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RELIABILITY-BASED ASSESSMENT OF SEISMIC POUNDING RISK BETWEEN ADJACENT BUILDINGS

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Abstract. *Earthquake ground motion excitation can induce pounding in adjacent buildings with inadequate separation distance. This hazard is particularly relevant in densely inhabited metropolitan areas, due to the very limited space among buildings.*

Existing procedures for minimum separation distance are based on approximations of the peak relative horizontal displacement between adjacent buildings, and are characterized by unknown safety levels. The present study proposes a reliability-based procedure for assessing the level of safety corresponding to a given value of the separation distance between adjacent buildings exhibiting linear elastic behavior. The seismic input is modeled as a nonstationary random process, and the first-passage reliability problem corresponding to the pounding event is solved employing analytical techniques involving the determination of some specific statistics of the response processes. Comparison of computed analytical results with numerical simulation results are also shown, in order to validate the proposed methodology.

The proposed procedure is employed for evaluating the reliability of simplified design code formulae used to determine building separation distances. Furthermore, the capability of the proposed method to deal with complex systems is demonstrated by assessing the effectiveness of the use of viscous dampers, according to different retrofit schemes, in reducing the pounding probability of adjacent buildings modeled as multi-degree-of-freedom systems.

1 INTRODUCTION

Earthquake ground motion excitation can induce pounding in adjacent buildings with inadequate separation distance. The corresponding risk is particularly relevant in densely inhabited metropolitan areas, due to the need of maximizing the land use and the consequent limited separation distance between adjacent buildings.

The problem of seismic pounding has been investigated by several researchers in the last two decades. A significant number of early studies focused on the definition of simplified rules, such as the Double Difference Combination (DDC) rule, for determining the peak relative displacement response of adjacent buildings at the potential pounding locations [1],[2],[3]. A critical separation distance (CSD) was defined and set equal to the mean peak relative displacement between adjacent buildings, by neglecting the associated probability of pounding. In the same context, considerable research effort was devoted to the assessment of the accuracy of code rules (e.g., the absolute sum (ABS) and square-root-of-the-sums-squared (SRSS) rule) [4] in determining the mean peak relative displacement response (i.e., the CSD) of adjacent buildings [5].

More recent studies have adopted a probabilistic approach for the assessment of the seismic pounding risk. In Lin [6], a method was proposed to estimate the first two statistical moments of the random variables describing the peak relative displacement response between linear elastic structures subjected to stationary base excitation. In Lin and Weng [7], a numerical simulation approach was suggested to evaluate the pounding probability, over a 50-year design lifetime, of adjacent buildings separated by the code-specified CSD. The latter study considered both the uncertainty affecting the seismic input intensity (by using a proper hazard model) and the record-to-record variability (by using artificially generated spectrum-compatible ground acceleration time histories as input loading). The buildings were modeled as multi-degree of freedom systems with inelastic behavior and deterministic properties. In Hong et al. [8], a procedure was developed to assess the fractiles of the CSD between linear elastic systems with deterministic and uncertain structural properties subjected to stationary base excitation. The previous study was later extended by Wang and Hong [9] to include nonstationary seismic input.

Despite the numerous studies available in the literature on seismic pounding, to the best of the authors' knowledge, a reliability-based methodology for the evaluation of the safety levels associated with specified CSDs is still needed. In addition, the gradual progress of seismic design codes from a prescriptive to a performance-based design philosophy generates a significant need for new advanced, accurate, and computationally efficient reliability-based methodologies for the assessment and mitigation of seismic pounding risk.

This paper presents a fully probabilistic methodology for assessing the seismic pounding risk between adjacent buildings with linear behavior. This methodology is consistent with and can be easily incorporated into a performance-based earthquake engineering (PBEE) approach such as the Pacific Earthquake Engineering Research center (PEER) framework [10],[11]. The presented methodology considers the uncertainty affecting both the seismic input (i.e., site hazard and record-to-record variability) and the parameters used to describe the structural systems of interest (i.e., material properties, geometry, and damping properties). The seismic input is modeled as a nonstationary random process. The seismic pounding risk is computed from the solution of a first-passage reliability problem. While the approach proposed is general, the methodology presented here is specialized to linear elastic systems subjected to Gaussian loading. Under these assumptions, approximate analytical solutions and efficient simulation techniques can be used to solve the relevant first-passage reliability problem. Thus, this methodology is appropriate for structural systems that remain in their linear elastic behavior range before pounding (which is a very common condition for low values of the CSDs and, thus, high seismic pounding risk), although it can be extended to account for nonlinear behavior of the considered structural systems.

2 PBEE FRAMEWORK FOR SEISMIC POUNDING RISK ASSESSMENT

The PEER PBEE framework is a general probabilistic methodology, based on the total probability theorem, for risk assessment and design of structures subjected to seismic hazard [10],[11]. The PEER PBEE methodology involves four probabilistic analysis components: (1) probabilistic seismic hazard analysis (PSHA), (2) probabilistic seismic demand analysis (PSDA), (3) probabilistic seismic capacity analysis (PSCA), and (4) probabilistic seismic loss analysis (PSLA). PSHA provides the probabilistic description of an appropriate ground motion intensity measure (*IM*), usually expressed as mean annual frequency (MAF) $\nu_{IM}(im)$ of exceedance of a specific value *im*. PSDA provides the statistical description of structural response parameters of interest, usually referred to as engineering demand parameters (*EDP*s), conditional to the value of the seismic intensity *IM*. PSCA consists in computing the probability of exceeding a specified physical limit-state, defined by structure-specific damage measures (*DM*s), and conditional to the values of the *EDP*s. Finally, PSLA provides the probabilistic description of a decision variable (*DV*), which is a measurable attribute of a specific structural performance and can be defined in terms of cost/benefit for the users and/or the society.

