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A NEW METHOD FOR PROBABILISTIC AFTERSHOCK RISK EVALUATION OF DAMAGED BRIDGE

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Abstract. *A critical issue in the emergency management after the earthquake is the functionality of the main infrastructure (hospitals, road network, etc.) and the decision on their usability just after the main shock. At present, a decision is taken on the basis of a structure in-situ inspection; an analytical assessment is in contrast with the lack of time and data. For this reason in this paper a method that rationally combines information from the analytical approach and the in situ inspection is proposed. In particular an effective tool to speed-up the decision making phase concerning the evaluation of the seismic risk of mainshock-damaged structures due to aftershocks has been proposed. The risk is calculated combining the aftershock hazard using the Omori law and the fragility curves of the structure calculated using IDA technique and updated using in-situ inspection data. The procedure has been applied to a highway r.c. bridge. The results have highlighted a high sensitivity to the Bayesian updating especially when the damage predicted by the numerical analysis does not correspond to the real damage. The mean annual rates of collapse provided by the method has shown that the risk structure change dramatically when an aftershock sequence strike the bridge and this risk decreases with time allowing the authorities to decide if and when re-open the bridge to traffic.*

1 INTRODUCTION

The earthquakes, as well known, are cluster phenomena: thus the sequence of a seismic event is non-poissonian. In order to simplify the model, the phenomenon is usually represented by a superposition of two different phenomena: the mainshock, considered as stationary, that is the major intense event of each cluster; the second one (aftershock), strongly non-stationary, follows the mainshock event. Although the mainshock may be preceded by precursor events (foreshocks), usually these latter are low-intensity shocks, whereas, the sequence of the aftershocks due to a high magnitude event is made of numerous earthquakes, some of which may be have an intensity similar to the mainshock one.

For a long time, the evaluation of the seismic risk was based on mainshocks. This is justified by the fact that a mainshock is usually the most violent event, capable to induce relevant damages to the building and to catch the people inside the buildings, and in case of collapse, to provoke the most number of victims. Aftershocks, although as much violent, act on empty buildings, and their potentiality in terms of victims is much lower. Nevertheless, the aftershock risk is relevant for two aspects: a) in the estimation of the risk based not only on the human life preservation, but also on the direct and indirect economic consequences of an earthquake, the damages induced by aftershocks may increase highly the cost of a seismic event; b) in assuming decisions about the feasibility of buildings and transitability of the streets, the aftershock risk represents the main parameter.

Usually, these decisions are taken in the field, on the basis of simple vision inspections and on a synthetic evaluation of the situation, based on the experience of the inspector, often not so wide given that the seismic events are likely rare events.

The decision about the feasibility of a construction, in a post-earthquake scenario, has relevant consequences, because if an imprudent choice can produce further victims, a conservative choice may produce a social inconvenience and useless costs for the housing of the displaced persons or for the interruption of the productive activities.

For some years some researcher and institutions [1, 2, 3, 4, 5, 6, 7] have suggested a more rational and analytical approach to the problem of the feasibility of the seismically damaged constructions, in particular in the period immediately after the mainshock.

The application of analytical methods, for a rapid estimation of the feasibility of buildings, introduces so many difficulties that the final decision cannot ignore the judgment of the detector; in fact, the buildings are numerous and complex structure for which is difficult to define general damage levels that account for the behavior of non-structural elements (partitions, ceilings, etc..), as well as the risk related to the surrounding situation (e.g. collapse of adjacent buildings). More affordable seems to be a rational and analytical procedure to manage the decisions about the transitability of bridges; in fact, bridges are more simple structures and less numerous, for which appears more plausible to organize a customized procedure to use in emergency situations.

The bridges are strategic structures because their unavailability can cause the interruption of the road ways, prevent the ordinary means to reach the zone struck by the seismic event. This condition becomes particularly penalizing during the post-earthquake emergency, when high amount of transportation means have to be employed for the rescue operations. The decision to close to the traffic a bridge of an important roadway, which connect the most struck zones, is therefore very delicate, and it should be taken on the basis of the most possible rational and objective criteria. In a recent paper [8] some authors have studied the problem to correlate the intensity of the event with the vertical load carrying capacity (traffic) of the bridge. The results are given in terms of fragility curves for different percentages of traffic reduction. Nevertheless the correlation of the residual vertical load carrying capacity with the seismic

damage is very difficult and uncertain, both for the limits of the current models and because the live loads often represent a limited fraction of the bridge weight.
More promising seems to be the way proposed by [9, 10] to relate the decision of the usability of a bridge to the risk due to aftershocks. For this purpose, is necessary to perform an Afterschock Probabilistic Seismic Hazard Analysis (APSHA) [3]; as soon as the magnitude and position of the main event is known, this analysis, as similar as the ordinary Probabilistic Seismic Hazard Analysis (PSHA), provides the probability that, in the unit time, an earthquake occurs with an intensity greater than a given one.
The analysis is completed with a convolution between the hazard and the vulnerability of the structure; in the post-earthquake the structures will be probably damaged, therefore fragility curves depending on the damage induced by the mainshock have necessary to build.
A relevant difficulty consists in the evaluation of the damage level suffered by the structure. In fact, an inspection will allow to detect only some visible effects due to the seismic action (e.g. cracks, spalling of concrete cover, buckling of reinforcement, etc..). These observations are certainly correlated with the damage suffered the structure but with a high dispersion.
Alternatively, it is possible to proceed analytically; being known position and magnitude of the event, the seismic intensity of the site is determined (e.g. the median spectrum) using an attenuation law; however, also in this case the dispersion of the intensity, of the shape of the spectra and the reliability of the model render the estimation of the suffered damage very uncertain; furthermore, the result of a predictive model based on the analysis cannot to conflict with the observation.
In a recent work [10], the authors use both the methods synergically. The predicted damage, on the basis of the attenuation curves and the fragility curves of the undamaged structure, is used as prior estimation of the probability, which are updated on the basis of the observed damage, using as likelihood function the probability function conditioned to the observed one; this function can be built using numerous experimental tests [11].
Another difficulty regards the use of the results in the decision process about the usability of the structure (e.g. the functionality of the bridge). In fact, differently from the mainshock, the aftershock process is strongly non-stationary, therefore, the hazard (and then the risk) vary day by day, decreasing rapidly with the time, measured starting from the mainshock event.
The risk threshold under which is possible to use the construction has been highly discussed. A reasonable criteria is based on the comparison between the prior risk of the undamaged structure, due to the mainshock, with the aftershock risk, but the non-stationarity of the aftershock process renders very problematic the comparison.
Different proposals have been done to make comparable the above two quantities. Yeo and Cornell [12] compare the cost (discounted) required to save a human life with respect to the mainshock risk (in an infinite time) with the same cost associated with the aftershock; with some approximations the conversion factor is the social discount factor (3-5%). More simply, Pinto and Franchin [9] compare the average risk (annualized) in the residual time between the data of the decision and the end of the aftershock sequence. Both the criteria will be applied and compared.

2 DESCRIPTION OF THE METHOD

In this section a rational method for probabilistic aftershock risk evaluation of damaged bridges is presented. Basically, the procedure consists of the following four steps:

- Evaluation of the fragility curves of damaged and undamaged bridge
- Damage evaluation phase

- Aftershock hazard analysis
- Evaluation of the aftershock risk

The main goal of the method is to provide the mean annual rate of collapse for aftershocks of a mainshock-damaged bridge. The method, already proposed by the authors [10], is here improved introducing a new technique for the evaluation of the fragility curves, which is based on the Latin Hypercube Sampling technique. All the relevant aspects of the proposed procedure is illustrated in the following sections.

2.1 Evaluation of the fragility curves

The first phase of the procedure, to be carried out before the earthquake occurs, consists of evaluating the fragility curves of undamaged and damaged structure. A parameter of damage measure has to be adopted (in the following the maximum drift will be adopted) and a series of dynamic analysis on a proper structural model has to be executed in order to calculate the probabilities functions $\overline{F}_{\phi,Y}(\phi_i, y)$ of the damage ϕ_i, versus the local intensity y_i of the earthquake. The IDA procedure [13] is the most used method for the evaluation of the fragility curves.

