COMPDYN 2011
3rd ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis, V. Plevris (eds.)
Corfu, Greece, 25–28 May 2011

NON-STATIONARY PROBLEMS OF ELASTIC WAVEGUIDES WITH INCLUSIONS

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Keywords: Elastic waveguide, Localizing oscillations, Trapped modes, Film delamination.

Abstract. *In modern constructions, thin-layer coats are often used as protecting or strengthening elements. Deformations of such constructions may cause significant stresses on the interface between the base and the coat because of the difference in their physical-mechanical properties, which leads to the destruction or delamination of the cover. Of special interest is strength analysis under dynamical or vibrational impacts because of the possibility of localizing oscillations in a neighborhood of the initial inhomogeneities (such as inclusions, defects, construction elements, etc.). In this paper, on the example of the delamination of a string from an elastic substrate, the possibility of localizing oscillations on a delamination defect is demonstrated and the effect of this localization on the growth of the delamination zone is analyzed. A simplified setting of the problem is considered. The possibility of localizing oscillations on a delamination defect is demonstrated and an approximate analytical solution is constructed, which takes into account only the first symmetric form of oscillations describing the development of the initial delamination. A numerical modeling of the problem is performed, and the results of modeling are compared with the approximate analytical solution.*

1 INTRODUCTION

In modern constructions, thin-layer coats are often used as protecting or strengthening elements. Deformations of such constructions may cause significant stresses on the interface between the base and the coat because of the difference in their physical-mechanical properties, which leads to the destruction or delamination of the cover. Delamination of multilayer constructions under static and dynamic (shock) loads have an extensive bibliography. We mention the fundamental works [1-8]. The impact of static or shock loads on the emergence and development of delamination in multilayer constructions has been studied fairly well, but much less is known about destructions of this type under nonstationary (vibrational) loads. The interest in the latter is caused by the fact that even small variable actions may cause the localization of oscillations in a neighborhood of inhomogeneities (such as inclusions, defects, construction elements, etc.) [9] and lead to the emergence and growth of defects, which strongly affects the reliability and functionality of the whole construction.

An example of such coated constructions in which delamination destructions are observed is the blades of a wind turbine, which are becoming increasingly popular because of the interest in alternative energy sources. In recent years, the increase in the power of wind turbines has enhanced requirements to rotor blades.

During the operation of the rotor, the top coat of a blade, which is a thin film, may detach on some part of its length. To estimate the functionality of the construction, it is important to know the further scenario of the behavior of such a film with a detached fragment. Such defects may appear and grow for several reasons, which include wind loads and stationary oscillations arising in blades during their operation.

This paper studies the possibility of oscillation localization near delaminations from cracks and the impact of localization on the growth of the defects. We suggest a model for analyzing conditions under which a delamination zone grows or ceases to grow.

2 STATEMENT OF THE PROBLEM

Taking into account the fact that the thickness of a thin-layer coat is usually much less than the characteristic size of the substrate, in the first approximation, we replace a coat attached to a substrate by a film on an elastic substrate.

The main purpose of this paper is to show that, if an elastic substrate has an inhomogeneity (the coefficient of the elastic substrate vanishes for some part of the film length), then, under a non-stationary harmonic load, some part of the propagating wave energy is localized near the inhomogeneity. Depending on the parameters of the film and the elastic substrate, various scenarios of the film behavior are possible: the length delamination zone may increase unlimitedly, or it may cease to grow at some time.

Note that if there are only propagating waves in the film, then the wave processes only insignificantly affect the behavior of the delamination zone, because the excited wave process rapidly fades out. The situation changes if the waves in the film-waveguide localize near the defect. As is known, in the absence of dissipation, localized oscillations of certain frequencies may not attenuate for an infinitely long time [9], thereby substantially affecting the behavior of the delamination zone. The existence of standing waves localized near the delamination zone means that the corresponding spectral problem has not only continuous but also discrete spectrum of eigenfrequencies.

This paper considers the simplest mathematical model for the behavior of such a film under the action of a transverse non-stationary harmonic load. The film is attached to an elastic Win-

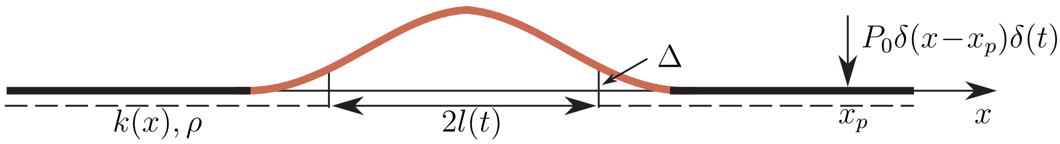

Figure 1: A scheme of film delamination

kler foundation, which models an element of a blade. In the framework of this model, the birth of a delamination is not studied; it is assumed that, at initial moment of time, a film fragment of length $2l_0$ is torn away from the elastic substrate.