The reliability-based procedure developed in this paper consists in computing the mean annual frequency (MAF) of pounding, ν_p, between two adjacent buildings. This procedure is a specialization for the seismic pounding problem of the first three steps of the general PEER PBEE framework (i.e., PSLA is out of the scope of this paper). It is noteworthy that the proposed approach is conceptually very different from the computation of the CSD, which does not explicitly provide the probability of pounding associated with a given separation distance. The computation of the MAF of pounding can be expressed as

$$\nu_p = \int_{edp} \int_{im} G_{DM|EDP}\left(dm \middle| edp\right) \cdot \mathrm{d}G_{EDP|IM}\left(edp \middle| im\right) \cdot \left|\mathrm{d}\nu_{IM}\left(im\right)\right| \tag{1}$$

in which, $G_{DM|EDP}\left(dm \middle| edp\right)$ = cumulative probability function of variable *DM* conditional to *EDP* = *edp*, and $G_{EDP|IM}\left(edp \middle| im\right)$ = cumulative probability function of variable *EDP* conditional to *IM* = *im*, where upper case symbols indicate random variables and lower case symbols denote specific values assumed by the corresponding random variable. The *IM* must be selected so that it can be readily related to the stochastic description of an appropriate random process model for the input ground motion. This selection must also account for sufficiency and efficiency of the *IM* in describing the effects of the ground motion excitation on the structural response [12]. However, an exhaustive selection of appropriate *IM*s for different types of structures and structural performances is outside the scope of this paper.

The maximum value $U_{rel,\max}$ of the relative distance $U_{rel}(t)$ between the adjacent buildings observed during the seismic event (i.e., for $t \in \left[0, t_{\max}\right]$, with t = time and $t_{\max}$ = duration of the seismic event) is assumed here as *EDP*. The probabilistic distribution of $U_{rel,\max}$ reflects the record-to-record variability of the ground motions expected to occur at the site for a given intensity, as well as the effects of the uncertainty in the parameters used to describe the structural model. Finally, the pounding event is assumed as the controlling limit-state in PSCA, by using the following limit-state function, g:

$$g = \Xi - U_{rel,\max} \tag{2}$$

in which Ξ = random variable describing the building separation distance, and the pounding event corresponds to $g \le 0$. Thus, $G_{EDP|IM}\left(epd \middle| im\right) = P\left[U_{rel,\max} \ge u \middle| IM = im\right]$ and $G_{DM|EDP}\left(dm \middle| edp\right) = P\left[g < 0 \middle| U_{rel,\max} = u\right]$. An important intermediate result of the procedure is the convolution of PSCA and PSDA, also called fragility analysis, which yields a fragility curve. Fragility curves describe the probability $P_{p|IM}$ of pounding conditional on the seismic intensity, i.e.,

$$P_{p|IM} = \int_{edp} G_{DM|EDP}\left(dm \middle| edp\right) \cdot \mathrm{d}G_{EDP|IM}\left(edp \middle| im\right) \tag{3}$$

The MAF of pounding, ν_p, can be used to compute the MAF of exceeding a specified value of *DV*, e.g., the MAF of repair cost due to pounding damage. The computation of the latter quantity requires the definition of a realistic loss model, based on appropriate structural response models (e.g., dynamic impact between adjacent systems) and damage models (e.g., damage produced by floor-to-floor and floor-to-column pounding). Structural response and damage models involve the definition of other *EDP*s and *DM*s, respectively, in addition to those already employed in this paper for assessing the pounding risk. Several structural response and damage models available in the literature could be employed to define an appropriate loss model [13],[14],[15],[16].

In addition, ν_P can be directly used to determine the pounding risk, $P_p\,(t_L)$, for a given structure over its design life (t_L = design lifetime, e.g., 50 years). Assuming that the occurrence of a pounding event can be described by a Poisson process and that the buildings are immediately restored to their original condition after pounding occurs, $P_p\,(t_L)$ can be easily computed as

$$P_p\left(t_L\right) = 1 - e^{-\nu_p \cdot t_L} \tag{4}$$

3 SEISMIC POUNDING RISK ASSESSMENT METHODOLOGY

Fragility analysis is the most computationally challenging component of the probabilistic PBEE framework. A simple and general approach for fragility analysis in seismic pounding assessment is provided by Monte Carlo simulation (MCS) [5],[7]. For any given value of *IM*, MCS-based fragility analysis requires (1) the definition of a set of ground motions that are selected from an appropriate database of real records or generated from an appropriate random process, (2) the sampling of the structural parameters that define the structural systems and of their separation distances, (3) the numerical simulation of the structural response for each ground motion time history and each set of structural parameters and separation distances, and (4) the evaluation of $P_{p|IM}$ as the ratio between the number of failures and the number of samples. However, the computational cost associated with MCS can be very high and even prohibitive when small failure probabilities need to be estimated by numerically simulating the time history response of complex multi-degree-of-freedom (MDOF) systems.

In this paper, an efficient combination of analytical and simulation techniques is proposed for the calculation of $P_{p|IM}$ under the assumptions of linear elastic behavior for the buildings and of Gaussian input ground motion. The methodology is described first for linear elastic systems with deterministic structural properties and separation distance, and then generalized to stochastic linear systems.

It is noteworthy that, for low values of the building separation distance ξ, the buildings are expected to behave elastically before pounding occurs, while the assumption of linear behavior of the buildings before pounding becomes less realistic for larger values of ξ. If the buildings are expected to enter their nonlinear behavior before pounding, the methodology described in the remainder of this paper needs to be extended to nonlinear systems, e.g., by using statistical linearization techniques [17] or subset simulation [18]. This extension is out of the scope of this paper.