Following this method a series of increasing damage levels is established (ϕ_1, ϕ_2,..) and a wide sample of accelerograms are chosen together with the representative parameter of the local intensity (e.g. PGA or spectral ordinate).

Nonlinear dynamic time-history analysis are performed using the selected set of ground motions, scaling each record, in an iterative procedure, until the intensity that causes precisely the damage level ϕ_i is found.

It is now possible, for each accelerogram, to induce in the structure the damage level j^{th}, and then, starting from that point, to apply another series of accelerograms with increasing intensity, in order to simulate the effects of the aftershocks. It is necessary to repeat the IDA procedure for a number of times equal to the product between the number of damage levels and the number of the accelerograms selected for simulate the mainshock. Altogether, the required number of analysis is rather large and significantly increase when the probability $\overline{F}_D(\phi_i \mid \phi_j, y)$ that the structure, with an initial damage level ϕ_j, reaches or exceeds the damage level ϕ_i caused by an aftershock, has to be evaluated; the latter is given by $n_{\mathtt{an}} = n_{\mathtt{dl}} \times n_{\mathtt{am}} \times n_{\mathtt{aa}} \times n_{\mathtt{il}}$, in which $n_{\mathtt{dl}}$ is the number of the considered damage levels, $n_{\mathtt{am}}$ is the number of the accelerograms used for the mainshock, $n_{\mathtt{aa}}$ is the number of the accelerograms used for the aftershocks and $n_{\mathtt{il}}$ is the number of the intensity levels considered in the IDA procedure.

Therefore the IDA technique can only be applied to simple structural schemes for which the time of calculation remains acceptable; the extension to more complex structures will require a more efficient procedure.

With the aim of extending the procedure to more complex structures, the regression analysis is here used to evaluate the fragility curves, significantly reducing the number of analysis and, consequently, the computation time. A set of accelerograms is selected together to a range of variation of the PGA and the Latin Hypercube Sampling technique (LHS) is applied to select the sample used for the structural analysis.

This technique uses a stratified sampling scheme that generates a sample containing n values on each of p variables; these are obtained by dividing the range of variation of each variable into n equiprobable and non-overlapping intervals; the values are randomly distributed with one from each interval (0,1/n), (1/n,2/n), ..., (1-1/n,1), and then they are randomly permuted.

The LHS technique is used to extract a significant sample of variables on which to perform the analysis and then the regressions. In particular, in order to determine the fragility curves of the undamaged structure, n_m couples of variables (accelerogram, intensity y_i) are extracted and the damage level ϕ_i, obtained in the analysis, is used to build n_m couples (y_i, ϕ_i).

To obtain the "fragility surface" of the damaged structure, n_a, quadruples of variables (A_{mi}, y_{mi}, A_{ai}, y_{ai}) are extracted, where A_{mi} and A_{ai} indicate the i^{th} mainshock and the i^{th} aftershock accelerogram respectively, whereas y_{mi} and y_{ai} are the corresponding intensities. From the analysis of the structure subjected to this couple of accelerograms, the corresponding two levels of damage ϕ_{mi} and ϕ_{ai} are recorded, on condition that $\phi_{ai} \geq \phi_{mi}$. Consequently the following n_a triples (y_a, ϕ_{mi}, ϕ_{ai}) are obtained. The objective is to find the distribution function of the damage $\phi_i,|y_i$ caused to the undamaged structure by the mainshock and then the distribution function of the damage $\phi_{ai},|\ \phi_{mi}, y_{ai}$ caused by the aftershock to the structure with a pre-existing damage level $\phi_{mi},$.

For both the analyses the following polynomial regression model is used:

$$\log\phi = \mathbf{X}\boldsymbol{\beta} + \varepsilon \tag{1}$$

where $\mathbf{X} = \begin{bmatrix} 1 & x_1 & x_2 & \dots & x_1^2 & x_1x_2 & \dots \end{bmatrix}$ is the vector (1 × m) of the polynomial variables, whereas β is the coefficient vector and ε is a random variable.

The use of the logarithms of ϕ depends on hypothesis that this quantity (non-negative) is better described by a lognormal random variable. The random variable ε is instead Gaussian.

In the examined case the variables in **X** are the intensity y_m in the first analysis and the pairs (y_a, ϕ_m) of the intensity of the aftershock and the initial damage.

If $\hat{\mathbf{\Phi}}$ is a vector (n_a × 1) built using the logarithms of the calculated damages and $\hat{\mathbf{X}}$ is the matrix (n_a × m) of the polynomial variables calculated in the sample points, it found, by the regression analysis, that log ϕ is a random variable with mean:

$$E\left[\log\phi\right] = \hat{\mathbf{X}}\boldsymbol{\beta} \tag{2}$$

with

$$\boldsymbol{\beta} = \left(\hat{\mathbf{X}}^T\hat{\mathbf{X}}\right)^{-1}\hat{\mathbf{X}}^T\hat{\mathbf{\Phi}} \tag{3}$$

and variance

$$\sigma^2{}_{\log\phi} = s^2\frac{\left\|\hat{\mathbf{X}}^T\hat{\mathbf{X}} + \mathbf{X}^T\mathbf{X}\right\|}{\left\|\hat{\mathbf{X}}^T\hat{\mathbf{X}}\right\|} \tag{4}$$

where

$$s^2 = \frac{\left(\hat{\mathbf{\Phi}} - \hat{\mathbf{X}}\hat{\boldsymbol{\beta}}\right)^T\left(\hat{\mathbf{\Phi}} - \hat{\mathbf{X}}\hat{\boldsymbol{\beta}}\right)}{\nu - 2} \tag{5}$$

The term logϕ follows the t-Student distribution with ν = n–m degrees of freedom. It is observed that if the dimensions m of the sample are high, the t-student distribution approximates the Gauss distribution, so ϕ becomes lognormal.

2.2 Damage evaluation

After a seismic event, the magnitude m_0 and the position r of the epicenter are quickly available; with such a data and using a proper attenuation law [14], 15, 16] it is possible to find the average value and the standard deviation of the selected parameter of local intensity as function of the magnitude and the epicentral distance (or minimum distance from the fault). Assuming for y a log-normal distribution, the conditional probability density of the intensity y is:

$$f_{Y|M,R}\left(y \mid m_0, r_0\right) = \frac{1}{\sqrt{2\pi}\zeta_0 y}\exp\left[-\frac{\left(\ln y-\mu_0\right)^2}{2\zeta_0^2}\right] \tag{6}$$

in which $\zeta_0 = \sqrt{\ln\left(1+\left(\sigma_y / \bar{y}\right)^2\right)}$ and $\mu_0 = \ln \bar{y} - \zeta_0^2/2$.

Consequently the probability of exceeding the damage level ϕ_i conditioned to the occurrence of an event of magnitude m_0 and distance r_0 can be calculated by integrating the product between seismic hazard and fragility:

$$\bar{F}_{\Phi}\left(\phi_i \mid m_0, r_0\right) = \int_0^{\infty}\bar{F}_{\Phi|Y}\left(\phi_i \mid y\right) f_{Y|M,R}\left(y \mid m_0, r_0\right) dy \tag{7}$$

This (probabilistic) damage estimation does not account for what is really occurred in the structure. As stressed above, the post-earthquake observation of the damage does not allow, at least for moderate intensity events, to evaluate the damage index ϕ_i used in the analytical model without a permanent instrumentation placed on the structure.

To evaluate the CDF of the damage index ϕ conditioned to the observed damage θ_k, $F_{\phi}\left(\phi \mid \theta_k\right)$, a relationship between the two variables must be known; on the basis of laboratory tests, Berry and Eberhard [11] have developed relationships that provides, as function of the characteristics of the reinforced concrete element (normal force, sectional area, length, percentage of reinforcement, etc..), the average value and the coefficient of variation of the damage index ϕ_i (drift) corresponding to the observed damage θ_k, ($k = 1,2,3$), where θ_1 = spalling, θ_2 = buckling of the steel bars, θ_3 = failure of the steel bars.