We assume that the delamination emerges not near the blade boundaries, which is confirmed by in-situ observations. The tension of the detached part of the film differs from the tension of the film on the elastic substrate, and when the detached part oscillates, the interaction between the film and the elastic substrate has the character of a one-sided contact, and the interaction of the film with the substrate has a relay characteristic. These two factors significantly complicate constructing a solution; for this reason, at the present stage of modeling, we assume that tension is constant over the entire film and the detached part does not interact with the elastic substrate. Vibro-impact loads in the delamination zone and a variable tension of the film will be taken into account at the next modeling stage.

As the simplest model we consider the following case, in which the problem under consideration reduces to a one-dimensional problem: the delamination zone is contained in the domain $-l_0 < x < l_0$, $-\infty < y < \infty$, and the force is applied along the line $x = x_p$, $-\infty < y < \infty$. In this case, the problem reduces to the problem about oscillations of a string on an elastic substrate with variable stiffness $k[x, l(t)]$: namely, stiffness vanishes on the detached part of the string, and on remaining part of the string, it equals k_0. The equation describing the displacement of a string on an elastic substrate under a load applied at a point $x = x_p$ has the form

$$\begin{aligned} &\rho u_{tt} - T u_{xx} + k[x, l(t)]u = -P(x,t), \quad -\infty < x < \infty, \\ &u,\ u_x \to 0 \qquad |x| \to \infty \end{aligned} \tag{1}$$

Here, ρ is the specific density of the string, u is the vertical displacement of the string, T is the tension of the unperturbed string, and $P(t)\delta(x - x_p)$ is the exciting force. We assume that, at the initial moment of time $t = 0$, there is a film fragment of length $2l_0$ detached from the elastic substrate. For the delamination criterion we take the following deformation criterion: when the displacement of at least one end of the detached part attains a critical value Δ, the delamination zone grows with rate β (see Fig. 1). The equation describing the growth of the delamination zone has the form [8]

$$\frac{\mathrm{d}l}{\mathrm{d}t} = \beta\Big\{H\Big[u(x,t)|_{x=l_-(t)} - \Delta\Big] + H\Big[u(x,t)|_{x=l+(t)} - \Delta\Big]\Big\} \tag{2}$$

Here, $l_-(t)$ is the coordinate of the left end of the detached fragment, $l_+(t)$ is the coordinate of the right end of the detached fragment, $2l(t) = l_- + l_+$ is the length of the detached fragment, $u(x,t)$ is the displacement of the string point with coordinate x at a moment t, H is the Heaviside function, β is the coefficient determining the growth rate of the delamination, and Δ is the critical displacement, under which the film separates from the elastic substrate.

Under certain conditions, oscillations may localize on the initial delamination fragment of length $2l_0$ (see [9]), and the amplitude of oscillations at the endpoints of the delamination zone may surpass the critical displacement value Δ at some moment of time, which leads to the growth of the delamination zone. The increase in the length of the detached film fragment, in turn, affects the localization of oscillations near the defect, so that the passage to other forms of localized oscillations may occur. Therefore, we must solve the system of equations (1) and (2) under the initial conditions

$$l|_{t=0} = l_0; \quad u, u_t|_{t=0} = 0 \tag{3}$$

and the following boundary conditions for any fixed t:

$$u,\ u_x \to 0 \qquad |x| \to \infty \tag{4}$$

Thus, we have obtained a connected problem about non-stationary oscillations of a string on an elastic substrate and the growth of the zone of the delamination of a part of this string from an elastic substrate, which is described by the system of equations (1)-(4).

3 STEADY-STATE OSCILLATIONS OF A STRING. LOCALIZED MODES

First, consider the case where the length of the delamination domain is constant: $2l = 2l_0$. The corresponding spectral problem $u(x,t) = v(x)\mathrm{e}^{\mathrm{i}\omega t}$ takes the form

$$\begin{aligned} &Tv_{xx} - [k(x) - \rho\omega^2]v = 0, \quad -\infty < x < \infty, \\ &v,\ v_x \to 0 \qquad |x| \to \infty \end{aligned} \tag{5}$$

Here, ω is the frequency and $k = k0[H(x+l_0) - H(x-l_0)]$. The specific density of the string is assumed to be constant. We denote the sound speed in the string by $c = \sqrt{T/\rho}$ and the cut-off frequency by $\omega_b = \sqrt{k_0/\rho}$.