3.1 Linear systems with deterministic structural properties

The computation of the conditional failure probability $P_{p|IM}$ can be expressed in the form of a single-barrier first-passage reliability problem as [5],[9]

$$P_{p|IM} = P\left\{ \max_{0 \le t \le t_{\max}} \left[U_{rel}(t) \right] \ge \xi \middle| IM = im \right\} \tag{5}$$

in which $U_{rel}(t) = U_{\mathrm{A}}(t) - U_{\mathrm{B}}(t)$, $U_{\mathrm{A}}(t)$ and $U_{\mathrm{B}}(t)$ = displacement response of the adjacent buildings A and B at the (most likely) pounding location, and ξ = deterministic value of the building separation distance (Fig. 1).

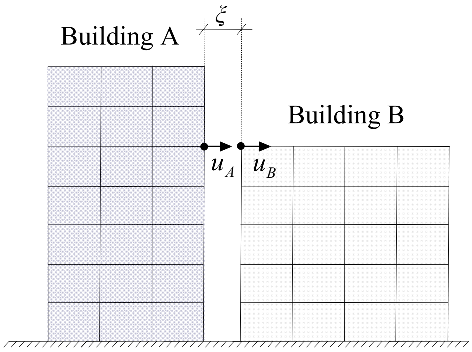

Fig. 1. Geometric description of the pounding problem between adjacent buildings.

Under the hypotheses of deterministic linear elastic systems subjected to Gaussian loading processes and deterministic threshold, several analytical approximations of $P_{p|IM}$ exist in the literature [19],[20],[21],[22]. These analytical approximations require computing the following statistics of the relative displacement process $U_{rel}(t)$ for a given $IM = im$: $\sigma^2_{U_{rel}}(t)$ = variance of $U_{rel}(t)$, $\sigma^2_{\dot{U}_{rel}}(t)$ = variance of the relative velocity process $\dot{U}_{rel}(t)$, $\rho_{U_{rel}\dot{U}_{rel}}(t)$ = correlation coefficient between $U_{rel}(t)$ and $\dot{U}_{rel}(t)$, $\omega_{c,U_{rel}}(t)$ = time-variant central frequency of $U_{rel}(t)$, and $q_{U_{rel}}(t)$ = bandwidth parameter of $U_{rel}(t)$.These statistics can be obtained from the spectral characteristics of order zero to two of process $U_{rel}(t)$ [23],[24],[25].

Following the methodology described in Barbato and Conte [24], a state-space formulation of the equations of motion for the two buildings is employed to compute exactly and in closed-form the required spectral characteristics. The seismic input is modeled as a time-modulated Gaussian colored noise process. For this specific input ground motion process, the spectral characteristics of the displacement processes (and of any response process obtained as a linear combination of the displacement processes) are available in exact closed-form for SDOF systems and both classically and non-classically damped MDOF systems [25].

The equations of motion for the linear system constituted by two non-connected adjacent buildings can be expressed as follows:

$$\mathbf{m}\cdot\ddot{\mathbf{U}}(t)+\mathbf{c}\cdot\dot{\mathbf{U}}(t)+\mathbf{k}\cdot\mathbf{U}(t)=\mathbf{p}\cdot F(t) \tag{6}$$

in which $\mathbf{m}=\begin{pmatrix}\mathbf{m}_\mathrm{A} & \mathbf{0}\\ \mathbf{0} & \mathbf{m}_\mathrm{B}\end{pmatrix}$, $\mathbf{c}=\begin{pmatrix}\mathbf{c}_\mathrm{A} & \mathbf{0}\\ \mathbf{0} & \mathbf{c}_\mathrm{B}\end{pmatrix}$, $\mathbf{k}=\begin{pmatrix}\mathbf{k}_\mathrm{A} & \mathbf{0}\\ \mathbf{0} & \mathbf{k}_\mathrm{B}\end{pmatrix}$, $\mathbf{U}=\begin{pmatrix}\mathbf{U}_\mathrm{A}\\ \mathbf{U}_\mathrm{B}\end{pmatrix}$, $\mathbf{m}_i$, $\mathbf{k}_i$, $\mathbf{c}_i$ and $\mathbf{U}_i$ = mass matrix, damping matrix, stiffness matrix, and vector of nodal displacements of building *i*, respectively (i = A, B), $\mathbf{p}$ = load distribution vector, $F(t)$ = scalar function describing the time-history of the external loading (input random process), and a superposed dot denotes differentiation with respect to time. It is noteworthy that connections between the two buildings (e.g., damping devices interposed between the building to mitigate seismic pounding risk) can be easily modeled by introducing the appropriate terms in matrix $\mathbf{c}$. The response process of interest $U_{rel}(t)$ can be related to the displacement response vector $\mathbf{U}(t)$ by means of a linear operator $\mathbf{b}$ as $U_{rel}(t)=\mathbf{b}\cdot\mathbf{U}(t)$.

The probability of pounding conditional on $IM = im$ is given by

$$P_{p|IM}=1-P\left[U_{rel}(t=0)<\xi\middle|IM=im\right]\cdot\exp\left\{-\int_0^{t_{\max}}h_{U_{rel}|im}(\xi,\tau)\,d\tau\right\} \tag{7}$$

in which $P\left[U_{rel}(t=0)<\xi\middle|IM=im\right]$ = probability that the random process $U_{rel}(t)$ is below the threshold ξ at time $t=0$, and $h_{u_{rel}|im}(\xi,\tau)$ = time-variant hazard function (i.e., up-crossing rate of threshold ξ conditioned on zero up-crossing before time *t*) conditional on IM = im. For systems with at rest initial conditions, $P\left[U_{rel}(t=0)<\xi\middle|IM=im\right]=1$.

To date, no exact closed-form expressions exist for the time-variant hazard function $h_{U_{rel}|im}(\xi,t)$. However, several approximate solutions are available in the literature, e.g., Poisson's (P), $h^{(\mathrm{P})}_{U_{rel}|im}(\xi,t)=\nu_{U_{rel}|im}(\xi,t)$, classical Vanmarcke's (cVM), $h^{(\mathrm{cVM})}_{U_{rel}|im}(\xi,t)$, and modified Vanmarcke's (mVM), $h^{(\mathrm{mVM})}_{U_{rel}|im}(\xi,t)$, approximations [22],[26]. These analytical approximations can be readily computed based on the closed-form expressions of the spectral characteristics of process $U_{rel}(t)$, as shown in Barbato and Vasta [25]. In addition, for linear elastic systems subjected to Gaussian loading, $P_{p|IM}$ can be efficiently and accurately estimated by using the Importance Sampling using Elementary Events (ISEE) method [27].