Assuming that the probability of ϕ conditioned to the damage θ_k can be expressed as lognormal, the previous data permit to determine the CDF of the drift ϕ conditioned to θ_k, $F_{\phi}\left(\varphi \mid \theta_k\right)$. The probability $p\left(\theta_k \mid \varphi\right)$ can be obtained observing that, if ϕ_k is the drift for the damage level θ_k, then $\phi < \min_k\left(\phi_k\right)$ = no damage, $\phi_1 \le \phi < \min\left(\phi_2, \phi_3\right)$ = damage θ_1, $\max\left(\phi_1, \phi_2\right) \le \phi < \phi_3$ = damage θ_2, and finally $\max_k\left(\phi_k\right) \le \phi$ = damage θ_3. Consequently, if θ_0 stands for no damage condition, we can assume:

$$\begin{aligned}
p\left(\theta_0 \mid \phi\right) &= \Pr\left[\min_k\left(\phi_k\right) > \phi\right] = \prod_{k=1}^{3}\left[1 - F_{\phi_k}\left(\phi \mid \theta_k\right)\right] \\
p\left(\theta_i \mid \phi\right) &= \prod_{j=1}^{i} F_{\phi_1}\left(\phi \mid \theta_j\right) \prod_{k=i+1}^{3}\left[1 - F_{\phi_k}\left(\phi \mid \theta_k\right)\right] \quad (i = 1,2) \\
p\left(\theta_3 \mid \phi\right) &= \Pr\left[\max_k\left(\phi_k\right) \le \phi\right] = \prod_{k=1}^{3} F_{\phi_k}\left(\phi \mid \theta_k\right)
\end{aligned} \tag{8}$$

where $p(\theta_k \mid \phi)$ is the probability of the observed damage θ_k conditioned to the drift ϕ. If we consider $p_0(\phi) = f_\Phi(\phi \mid m_{0,} r_0)$ a prior probability distribution, estimated before the direct observation of the damage, and θ_k is the level of the real observed damage, applying the Bayes theorem using $p(\theta_k \mid \phi)$ as likelihood function, the updated estimation of the probability can be obtained as:

$$p(\phi \mid \theta_k) = \frac{p(\theta_k \mid \phi)\, p_0(\phi)}{\int p(\theta_k \mid \phi)\, p_0(\phi)\, d\phi} \tag{9}$$

This probability function of the damage index ϕ combines the results of a predictive analysis carried out on the numerical model of the structure with the information gathered from the visual observation of the residual damage of the structure.

2.3 Aftershock hazard analysis

Since it is supposed that the main event has occurred and then magnitude, epicentral distance and (possibly) fault line and position are known, it is possible to carry out an Aftershock Probabilistic Hazard Analysis (APSHA), similarly to the procedure illustrated in [2].

In this paper we will suppose that the length of the rupture zone (L_R), if not directly known, can be estimated as function of the magnitude as indicated in [17]:

$$L_R = 10^{(a+bm_0)} \tag{10}$$

where a and b are parameters depending on the fault rupture type and L_R is expressed in Km.

Unlike the standard hazard analysis (PSHA), in which the seismic events is assumed Poissonian and stationary, the aftershocks process is highly non-stationary; the mean instantaneous daily rate of events at time t with magnitude greater or equal to m following a mainshock of magnitude m_m, is given by the “modified Omori Law” [18] .

$$\gamma(t, m, m_m) = \frac{10^{a+b(m_m - m)}}{(t+c)^p} \tag{11}$$

where a,b,c, and p are regional parameters that can be estimated on the basis of the past seismic events [19] .

Moreover, by introducing a truncation of the magnitude m such that $m_1<m<m_m$, the average number of daily events with intensity greater than m is given by [2]:

$$\gamma(t, m, m_m) = \gamma_0(t, m_1, m_m)\frac{e^{-\beta m} - e^{-\beta m_m}}{e^{-\beta m_1} - e^{-\beta m_m}} = \gamma_0(t, m_1, m_m)\left[1 - F_M(m)\right] \tag{12}$$

where $\beta = b\ln 10$ and $\gamma_0(t, m_1, m_m)$ is the average number of the daily events at time t with magnitude $m_1<m<m_m$ and

$$\gamma_0(t, m_1, m_m) = \frac{10^{a+b(m_m - m_1)} - 10^a}{(t+c)^p} \tag{13}$$

The mean number of aftershocks with magnitudes m such that $m_1<m<m_m$ in the time interval [t; $t + T$] following a mainshock of magnitude m_m is given by:

$$\gamma(t,T;m_m)=\int_t^{t+T}\gamma_0(\tau,m_1,m_m)\,d\tau=\frac{10^{a+b(m_m-m_1)}-10^a}{p-1}\left[(c+t)^{1-p}-(c+t+T)^{1-p}\right] \tag{14}$$

For an aftershock of magnitude *m* and epicenter *P*, the local intensity at the site is deduced by the attenuation law: $\log y=\psi(r,m)+\varepsilon$, in which ψ is a function of the magnitude *m* and of the distance *r* between the structure and the epicenter (or the minimum distance from the fault), whilst ε is a zero-mean Gaussian variable with standard deviation σ_ε. The probability density of *Y* for a given event conditioned to *m* and *r* is then:

$$f_{Y|R}(y\,|\,r,m)=\frac{1}{y}\varphi\left(\frac{\ln y-\psi(r,m)}{\sigma_\varepsilon}\right) \tag{15}$$

where $\varphi(.)$ is the standard normal PDF.

In order to obtain the probability of *Y* unconditioned with respect to the magnitude, the equation (15) must be multiplied by the PDF of the magnitude of the aftershocks, $f_M(m)=dF_M(m)/dm=\beta e^{-\beta m}/\left(e^{-\beta m_1}-e^{-\beta m_m}\right)$ and integrated over *m*:

$$f_{Y|R}(y\,|\,r)=\frac{1}{y}\int_{m_1}^{m_m}\varphi\left(\frac{\ln y-\psi(r,m)}{\sigma_\varepsilon}\right)f_M(m)\,dm \tag{16}$$

As frequently occurs, when the attenuation law is a linear function of *m*, $\log y=\psi(r)+qm+\varepsilon$, the integral (16) can be developed in closed form [25]:

$$\begin{aligned}
f_{Y|R}(y\,|\,r)&=\frac{1}{\sqrt{2\pi}\sigma_\varepsilon y}\int_{m_1}^{m_m}\exp\left(-\frac{\left(\ln y-\psi(r)-qm\right)^2}{2\sigma_\varepsilon^2}\right)\frac{\beta e^{-\beta m}}{e^{-\beta m_l}-e^{-\beta m_m}}\,dm=\\
&=\frac{e^{\beta\left(m_l+m_m+\psi(r)/q+\sigma_\varepsilon^2\beta/q^2\right)}}{2\left(e^{-\beta m_l}-e^{-\beta m_m}\right)y^{\beta/q+1}q\beta}\left[\operatorname{erf}\left(\frac{-\beta\sigma_\varepsilon^2-q\left[qm_l-\ln(y)+\psi(r)\right]}{\sqrt{2}\sigma_\varepsilon q}\right)+\right.\\
&\quad\left.-\operatorname{erf}\left(\frac{-\beta\sigma_\varepsilon^2-q\left[qm_m-\ln(y)+\psi(r)\right]}{\sqrt{2}\sigma_\varepsilon q}\right)\right]
\end{aligned} \tag{17}$$

If *R* denotes the region where the aftershocks may occur (in the following it will be assumed that coincides with the length of mainshock fault, similarly to [2]), and $f_R(r)$ is the probability density function of the distances *r* from the site, the probability density of an event with intensity *y*, conditioned to the occurring of the event, is:

$$f_Y(y)=\int_R f_{Y|R}(y\,|\,r)\,f_R(r)\,dr \tag{18}$$

2.4 Evaluation of the aftershock risk

The functions $\overline{F}_D(\phi_i\,|\,\phi_j,y)$ and $p(\phi_j\,|\,\theta_k)$ previously defined have the following meaning; the first one provides the probability that the structure, stricken by the mainshock with a damage level ϕ_j, could exceeds the damage level $\phi_i\geq\phi_j$ because of an aftershock, while the second one is the probability that the structure, stricken by the mainshock, has suffered the damage ϕ_j.