This spectral problem is a special case of the problem about oscillations of a string with a distributed elastic-mass inclusion, which was considered in detail in [9]. It was shown that problem (5) has not only continuous spectrum of oscillation eigenfrequencies, which begins with the cut-off frequency ω_b, but also discrete spectrum, which goes before the cut-off frequency consists of finitely many eigenfrequencies. The eigenforms of oscillations corresponding to the discrete spectrum, known as localized (trapped) modes, are localized near the inclusion and do not carry energy away to infinity.

The symmetric localized modes corresponding to the eigenfrequencies ω_{sn}, which are determined by the dispersion relation

$$\tan\frac{l_0}{c}\omega = \frac{\sqrt{\omega_b^2 - \omega^2}}{\omega}, \tag{6}$$

have the form

$$v_i^s = \begin{cases} \cos\gamma_1 x,\ |x| < l_0, \\ \\ \cos\gamma_1 l_0\,\mathrm{e}^{-\gamma_0|x-l_0|},\ |x| > l_0 \end{cases} \tag{7}$$

where $\gamma_1 = \omega/c$ and $\gamma_0 = \sqrt{\omega_b^2 - \omega^2}/c$.

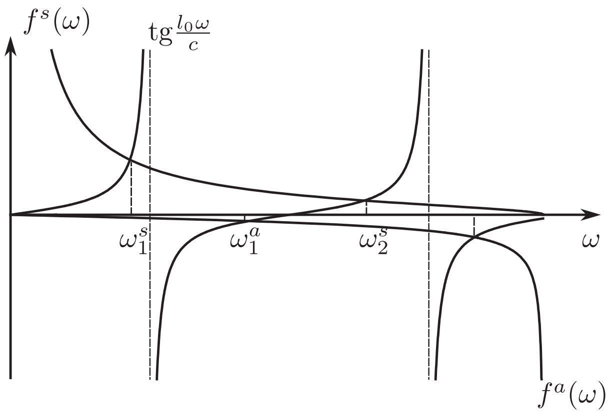

Figure 2: The eigenfrequencies.

The antisymmetric localized modes are determined by the spectrum of frequencies determined from the dispersion relation

$$\tan\frac{l_0}{c}\,\omega = -\frac{\omega}{\sqrt{\omega_b^2-\omega^2}} \tag{8}$$

and have the form

$$v_i^a = \begin{cases} \sin\gamma_1 l_0\, e^{-\gamma_0|x-l_0|},\ x > l_0 \\ \sin\gamma_1 x,\ |x| < l_0, \\ -\sin\gamma_1 l_0\, e^{\gamma_0|x+l_0|},\ x < -l_0. \end{cases} \tag{9}$$

The scheme of the determination of the roots of the frequency equations (6) and (8) is shown in Fig.2. The abscissas of the intersection points of the curves determining the right- and left-hand sides of the frequency equations give the required eigenfrequencies, provided that the parameter values ω_b, c, and l_0 are known. Note that the first eigenfrequency corresponds to a symmetric localized mode, the second corresponds to an antisymmetric mode, etc. The number of eigenfrequencies before the cut-off frequency is determined by the parameters of the waveguide and the length of the delamination zone.

The eigenfunctions corresponding to frequencies $\omega > \omega_b$ from the continuous spectrum (we call them traveling modes) are determined in [9].

It can be shown that the localized modes (7)-(9) and traveling modes are orthogonal in the sense of generalized functions [10].

4 THE NON-STATIONARY PROBLEM: EXPANSION IN EIGENFORMS

As above, we assume that the length of the delamination zone is constant and equals $2l_0$. Let $v_i(x)$ denote the localized modes (eigenfunctions) corresponding to the discrete eigenfrequencies ω_i $(i = 1, 2, ..., N)$, and let $v_\omega(x)$ be the traveling modes (eigenfunctions) corresponding to frequencies from the continuous spectrum.

Then we seek a solution of (1) in the form of an expansion in the eigenfunctions of the spectral problem (5) [10]

$$u(x,t) = \sum_{i=1}^{N} v_i(x) q_i(t) + \int_{\omega_b}^{\infty} v_\omega(x) q_\omega(t) \mathrm{d}\omega. \tag{10}$$

where $q_i(t)$ and $q_\omega(t)$ are unknown functions so-called generalized coordinates.