3.2 Linear systems with uncertain structural properties and separation distance

In addition to the uncertainty in the seismic input, significant uncertainty can be found in geometrical, mechanical, and material properties characterizing the structural systems and their models. Hereinafter, the uncertainty in geometrical, mechanical, and material properties of the structural models, as well as in their separation distance, Ξ, is referred to as model parameter uncertainty (MPU). MPU can significantly modify the structural performance and, thus, must be considered in the assessment of seismic pounding risk.

In order to include the effects of MPU, the total probability theorem is employed to compute the conditional probability of pounding as follows:

$$P_{p|IM} = \int_{\mathbf{X}} P_{p|IM,\mathbf{X}}(\mathbf{x}) \cdot f(\mathbf{x}) \cdot \mathrm{d}\mathbf{x} = E_{\mathbf{X}}\left[P_{p|IM,\mathbf{X}}\right] \tag{8}$$

in which $\mathbf{X}$ = vector of uncertain model parameters (including the uncertain separation distance Ξ) with joint probability density function $f_{\mathbf{X}}(\mathbf{x})$, and $P_{p|IM,\mathbf{X}}(\mathbf{x})$ = probability of pounding conditional on $\mathbf{X}$ and *IM*.

MCS, or any variance reduction technique such as stratified sampling, can be employed to evaluate $P_{p|IM}$ in Eq. (8). For example, Latin hypercube sampling (LHS) can be employed for its computational efficiency [28]. The samples of $\mathbf{X}$ generated by using LHS can be used to define a set of deterministic linear elastic models with deterministic separation distance, for which the conditional probability of pounding can be computed as in Eq.(7).

4 APPLICATION EXAMPLES

In this section, the proposed methodology is applied to: (1) compute the pounding risk for SDOF systems with deterministic model parameters, (2) evaluate the reliability of simplified design code formulae used to determine building separation distance, and (3) to evaluate the effectiveness of different retrofit solutions using viscous dampers in reducing the pounding risk for deterministic MDOF models of multistory buildings. In all the application examples considered here, the input ground acceleration is modeled by a time-modulated Gaussian process. The time-modulating function, $I(t)$, is represented by the Shinozuka-Sato's function [29], i.e.,

$$I(t) = c \cdot \left(e^{-b_1 \cdot t} - e^{-b_2 \cdot t}\right) \cdot H(t) \tag{9}$$

in which $b_1 = 0.045\pi$ s^{-1}, $b_2 = 0.050\pi$ s^{-1}, $c = 25.812$, and $H(t)$ = unit step function. A duration t_{max} = 30s is considered for the seismic excitation.

The power spectral density (PSD) of the embedded stationary process is described by the widely-used Kanai-Tajimi model, as modified by Clough and Penzien [30], i.e.,

$$S_{CP}(\omega) = S_0 \frac{\omega_g^4 + 4\xi_g^2\omega^2\omega_g^2}{\left[\omega_g^2 - \omega^2\right]^2 + 4\xi_g^2\omega^2\omega_g^2} \cdot \frac{\omega^4}{\left[\omega_f^2 - \omega^2\right]^2 + 4\xi_f^2\omega^2\omega_f^2} \tag{10}$$

in which S_0 = amplitude of the bedrock excitation spectrum, considered to be a white process, ω_g and ξ_g = fundamental circular frequency and damping factor of the soil, respectively, and ω_f and ξ_f = parameters describing the Clough-Penzien filter. The values of the parameters employed for all the applications are ω_g = 12.5rad/s, ξ_g = 0.6, ω_f = 2rad/s, and ξ_f = 0.7. The PSD function in Eq. (10) is shown in Fig. 2(a) for $S_0 = 1$.

The peak ground acceleration, *PGA*, is assumed as *IM*. In order to derive the fragility curves in terms of the selected *IM*, the relationship between the parameter S_0 of the Kanai-Tajimi spectrum and the *PGA* at the site is assessed empirically. A set of 500 synthetic stationary ground motion records are generated using the spectral representation method [31] based on the PSD function given in Eq. (10) with $S_0 = 1$. Each ground motion realization is then modulated in time using the function defined in Eq. (9). The peak ground acceleration corresponding to $S_0 = 1$, $PGA_{S_0=1}$, is estimated as the mean of the PGAs of the sampled ground motion time histories. The values of S_0 corresponding to different values of PGA are obtained as follows:

$$S_0 = \left(\frac{PGA}{PGA_{S_0=1}} \right)^2 \tag{11}$$

In this study, the site hazard curve is expressed in the approximate form used in Cornell et al. [32], i.e.,

$$v_{IM}(im) = P\left[IM \geq im \middle| 1yr \right] = k_0 \cdot im^{-k_1} \tag{12}$$

in which k_0 and k_1 = parameters obtained by fitting a straight line through two known points of the site hazard curve in logarithmic scale. The site hazard curve is taken from Eurocode 8-Part 2 [33], assuming that, for the site of interest, PGA = 0.3g corresponds to a return period of 475 years. Using $k_1 = 2.857$ [34], the site hazard curve becomes (see Fig. 2(b))

