting of strain

Ratcheting, which is one of the characteristic of the material, is presented here for the asymmetrical stress cycling. The ratcheting deformation accumulates continuously with the applied number of cycles. It can be inferred that the strain limits of the cycles are displaced progressively along the strain axis from one cycle to the next one.

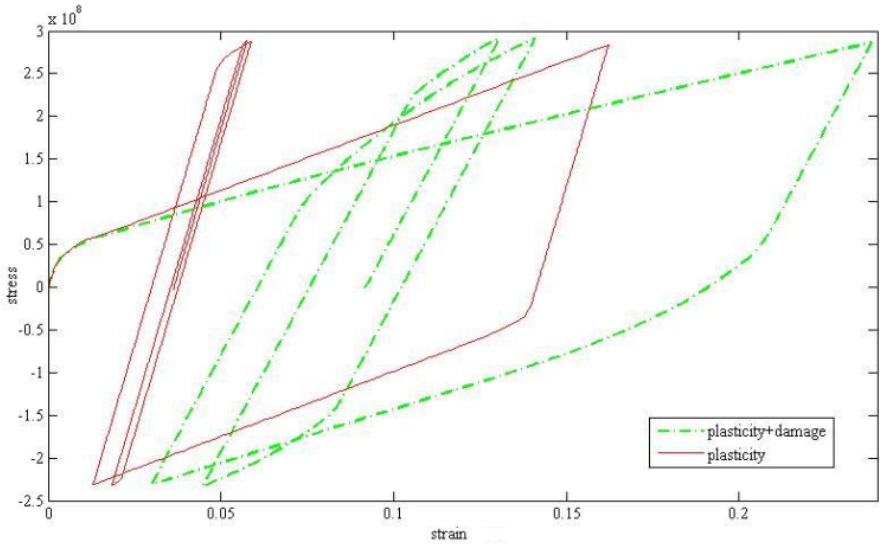

Figure 5: Stress-strain response to cyclic loading resulting in ratcheting of strain.

5 CONCLUSION

In this study, we have presented the phenomenological model of classical plasticity, which is capable of accounting for both isotropic and kinematic hardening effects, and damage model, which is described in an analogous way to the classical plasticity model. The coupled plasticity-damage model points out the irreversible deformation and change of elastic response. The defined elasto-plastic damage model is adapted to the stress based formulation. The mechanism of damage and elasto-plasticity behavior depends on the stress field. For that purpose, the mixed variational type of Hellinger-Reissner is used to develop the finite element approach.

Numerical computations and results are shown in order to illustrate the effectiveness of the algorithmic procedure. The performed analysis allows to have better understanding on the inelastic behavior, which depend on the cyclic loading conditions.

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