Therefore, if an aftershock occurred, the probability that a damage $\phi\geq\phi_i$ is:

$$Pr(\phi \geq \phi_i) = \int_0^{\phi_i} \int_0^{\infty} \overline{F}_D(\phi_i \mid \phi_j, y) f_Y(y)\, p(\phi_j \mid \theta_k)\, dy\, d\phi_j \tag{19}$$

At time *t,* the daily mean number of the shaking that induce in the structure a level of damage greater or equal to ϕ_i is therefore $\gamma(\phi_i \mid t) = \gamma_0(t, m_1, m_{\mathrm{m}}) \Pr(\phi \geq \phi_i)$. If an aftershock cluster is represented by a Poisson process, the probability (risk) that the damage is exceeded in the time $[t, t+T]$ is:

$$\Pr(\phi \geq \phi_i \mid t, T) = 1 - e^{-N_{\mathrm{a}}(\phi_i, t, T)} \tag{20}$$

where:

$$\begin{aligned} N_{\mathrm{a}}(\phi_i, t, T) &= \int_t^{t+T} \gamma(\phi_i \mid \tau)\, d\tau = \\ &= \int_t^{t+T} \gamma_0(\tau, m_1, m_{\mathrm{m}}) \Pr(\phi \geq \phi_i)\, d\tau = \Pr(\phi \geq \phi_i) \int_t^{t+T} \gamma_0(\tau, m_1, m_{\mathrm{m}})\, d\tau = \\ &= \Pr(\phi \geq \phi_i) \frac{10^{a+b(m_{\mathrm{m}}-m_1)} - 10^a}{p-1} \left[(c+t)^{1-p} - (c+t+T)^{1-p} \right] \end{aligned} \tag{21}$$

is the mean number of the events that produce the damage φ_i in the time interval [*t*, *t*+ *T*].

3 DECISION-MAKING ANALYSIS

The probability evaluated with equation (20) represents the risk, due to an aftershock, that a damage level ϕ_i in the structure is exceeded during the time interval *T*, starting from the time *t*; in the following, ϕ_i will be assumed as collapse condition. This data is useful to assume a rational decision on the usability of the construction (e.g. the functionality of a bridge). A reasonable criterion to orient such a decision consists of comparing the risk due to the aftershocks (provided by eq. (17)) with the risk due to the mainshock. Nevertheless, this comparison introduces considerable conceptual difficulties.

In fact, in the Probabilistic Seismic Risk Analysis (PSRA), the process of the events is assumed to be stationary, therefore the mean number of the events that produce the damage ϕ_i in the time *T* is simply, $\gamma_{\mathrm{m}}(\phi_i)T$, where $\gamma_{\mathrm{m}}(\phi_i)$ is the mean number of events in the unit time (generally one year). The comparison among these two risks can simply be done comparing the frequencies, independently from the used time unit, because of the simple proportionality with the time *T*. In case of aftershocks, the Omori law shows that the process is strongly non-stationary, and the average frequency decades as *~1/t*; for instance, using the values of the coefficients of the Omori law obtained by [20] for the Italian earthquakes, after 7 days the activity is reduced to less than the 17% of that of the first day, while after one month it is a little bit more than 4%. Similar results, but even more pronounced, are found using the values of Reasemberg and Jones for the southern California.

The dependency of the aftershocks risk from the initial time *t* can be employed for establishing a date over which the risk decrease up to a level comparable with the mainshock risk and therefore considered as acceptable; for this purpose the problem of establishing for what time *T* the risk has to be measured, must be overcome. For instance, it may be assumed *T*=1 year in order to make N_a directly comparable with the annual frequency of collapses due to mainshocks. But if a lower value of *T* is assumed (e.g. 6 months), N_a is reduced, at least in the first ten days, less than 20% (Lolli and Gasparini data [20]), while the expected mean number of the mainshocks halves. The choice of T=1 year appears therefore arbitrary and related only to common criteria.

More realistic appears the approach proposed by [9] consisting in using the maximum duration of the aftershocks sequence expressed as $t_u = 60 + 60(m_{\mathbf{m}} - 4)\text{days}$ [20]. In this way however the duration of the time period $T = t_u - t$ varies with t. For a direct comparison between this quantity and the annual stationary frequency, an "equivalent" annual frequency expressed as: $[365/(t_u - t)]N_{\mathbf{a}}(\varphi_i, t, t_u - t)$ has to calculated. Another interesting approach is that carried out from Yeo and Cornell [12]; in this work the authors assume as equivalence criteria the condition that the expected investment into life-safety technologies for saving an arbitrary building occupant in the future is the same for both cases. As result, an equivalent, time-independent collapse rate is obtained multiplying the total number of collapses in the interval $(t - \infty)$ by the “inflation-adjusted discount rate”.

4 INCREMENTAL DYNAMIC ANALYSIS AND REGRESSION METHODS FOR THE EVALUATION OF FRAGILITY CURVES

To validate the application of the regression method for fragility curves evaluation the results obtained on a case study have been compared with those obtained by the IDA procedure.

In a previous author's work [9] the developed procedure with IDA has been applied to a simply supported highway viaduct (Vallone del Duca) of the A16 Napoli-Canosa highway, composed by two independent roadways, each one composed by three simply supported reinforced concrete beams with a span of 32 m.

For the fragility calculation, a set of 10 ground motion records has been selected from the *PEER* database; the same set of records has been used to represent the mainshocks and aftershocks.

A first cycle of analysis (*IDA*), representing the mainshock event, has been carried out to evaluate the fragility curves for the undamaged structure and the PGA leading to predefined damage levels; 350 analysis have been executed: 10 accelerograms × 35 intensity levels.

A second cycle of analysis (*IDA*) has been performed to evaluate the fragility curves of the damaged structure due to the aftershocks leading to collapse; in this cycle 56000 analysis have been executed combining 16 mainshock damage level for each of the 10 accelerograms with 35 intensity levels for each of the 10 aftershock records. The regression significantly reduces the analysis number; in this case 160 analysis have been performed for the mainshock fragility evaluation and 1600 for the aftershock ones.

Figure 1 and Figure 2 show the comparison between the mainshock and aftershock fragility curves obtained by IDA and those obtained by the regression method; it can be noticed that the IDA curves presents some differences with respect to approximate curves obtained with a small size sample; therefore IDA curves can be well approximated using the regression.

Figure 3 shows the comparison between the mean annual frequency of collapse due to mainshocks and to aftershocks evaluated by the two methods; for the aftershocks the risk has been evaluated as mean daily rate, calculated using equation (21) and assuming T = 1 day. Then, it is transformed in annual rate multiplying by 365. The comparison show a good agreement from the results of two methods, with the clear advantage, for regression method, of a substantial reduction of the computational time.