$$v_{PGA}(pga) = 6.734 \cdot 10^{-5} \cdot pga^{-2.857} \tag{13}$$

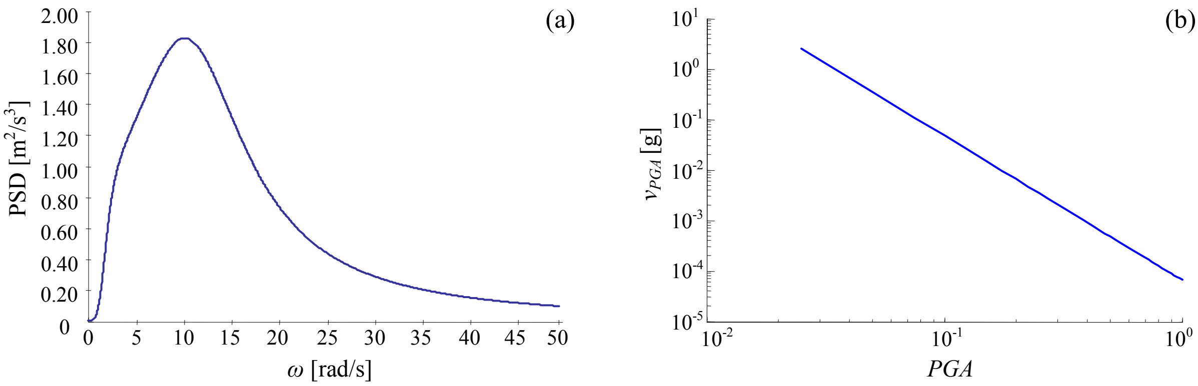

Fig. 2. Input ground motion: (a) PSD function of the embedded stationary process, and (b) site hazard curve.

4.1 Pounding risk for linear SDOF systems with deterministic model parameters

The first application example consists in the assessment of the pounding risk between two adjacent buildings modeled as deterministic linear elastic SDOF systems with periods t_A and t_B, and damping ratios $\zeta_A = \zeta_B = 5\%$. The conditional probability of pounding $P_{p|IM}$ is calculated using the approximate analytical hazard functions $h^{(\mathrm{P})}_{U_{rel}|im}(\xi,t)$, $h^{(\mathrm{cVM})}_{U_{rel}|im}(\xi,t)$, and $h^{(\mathrm{mVM})}_{U_{rel}|im}(\xi,t)$, for a deterministic distance between the buildings $\xi = 0.1$m and for two different combinations of natural periods of the two systems, i.e., (1) t_A = 0.5s and t_B = 1.0s, referred to as well separated natural periods (Fig. 3(a)), and (2) t_A = 0.9s and t_B = 1.0s, referred to as close natural periods (Fig. 3(b)). The obtained conditional probabilities are presented in Fig. 3 as fragility curves and compared with the corresponding results obtained using ISEE method [27], which are assumed as reference solution.

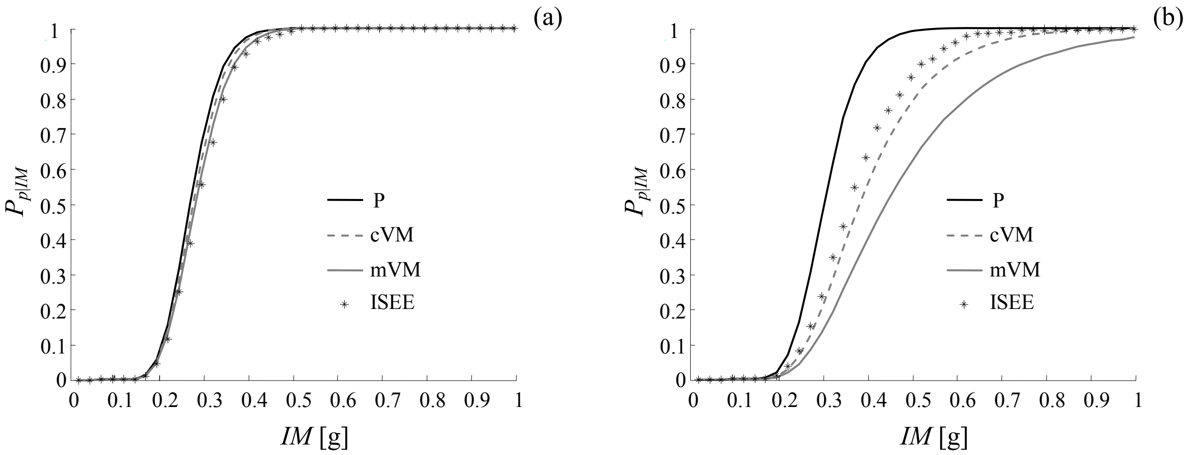

Fig. 3. Fragility curves for $\xi = 0.1$m: (a) $t_A = 0.5$s and $t_B = 1.0$s, and (b) $t_A = 0.9$s and $t_B = 1.0$s.

In the case of well separated natural periods for the structures (Fig. 3(a)), the fragility curves estimated using the P, cVM, and mVM approximations are very similar and close to the fragility curves obtained using the ISEE method. In the case of close natural periods (Fig. 3(b)), the fragility curves estimated with the approximate analytical methods show significant differences, and only the cVM approximation provides results that are close to the fragility curves estimated using the ISEE method. The observed result can be explained by recognizing that the relative displacement process $U_{rel}(t)$ can be interpreted as a response process of a two-degree-of-freedom system. This multi-modal characteristic of $U_{rel}(t)$ can significantly affect the accuracy of the different approximations of the time-variant hazard function $h_{U_{rel}|im}(\xi,t)$ [35]. In the case of well separated natural periods, the contribution of the higher period vibration mode to $U_{rel}(t)$ is significantly larger than the contribution of the lower period vibration mode. By contrast, in the case of close natural periods, both vibration modes provide a significant contribution to the response process.

Fig. 4 shows the MAF of pounding, ν_p, as a function of the building separation distance ξ (in semi-logarithmic scale) for the cases of well separated natural periods (Fig. 4 (a)) and of close natural periods (Fig. 4(b)), respectively. The estimates of the MAF of pounding obtained using the analytical approximations (P, cVM, and mVM) of the hazard function are compared to the corresponding estimate obtained using the ISEE method. Fig. 5 plots (in semi-logarithmic scale) the pounding risk for a design lifetime of 50 years, evaluated according to Eq. (4), for the same two cases of well separated and close natural periods. Considerations similar to the ones made for the fragility curves can be made also for the MAF of pounding and the 50-year pounding risk, i.e., the analytical approximations provide very accurate results for the case of well separated natural periods and less accurate results for the case of close natural periods, with the exception of the cVM approximation, which is accurate in both cases.