Substituting (10) into (1), multiplying by eigenfunctions, integrating with respect to x, and applying the dispersion relations (6) and (8), we obtain

$$\ddot{q}_i + \omega_i^2 q_i = \frac{Q_i(t)}{M_i}, \quad q_i, \dot{q}_i|_{t=0} = 0, \quad i = 1, 2, ..., N \tag{11}$$

$$\ddot{q}_\omega + \omega^2 q_\omega = \frac{Q_\omega(t)}{M_\omega}, \quad q_\omega, \dot{q}_\omega|_{t=0} = 0. \tag{12}$$

Here, Q_i are generalized forces acting on the forms from the discrete spectrum, the Q_ω are generalized forces acting on the forms from the continuous spectrum, and the M_i and M_ω are generalized masses, which are determined as follows:

$$Q_i = \int_{-\infty}^{-\infty} v_i(x) P(x,t) \mathrm{d}x, \quad M_i = \frac{1}{c^2} \int_{-\infty}^{-\infty} v_i^2(x) \mathrm{d}x$$

$$Q_\omega = \int_{-\infty}^{-\infty} v_\omega(x) P(x,t) \mathrm{d}x, \quad M_\omega = \frac{1}{c^2} \int_{-\infty}^{-\infty} v_\omega^2(x) \mathrm{d}x$$

Solving the system of equations (11)-(12), we obtain the following expression for the string deflection:

$$u(x,t) = \sum_{i=1}^{N} \frac{v_i(x)}{M_i\, \omega_i} \int_0^t \sin \omega_i (t-\tau) Q_i(\tau) \mathrm{d}\tau + \int_{\omega_b}^{\infty} \frac{v_\omega(x)}{M_\omega\, \omega} \int_0^t \sin \omega (t-\tau) Q_\omega(\tau) \mathrm{d}\tau \mathrm{d}\omega. \tag{13}$$

Thus, we have obtained a solution of Eq.(1) in the form of an expansion in localized and traveling modes for the string with a distributed inclusion. Using the Riemann-Lebesgue lemma, we can show that the integral determining the expansion in the continuous spectrum in formula (13) tends to zero as $t \to \infty$, and for large t, the solution of problem (1) is determined only by localized forms of oscillations:

$$u(x,t) \to \sum_{i=1}^{N} \frac{v_i(x)}{M_i\, \omega_i} \int_0^t \sin \omega_i (t-\tau) Q_i(\tau) \mathrm{d}\tau \quad \text{for} \quad t \to \infty. \tag{14}$$

Thus, an external action leads to the localization of waves in the film delamination zone, and in the absence of dissipation in the system, these localized waves may exist infinitely long.

5 FILM DELAMINATION

After the localization of oscillations on a defect of fixed length is analyzed, we proceed to study the initial problem (1)-(4) on film delamination, that is, on the growth of the initial delamination zone.

Suppose that, at the initial moment of time, the length of the zone of film delamination from the substrate equals $2l_0$. We assume that the value l_0 is such that there exists a unique eigenfrequency $\omega_0 < \omega_b$, which determines a symmetric eigenform according to (7). Then we seek a solution of the initial problem (1)-(4) in the form

$$u(x,t) = v_0(x) q_0(t). \tag{15}$$

Remark. In expansion (10) only the first term corresponding to a localized mode is retained; thus, the transition processes related to the propagation of traveling waves are not taken into account.

Thanks to the existence of the first symmetric form of oscillations, we can assume that $l_+ = l_-$ and write Eq.(2), which describes the growth of the delamination zone, in the form

$$\frac{\mathrm{d}l}{\mathrm{d}t} = \beta\, H\Big[u(x,t)|_{x=l(t)} - \Delta\Big]. \tag{16}$$

The length of the delamination zone varies in time, and now the symmetric form $v_0(x,l)$ depends on time and is defined on a variable interval, namely,

$$v_0(x,l) = \cos\lambda x\, H(l-x) + \cos\lambda l\,\mathrm{e}^{-\gamma(x-l)} H(x-l), \quad 0 < x < \infty,$$

where $\lambda = \omega_0/c$. Substituting (15) into the equation (1) for string oscillations, multiplying the obtained equation by $v_0(x,l)$, and integrating the result with respect to x, we arrive at a fairly complicated nonlinear equation with respect to $q_0(t)$ and $l(t)$. Discarding nonlinear terms, we obtain the following equation for the generalized coordinate $q_0(t)$:

$$\ddot{q}_0 + \omega^2(l) q_0 = \frac{Q_0(t)}{M_0}, \quad q_0, \dot{q}_0|_{t=0} = 0,$$

where

$$\omega^2(l) = \Big[2\omega_b^2 \int_l^{+\infty} v_0^2\,\mathrm{d}x + c^2 \int_{-\infty}^{+\infty} v_{0x}^2\,\mathrm{d}x\Big] \Big/ \int_{-\infty}^{+\infty} v_0^2\,\mathrm{d}x.$$