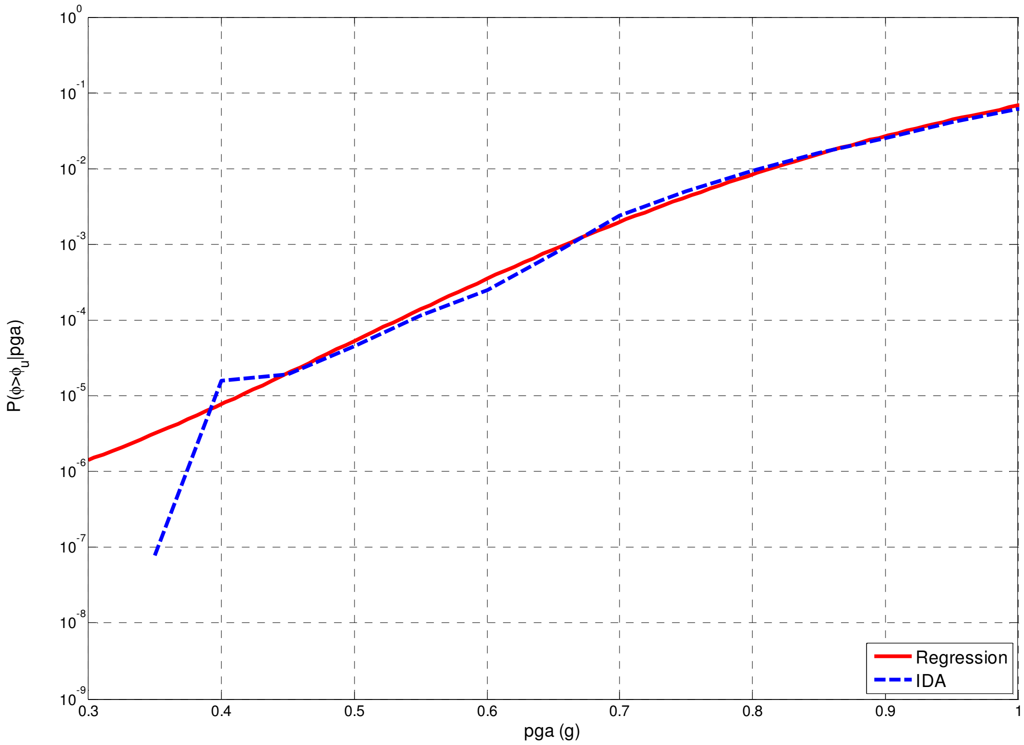


Figure 1: Comparison between the mainshock fragility curves obtained using IDA and the regression method

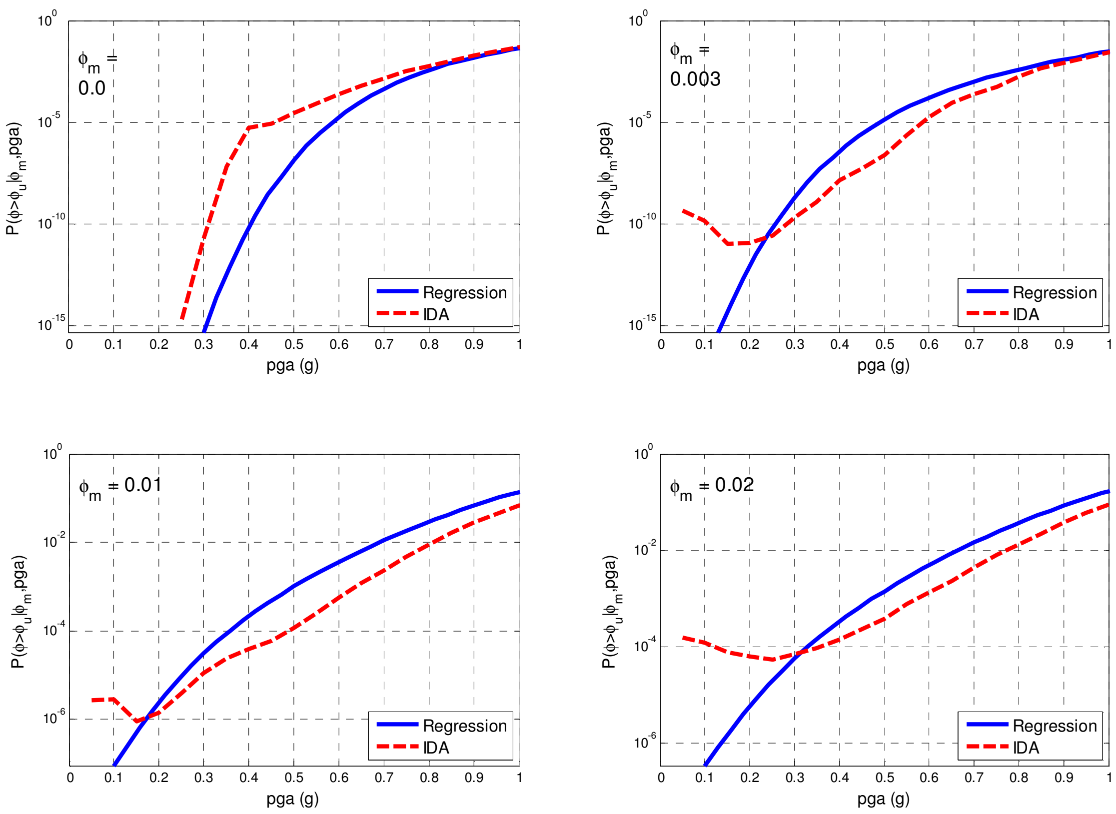


Figure 2: Comparison between the aftershock fragility curves obtained using IDA and the regression method

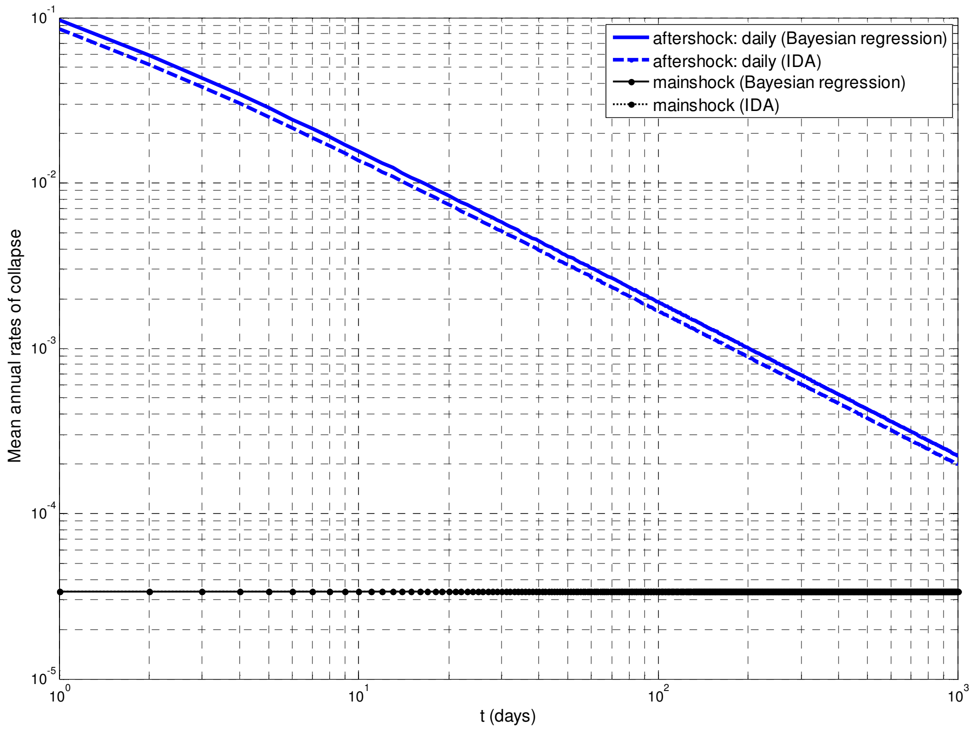


Figure 3: Comparison between the mean annual frequency of collapse evaluated with fragility with IDA and the regression method

5 APPLICATION

The bridge object of this study is an old reinforced concrete viaduct consisting of a thirteen-span bay deck with two independent roadways sustained by 12 couples of portal frame piers (Figure 4), each composed of two solid or hollow circular columns of variable diameter (120-160 cm), connected at the top by a cap-beam and at various heights by one or more transverse beams of rectangular section.

The height of the piers varies between 13.8m, near the abutments, to 41 m, at the center of the bridge. The deck is realized by two Π reinforced concrete beams 2.75m high, which are interrupted by some Gerber saddles placed at the second, seventh and twelfth bay respectively. The deck is connected to the piers by two steel bars inserted in the concrete.

The linear distributed weight of the deck is approximately 200 kN/m for each road-way. This means that to each pier receives a vertical load varying between 6600 kN and 8400 kN, while the length of the bays varies between 33 and 42 m.