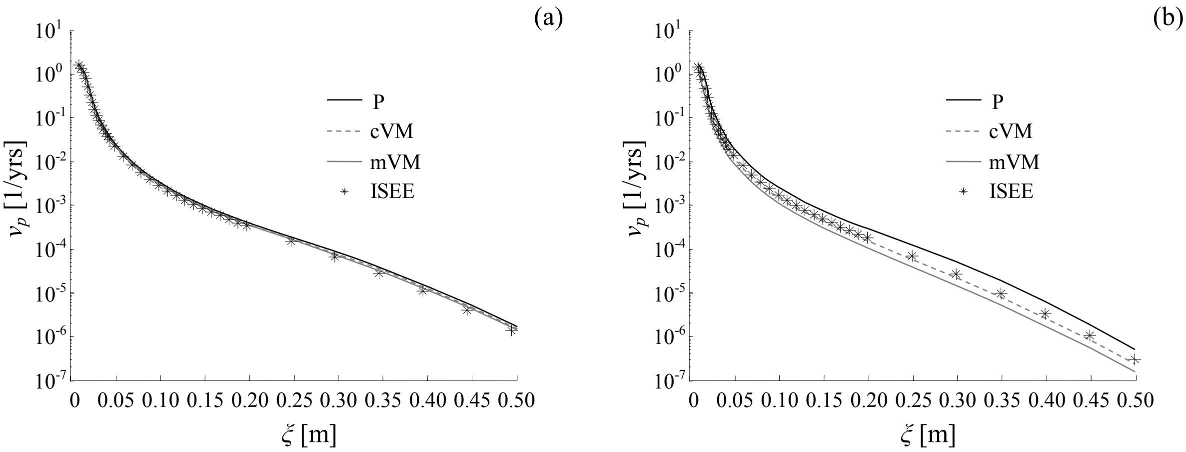

Fig. 4. MAF of pounding for varying separation distance: (a) t_A = 0.5s and t_B = 1.0s, and (b) t_A = 0.9s and t_B = 1.0s.

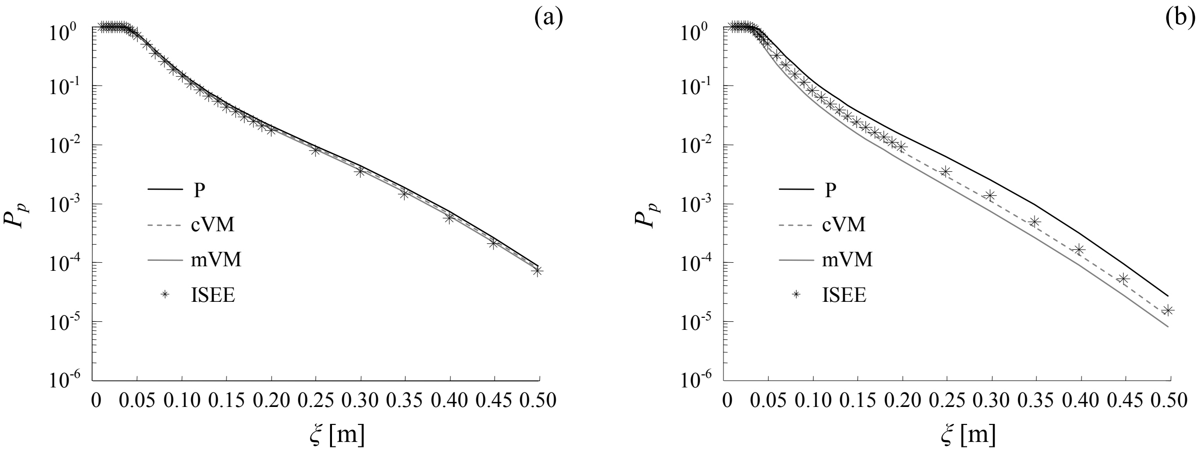

Fig. 5. 50-year pounding risk for varying separation distance: (a) t_A = 0.5s and t_B = 1.0s, and (b) t_A = 0.9s and t_B = 1.0s.

It is observed that the Poisson's approximation of the time-variant hazard function always yields conservative results, while the mVM approximation underestimates the risk computed using the ISEE method for the case of close natural periods. Similar results have been documented for the first-passage reliability problem of SDOF and MDOF systems subjected to time-modulated white and colored noise excitations [26].

4.2 Reliability of code formulae

The proposed methodology is applied here to evaluate the pounding risk corresponding to the separation distance prescribed by anti-seismic design codes. In order to avoid pounding between new adjacent buildings, current seismic design codes (e.g., [4],[33]) prescribe a minimum clearance to be provided between the structures. This minimum clearance between two adjacent buildings is assumed equal to the expected valued of the peak relative displacement (or CSD), for a given site-specific earthquake action and a given value of the seismic intensity (hazard level). Given the seismic input, the peak relative displacement is obtained by combining (using simplified combination rules) the values of the peak displacements of the two adjacent structural systems, which are computed using (deterministic) structural analysis. The most commonly employed rules are the ABS method or the slightly more accurate SRSS method. The major limit of these approximate rules is that they neglect the response phase differences between the adjacent structures. In order to overcome this drawback, the use of the Double Difference Combination rule for determining the CSD has been proposed and investigated by several researchers [1], [2],[3].