We assume that $\lambda\widetilde{l} = \omega_0[l(t) - l_0]/c \ll 1$. It is easy to show that, in this case, we have

$$\omega^2(l) = \omega_0^2 + o[(\lambda\widetilde{l})^2], \quad \widetilde{l} = l(t) - l_0.$$

Replacing $\omega(l)$ by ω_0 in the last equation, we reduce the initial problem to the system of equations

$$\ddot{q}_0 + \omega_0^2 q_0 = \frac{Q_0(t)}{M_0}, \quad q_0, \dot{q}_0|_{t=0} = 0, \tag{17}$$

$$\frac{\mathrm{d}l}{\mathrm{d}t} = \beta H\Big[v_0(l,\omega_0) q_0(t) - \Delta\Big], \quad l(t))|_{t=0} = l_0. \tag{18}$$

First, consider the simplest example of an external action, namely, the oscillations initiated by an impulse exciting force $P(t) = P_0\delta(t)$. In this case, the generalized force is

$$Q_0(t) = P_0 \cos\lambda l\,\mathrm{e}^{-\gamma_0(x_p - l)}\delta(t).$$

The generalized mass does not depend on the exciting force and has the form

$$M_0(t) = \rho[l + \sin 2\lambda l/2\lambda + \cos\lambda l/\gamma_0].$$

Thus, Eq. (17) can be rewritten in the form

$$q_0(t) = \frac{P_0 \mathrm{e}^{-\gamma_0(x_p - l)}}{M_0\omega_0} \cos\lambda l\, \sin\omega_0 t \tag{19}$$

Substituting (19) into (18), we obtain

$$\frac{\mathrm{d}l}{\mathrm{d}t} = \beta H\Big[\frac{P_0\varepsilon\,\cos^2\lambda l}{M_0(l)\,\omega_0}\,\sin\omega_0 t - \Delta\Big],\quad l(t)|_{t=0} = l_0 \tag{20}$$

where $\varepsilon = \mathrm{e}^{-\gamma_0(x_p - l)}$. We assume that the force is applied so that $\varepsilon < 1$. We seek an approximate solution to Eq.(20). Since $|\sin\omega_0 t| \le 1$, it follows from (20) that a necessary condition for the growth of the delamination zone has the form

$$\frac{P_0\varepsilon\,\cos^2\lambda l}{M_0(l)\,\omega_0} > \Delta. \tag{21}$$

If l_0 is such that condition (21) is violated (the amplitude of string oscillations does not attain the critical value), then the delamination zone cannot grow. If the initial conditions are such that inequality (21) holds, then the delamination zone begins to linearly grow at rate β at the moment $t = t_1$, which can be approximately determined from the expression

$$\sin\omega_0 t = \frac{\Delta\,\omega_0 M_0(l_0)}{P_0\varepsilon\cos^2\lambda l_0}$$

and

$$l(t) = l_0 + \beta\,t\,H(t - t_1).$$

The length of the delamination zone increases until time $t = t_2$, at which the argument of the Heaviside function (20) vanishes. The growth of the delamination zone resumes at time $t = t_3$, which can be found from the expression

$$\sin\omega_0 t = \frac{\Delta\,\omega_0 M_0(l_2)}{P_0\varepsilon\cos^2\lambda l_2}.$$

Such a step growth of the delamination zone continues until $l(t)$ reaches a certain critical value. Indeed, as the length of the delamination zone decreases, the amplitude of string oscillations decreases; thus, there exists a moment of time t_k and the corresponding length $l = l_k$ for which condition (20) ceases to hold, and the growth of the delamination zone stops.