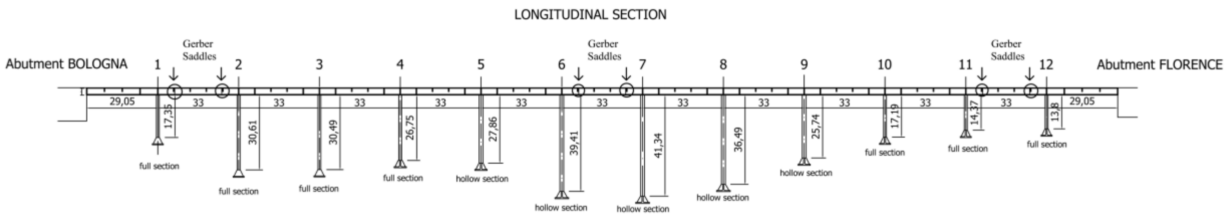


Figure 4: The Rio Torto highway viaduct

The examined pier (Figure 5) has a total height of 25.14 m and it is composed by two columns with hollow section of external and internal diameter equal to 160 cm and 120 cm respectively. The longitudinal and transverse reinforcement consists respectively of 16 steel bars ϕ20 mm and spiral stirrups ϕ 6 mm with spacing s=14cm. The transverse beam has a rectangular section 40 × 130 cm with a symmetrical longitudinal reinforcement realized with ϕ24 and ϕ20 mm and stirrups ϕ 8 mm with variable spacing (s=10cm at nodes and s=14 cm in the meddle). The cap-beam presents a U-shaped section with longitudinal reinforcement 4ϕ24+8ϕ20 mm and stirrups ϕ8 mm with variable step.

The average strengths of the material, specified in original design drawings and report, are: concrete compressive strength f_c=26 MPa and steel yield strength f_y =360 MPa. The deck weight of 200kN/m correspond to a frame load of 6600kN divided over the two columns.

The dynamic analysis have been performed using Opensees software [21]. The pier have been modeled as 2D-frame composed of nonlinear beam column elements, including uniaxial bending and axial force modeled by a fiber-discretized section; the P-delta effect is also included.

The axial/flexural response of transverse beams have been coupled at each integration point, by the section aggregator command, with the shear response modeled by an uniaxial hysteretic law. The shear force-deformation law is represented in Figure 7. The envelope is multilinear with characteristic points at cracking, peak and residual strength given evaluated according to [22].

The typical cyclic response, for a given axial force, here assumed as constant, is shown in Figure 6; Figure 7 shows the cyclic shear response of the end section of a transverse beam.

The viaduct here analyzed has been realized in a zone with moderate-to-high seismic activity. The seismogenic zone (913) in which the bridge is placed is indicated in Figure 8. The return period for the life safety condition is about 2000 years. The associated shaking map (from INGV) shows (Figure 9) that the expected PGA ranges between 0.23g and 0.25g, whereas considering the collapse prevention condition (probability of 2% in 50 years) PGA ranges between 0.30g and 0.35g.

In order to evaluate the aftershock hazard of the site, two mainshocks have been here considered, with epicenter placed at a distance R = 10 km from the site and magnitude M = 6.5 and M = 6. These latter have been adopted taking into account that the maximum value of the magnitude associated with an event within 20 km from the site in the parametric catalogue of damaging earthquakes in the Italian area [23] is M = 6.0; this value is compatible with a value of the *pga* corresponding to an average return period equal to 2475 (0.34g), resulting from a conventional PSHA analysis [23] and evaluated using the most recent data for seismogenic zones for the Italian territory [24].

For the evaluation of the aftershock rate (eq.(10)), the parameters suggested by Lolli and Gasperini [20] are here adopted: $a = -1.676$, $b = 0.96$, $c = 0.0295$, $p = 0.93$ and $m_1 = 4.5$; moreover, the fault length has been calculated as function of the magnitude m using eq. (10) as suggested in [17].

A Montecarlo simulation, using 1000 samples, has also been used for the integration over the distance R.

For the evaluation of the fragility curves, a set of 10 ground motion records has been selected from the *PEER* database, with the following parameters: 6 ≤ Magnitude ≤ 7, Source-Site distance ≤ 25km; the records have been normalized to the peak ground acceleration and scaled according to target spectrum provided by the Italian Code for an average return period of 2475 years. In this work the same set of records used to represent the mainshocks has also been used for the aftershocks.