In the application presented here, the values of the CSD according to the ABS, SRSS, and DDC rules are calculated following the procedure described in [5]. This procedure involves (1) generating a set of 500 samples of input ground motion time histories for the reference value of the peak ground acceleration, (2) computing the corresponding 500 peak displacement responses of systems A and B ($U_{\mathrm{A,max}}$ and $U_{\mathrm{B,max}}$), (3) computing the sample means $\bar{U}_{\mathrm{A,max}}$ and $\bar{U}_{\mathrm{B,max}}$ of $U_{\mathrm{A,max}}$ and $U_{\mathrm{B,max}}$, respectively, and (4) combining $\bar{U}_{\mathrm{A,max}}$ and $\bar{U}_{\mathrm{B,max}}$ using the ABS, the SRSS, and the DDC rule to derive estimate of the peak relative displacement $\bar{U}_{rel,\max}$.

Table 1 shows the values of the separation distance computed according to different combination rules and the corresponding 50-year probability of failure, computed based on the cVM approximation of the time-variant hazard function.

t_A = 0.5s and t_B = 1s				t_A = 0.9s and t_B = 1s			
	ABS	SRSS	DDC		ABS	SRSS	DDC
ξ [m]	0.1379	0.1049	0.1042	ξ [m]	0.1832	0.1298	0.0946
Pp	0.0620	0.1351	0.1376	*Pp*	0.0106	0.0334	0.0857

Table 1: Critical separation distance and corresponding 50-year pounding risk using different combination rules.

It is observed that the CSDs obtained using simplified combination rules yield inconsistent values of the failure probability, which are also strongly dependent on the natural periods of the two adjacent buildings. It is concluded that a methodology is still needed to determine the CSD between adjacent buildings corresponding to consistent safety levels for different combinations of the buildings' natural periods and location's seismic hazard.

4.3 MDOF models of multistory buildings retrofitted by means of viscous dampers

As a third application, the proposed methodology is employed to assess the risk of pounding between two adjacent multistory buildings modeled as linear MDOF systems, before and after retrofit with viscous dampers (Fig. 6). Different retrofit solutions are considered and their effectiveness in reducing the seismic pounding risk is compared (Fig. 6(b)). The considered buildings are steel moment-resisting frames with shear-type behavior. The properties of the buildings are taken from Lin [36]. Building A is a six-story building with story stiffness k_A = 548,183kN/m (equal for every story) and floor mass m_A = 454.545tons (equal for each floor), building B is a four-story building with story stiffness k_B = 470,840kN/m and floor mass m_B = 454.545tons. A Rayleigh-type damping matrix $\mathbf{c_R}$ is used to model the inherent buildings' damping and is built by considering a damping ratio ζ_R = 2% for the first two vibration modes of each system. MPU is not considered in this application. The fundamental vibration periods of building A and B are t_A = 0.751s and t_B = 0.562s, respectively.

The following six different retrofit solutions, based on the use of braces with purely viscous behavior [37], are considered: (1) braces located at each story of both buildings (retrofit scheme 1), (2) braces located at all stories of the tall building only (retrofit scheme 2), (3) braces located at all stories of the short building only (retrofit scheme 3), (4) braces located at the lower four stories of the tall building only (retrofit scheme 4), (5) braces located at the lower four stories of both buildings, and (6) a single brace located at the first story of the tall building only. The two buildings before retrofit are shown in Fig. 6(a), while the six retrofit schemes are shown in Fig. 6(b). The viscous braces provide an additional source of damping, modeled by means of a damping matrix $\mathbf{c_V}$. The total damping matrix for the two buildings' systems becomes $\mathbf{c} = \mathbf{c_R} + \mathbf{c_V}$. The damping coefficient corresponding to the dampers at each floor of buildings A and B is c_d = 10,000kN·s/m. The systems corresponding to retrofit schemes 4, 5, and 6 are non-classically damped and their analysis requires the use of the complex modal analysis technique [25].

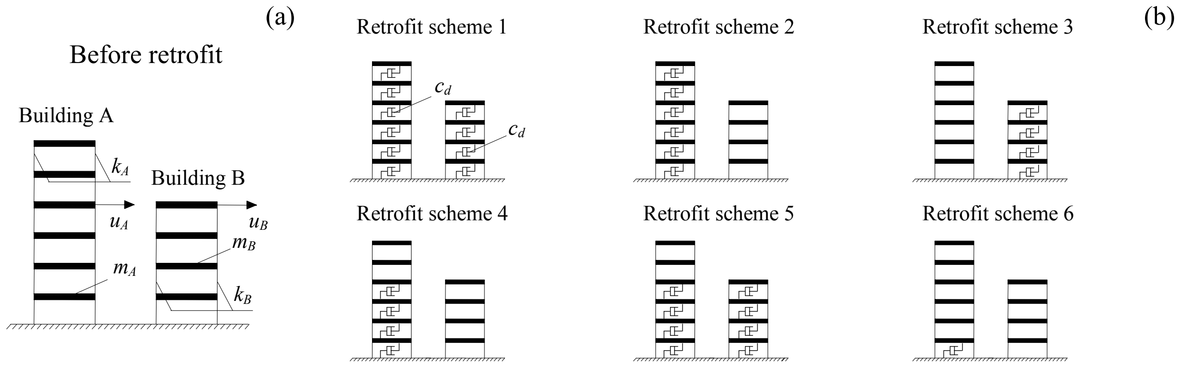

Fig. 6. Pounding between adjacent multistory buildings: (a) building A and B before retrofit, and (b) different retrofit schemes considered in this study.

Fig. 7(a) shows three different analytical estimates (P, cVM, and mVM approximations) of the 50-year probability of pounding between the two un-retrofitted buildings, for different values of the separation distance. Fig. also reports the 50-year probability of pounding obtained using the ISEE method, which is considered as reference solution. The analytical estimates provide a very good estimate of the pounding risk for a wide range of separation distances. In this particular case, the results obtained using the cVM hazard function give the best approximation of the ISEE results.