A similar picture is observed in the case where the exciting force is harmonic, in which this force has the form $P(t) = P_0\sin\nu t$, where ν denotes frequency. In this case, Eq. (17) has the approximate solution

$$q_0(t) = P_0\varepsilon\int_0^t \frac{\cos\lambda l(\tau)}{M_0(\tau)}\,\sin\nu(t-\tau)\sin\omega_0\tau\mathrm{d}\tau. \tag{22}$$

As well as in the case of impulse load, it can be proved that the moments of time ti determining the growth periods of the delamination zone, can be found from the equation

$$q_0(t)\cos\lambda l = \Delta$$

Thus, the approximate analytic solution of the problem constructed on the basis of only the symmetric eigenform (7) describes the step growth of the delamination zone, and the growth rate is determined by the value of the parameter β. An analysis of Eq.(18), which describes the

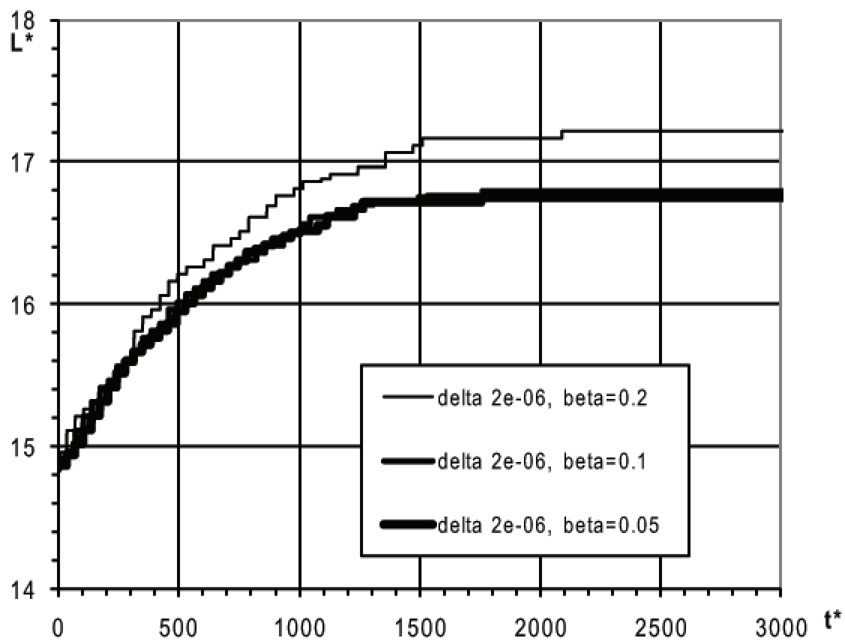

Figure 3: The behavior of the boundary of the delamination zone.

growth of the delamination zone, shows that various modes of growth are possible, which are determined by the behavior of the argument of the Heaviside function in (18).

A numerical modeling of the initial problem (1)-(4) was performed. Motion was excited by a harmonic force. Results of the numerical modeling of the growth of the delamination zone for the parameter values $\beta = 0.2, 0.1, 0.05$ and $\Delta = 2 \cdot 10^{-6}$ are shown in Fig.3.

An analysis of numerical results demonstrates the possibility of the localization of oscillations near a delamination defect under certain loads and problem parameter values and of the following three cases of the delamination development: (i) the absence of growth of the delamination zone for large values of the critical displacement Δ, provided that the amplitude of the displacements does not attain the critical value; (ii) the growth of the delamination zone up to a certain size, after which the growth stops (see Fig.3); (iii) for very small Δ, the growth of the delamination zone is virtually unbounded. Thus, the numerical solution of the system of equations (1)-(4) well agrees qualitatively with the approximate analytic solution.

6 CONCLUSIONS

The delamination of thin-layer elements caused by dynamic or vibrational loads, which is observed during the exploitation of real-life constructions, leads to the destruction of these constructions. Therefore, problems related to determining reasons for such destructions are of great practical interest. In this paper, for the example of the delamination of a string from an elastic substrate, the possibility of the localization of oscillations near a delamination defect was demonstrated and the impact of this localization on the growth of the delamination zone was analyzed. The problem considered here is only the first approximation of a fairly complicated problem. A simplified setting of the problem under consideration was suggested. In constructing an approximated analytic solution, only the first symmetric form of oscillations was taken into account. In the future, non-symmetric forms and the transformation of oscillations as the delamination zone grows must be taken into account. The simplest criterion for the growth of the delamination zone was chosen, which does not take into account the real rheology near the boundaries of the delamination zone. It is quite natural to consider the generalization of the obtained results to the case of the delamination of a two-dimensional film, a bar, and a thin plate.

However, even the simplified model makes it possible to explain the destructions observed in constructions with thin-layer coats under comparatively mild loads, which are caused by the localization of oscillations near various defects.

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