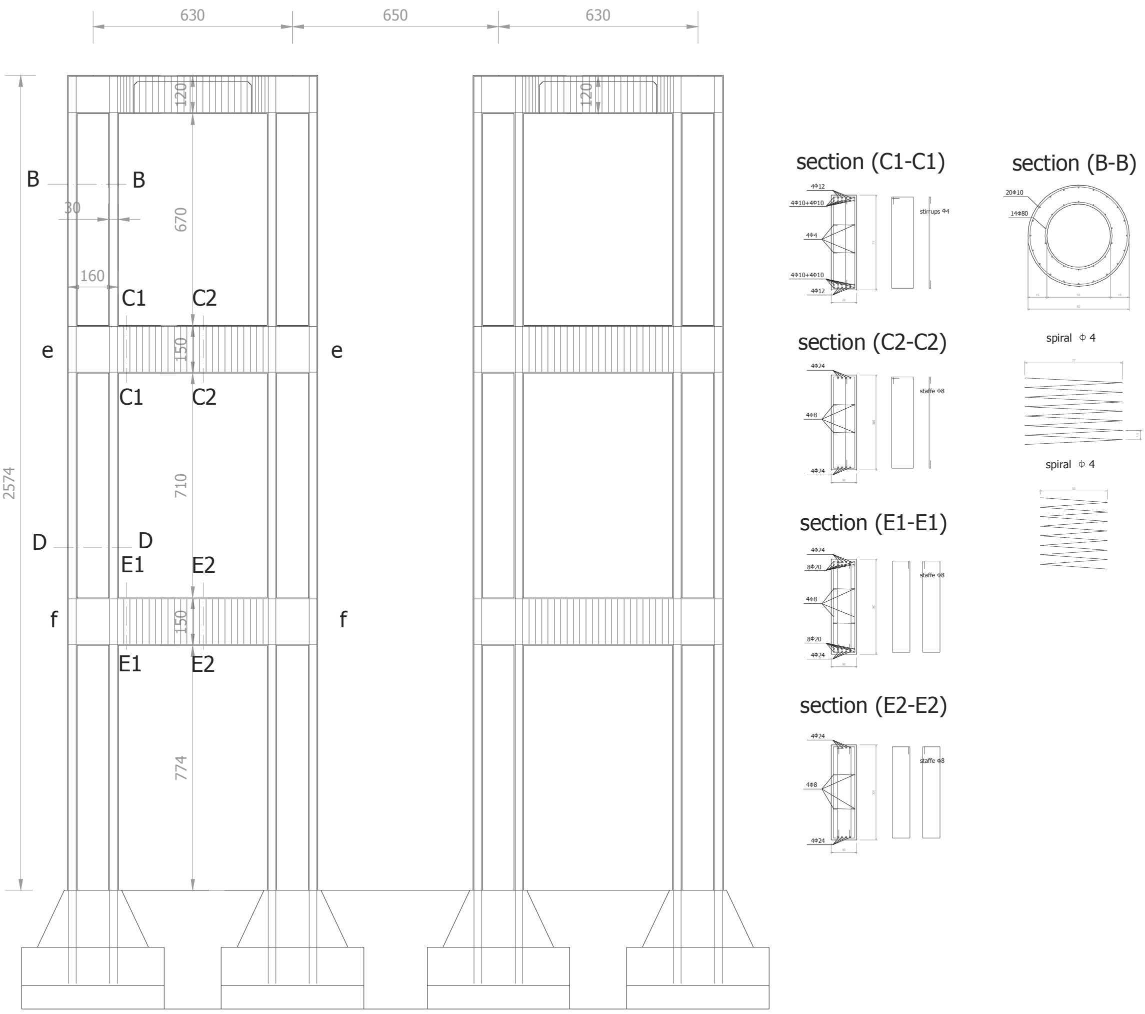


Figure 5: Geometry and reinforcement details of pier n. 9

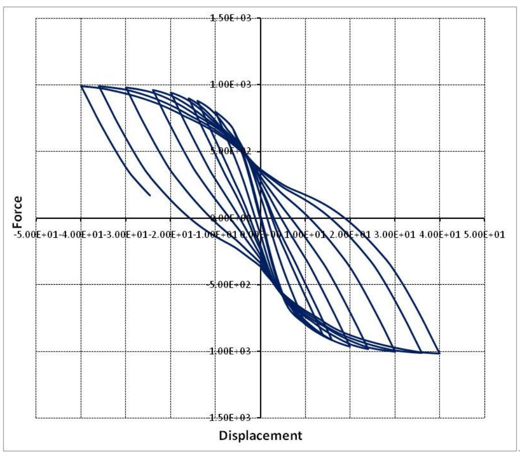


Figure 6: Cyclic behaviour of the pier

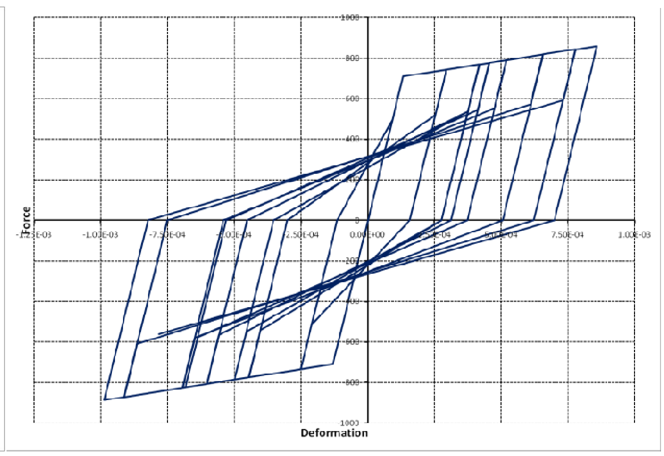


Figure 7: Cyclic shear force-deformation of a transverse

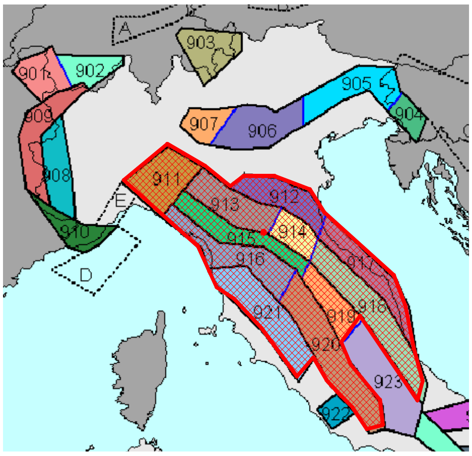


Figure 8: ZS9, Seismogenetic area for Italian territory.

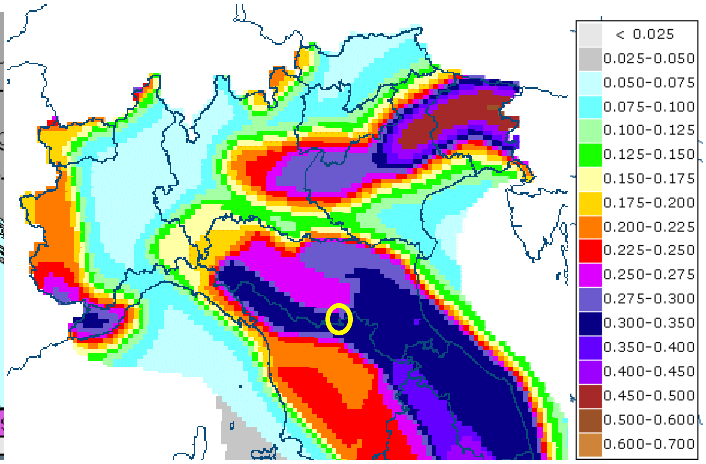


Figure 9: Seismic hazard for the probability of exceedance of the 2% in 50 years (mean return period 2475 years) [26].

In Figure 10 the daily rate of exceeding site PGA of the aftershocks is illustrated for different level of PGA as a function of the elapsed time from the initial rupture t (1, 5, 10 ,15 days from the mainshock).

To calculate the fragility curve of the intact structure subjected to a mainshock, 160 non-linear time history analysis have been carried out, by selecting, with the LHS technique, the records and the relevant PGA in the range 0.5g ÷ 1.5g. The lower limit of 0.5g on the PGA has been adopted taking into account that only the drifts that cause structural damage are considered.

To calculate the fragility curve of the mainshock-damaged structure, subjected to an aftershock that causes collapse (corresponding to a drift ϕ_u=0.025), a second cycle of 1600 analysis has been performed by selecting, with the LHS technique, the records of the mainshock and the aftershock and the relevant PGA in the range 0.1g ÷ 1.5g.

Figure 11 shows the fragility curves of the undamaged structure due to the mainshock $p_0(\phi) = f_\Phi(\phi \mid m_{0,} r_0)$, estimated before the direct observation of the damage, in comparison with fragility curves due to aftershocks $\overline{F}_D(\phi_i \mid \phi_j, y)$, for three levels of damage caused by mainshock (ϕ_1=0.003, ϕ_3=0.01, ϕ_3=0.02). The effect of the initial damage on the probability of failure due to an aftershock generally increases the probability of failure for each value of PGA.

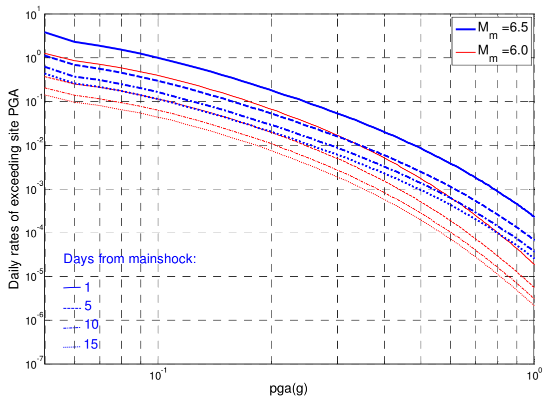


Figure 10: Daily rates of exceeding site PGA

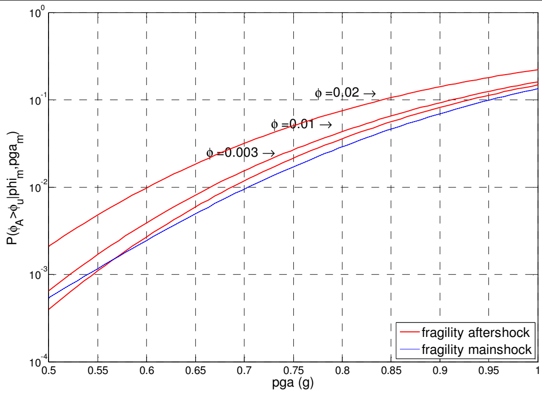


Figure 11: Fragility curves of the pier undamaged and for three levels of damage

The three level of damage considered in this work have been evaluated by experimental tests performed by some of the authors [27] in the laboratory of the Department of Structures of the University of Roma Tre. The tests consist of cyclically imposed displacement to a 1:4 scale of one of the piers; they have shown that the shear behaviour of the pier is extremely relevant; in particular, the shear failure of the transverse beam has a strong influence on global cyclic behaviour of the specimen, with a pronounced pinching effect that reduces the dissipation capability of the pier. The assumed drift values (ϕ_1 =0.003, ϕ_3 =0.01, ϕ_3 =0.02) correspond to the shear cracking of the transverse beam and flexural cracking at the base of the column.