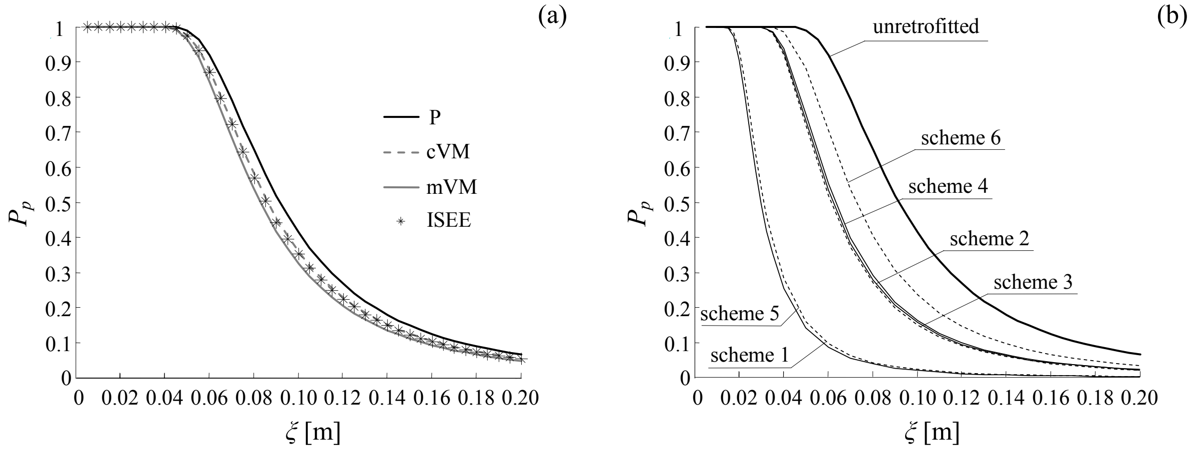

Fig. 7. Pounding risk between multistory buildings A and B: (a) comparison of different analytical solution and ISEE results, and (b) comparison of different retrofit schemes.

Fig. 7(b) shows compares the 50-year probability of pounding of the un-retrofitted buildings and of the buildings retrofitted following the six different retrofit solutions considered in this application example. The results presented in Fig. 7(b) are obtained using the cVM approximation of the hazard function.

It is observed that the use of viscous dampers can be very effective in reducing the risk of pounding between the two buildings. It is also found that the introduction of viscous braces according to scheme 3, scheme 5, and scheme 6 (corresponding to the dotted lines in Fig. 7(b)) is a very efficient retrofit solution, since it obtains a significant reduction of the pounding risk at a significantly lower retrofit cost when compared with other retrofit schemes. In particular, retrofit scheme 3 appears to achieve a very good compromise between retrofit cost and reduction of pounding risk.

CONCLUSIONS

This paper presents a fully probabilistic performance-based methodology for assessment of the seismic pounding risk between adjacent buildings. This methodology, which is consistent with the PEER PBEE framework, is able to account for all pertinent sources of uncertainty that can affect the pounding risk, e.g., uncertainty in the seismic input (i.e., site hazard and record-to-record variability) and in the parameters used to describe the structural systems of interest (i.e., material properties, geometry, damping properties, separation distance).

An efficient combination of analytical and simulation techniques is proposed for the calculation of the pounding risk under the assumptions of linear elastic behavior for the buildings and of non-stationary Gaussian input ground motion. The pounding problem is recast as a first-passage reliability problem, which is solved analytically by using the spectral characteristics (up to the second order) of the non-stationary stochastic process representing the relative displacement between the buildings. Three different analytical approximations of the time-variant hazard function are used: (1) the Poisson's approximation, (2) the classical Vanmarcke's approximation, and (3) the modified Vanmarcke's approximation. Results obtained by employing the importance sampling using elementary events method are assumed as reference solutions to evaluate the absolute and relative accuracy of the three analytical approximations considered here. The proposed formulation is very convenient in the case of linear elastic MDOF systems with both proportional and non-proportional damping, since the spectral characteristics of the relative displacement processes can be computed in exact closed form. The effects of uncertainty in the model parameters are efficiently included by means of the total probability theorem and the Latin hypercube sampling technique.

The proposed methodology is applied to investigate the risk of pounding between SDOF systems, both with deterministic and uncertain properties. With reference to this specific application example, the following observations are made. (1) The proposed combination of analytical and simulation techniques provides sufficiently accurate estimates of the pounding risk when the classical Vanmarcke's approximation is used to estimate the time-variant hazard function. (2) The accuracy of the analytical approximations of the time-variant hazard function depends on the ratio between the natural periods of the adjacent buildings. Higher accuracy is reached when the natural periods of the two buildings are well separated. (3) The Poisson's approximation of the time-variant hazard function yields always conservative estimates of the risk. (4) The design codes' simplified combination rules for calculating the critical separation distance yield inconsistent values of the pounding probability, which are also strongly dependent on the natural periods of the adjacent buildings.

In addition, the capabilities of the proposed method are demonstrated by assessing the effectiveness of the use of viscous dampers, according to different retrofit schemes, in reducing the pounding probability of adjacent multi-story buildings modeled as linear elastic multi-degree-of-freedom systems. Based on the results presented, the following considerations are made. (1) The analytical approximations provide very accurate estimates of the pounding risk, due to the fact that the fundamental periods of the two buildings are well separated. (2) The use of viscous dampers can dramatically reduce the risk of pounding between the two systems for any given separation distance. (3) The use of viscous braces in the lower levels of the taller building is a very efficient and cost-effective technique for minimizing the pounding risk.

Based on the results presented in this paper, it is concluded that the proposed methodology can be efficiently employed (1) for the assessment of pounding risk of adjacent buildings exhibiting linear elastic behavior before pounding, (2) for the computation of the mean annual frequency of pounding between adjacent buildings in the context of performance-based earthquake engineering, and (3) for the rational evaluation of the absolute and relative effectiveness of different retrofit solutions for adjacent building with high risk of seismic pounding.

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