In 12 the effect of the Bayesian updating on the probability function of the damage (drift) produced by the mainshock is shown; the continuous curve is the CDF of the drift due to the seismic event, evaluated using the attenuation law and the fragility curve of the undamaged structure (eq. (7)). The dashed curves represent the updated function [eq. (9)] assuming two different observed damages θ_1 and θ_2 with the relevant drift mean value from tests and assuming a lognormal distribution. A high sensitivity to the Bayesian updating, considering the limited level of damage here adopted, can be noticed. In particular, an event with magnitude M = 6.0 causes a calculated median drift of 0.52% that undergoes a small variation if a damage θ_1 is observed, but moves to 0.97% if a damage θ_2 is observed; an event with magnitude M = 6.5 causes drift with a median value of 0.65% that undergoes reduces to 0.6% or moves to 1.1% if a damage θ_1 or θ_2, respectively, are observed. This result shows as the Bayesian updating can strongly modify the damage curve, when the damage predicted by the numerical analysis does not correspond to the in-situ one, although the high dispersion of the observed damages.

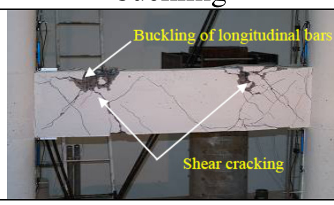
Observed damage	θ1	θ1	θ1
Drift mean value	0.003	0.01	0.02
Correlation coefficient	0.35	0.25	0.2
Physical description	Hairline cracks	Onset of concrete spalling - development of shear cracks	Wide-cracks widths - longitudinal bars buckling
			

Table 4.1. Mean value and correlation coefficient of the drift

Figure 3 shows the results of the analysis in terms of risk for the two assumed values of the mainshock magnitude. The continuous lines represent the mean daily rate of collapse, calculated using equation (20) assuming T = 1 day, transformed in annual rate multiplying by 365; the dashed lines represent the equivalent mean annual rate of collapse, computed using the variable observation time $T = t_u$-t (in this equation t_u is the time duration of the aftershocks [20]). In the same graph the lines with markers represent the equivalent rates of collapse evaluated applying a discount rates of 2.5% .

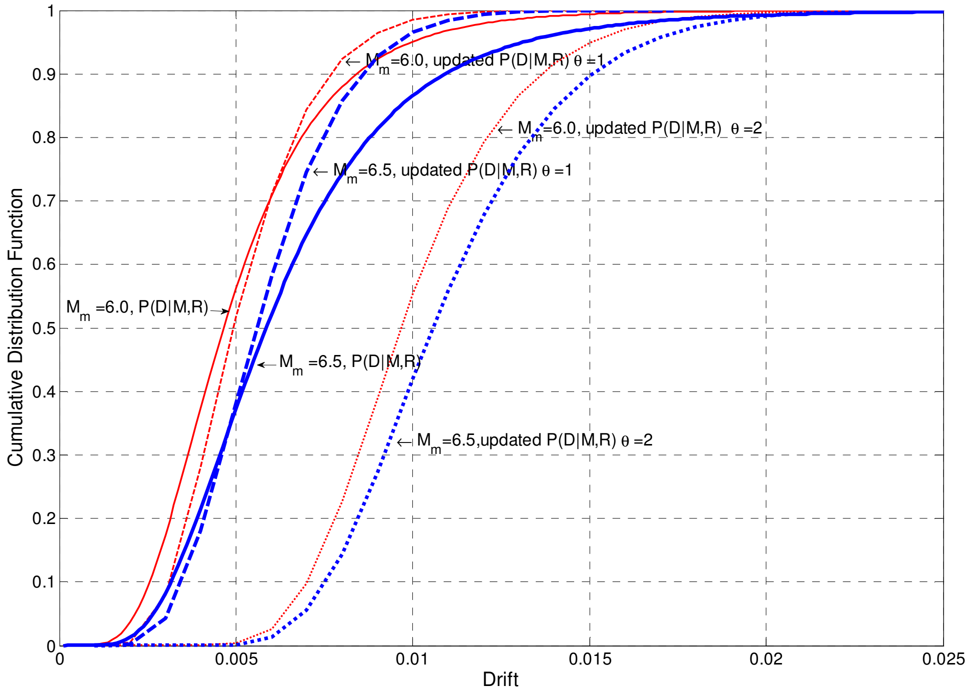


Figure 12: CDF of the drift for different observed damages θ_k

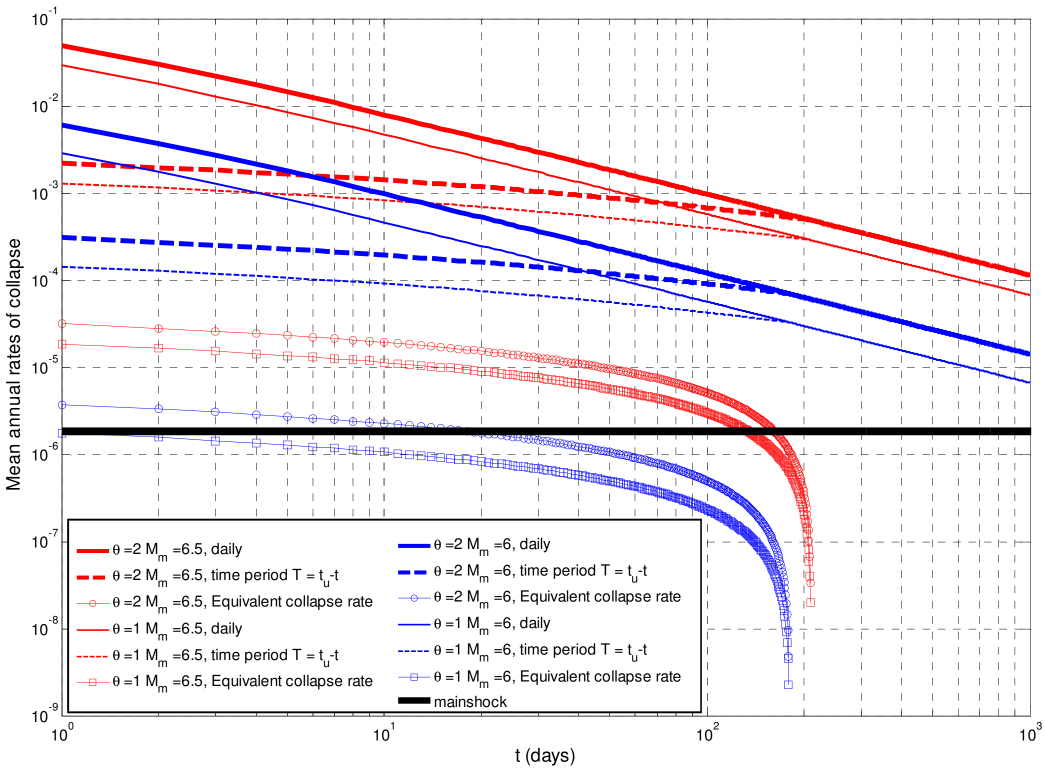


Figure 13: Mean annual frequency of collapse as a function of time elapsed from the mainshock

The equivalent mean annual rate of collapse computed with the first two criteria is much larger than the mean annual rate due to mainshock; in practice this curves intersect the mainshock level only at the end of the aftershock sequence. The criteria proposed in [12], instead, make the two risk comparable, and, for moderate earthquakes, the crossing time is shorter than the duration of the aftershocks sequence.

1. CONCLUSIONS

A critical issue in the emergency management after the earthquake is the functionality of the main infrastructure (hospitals, road network, etc.) and the decision on their usability just after the main shock. At present, a decision is take on the basis of a structure in-situ inspection; an analytical assessment is in contrast with the lack of time and data. For this reason in this paper a method that rationally combines information from the analytical approach and the in situ inspection is proposed. In particular an effective tool to speed-up the decision making phase concerning the evaluation of the seismic risk of mainshock-damaged structures due to aftershocks has been proposed. The risk is calculated combining the aftershock hazard using the Omori law and the fragility curves of the structure calculated using IDA improved by using LHS technique and updated with in-situ inspection data. The procedure has been applied to a highway r.c. bridge. The results has highlighted a high sensitivity to the Bayesian updating especially when the damage predicted by the numerical analysis does not correspond to the real damage. The mean annual rates of collapse provided by the method has shown that the risk structure change dramatically when an aftershock sequence strike the bridge and that this risk decreases with time allowing the authorities to decide if and when re-open the bridge to traffic.

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