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DISTRIBUTION OF SEISMIC EARTH PRESSURES ON RIGID RETAINING Walls

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Abstract. *The distribution of seismic earth pressures on rigid retaining walls is an important task in design. On the basis of the limit equilibrium approach and limit state analysis, this paper presents a solution to compute the seismic earth pressure on the back of a retaining wall. For this purpose, the equilibrium of forces acting on an element of the failed wedge is considered and the earth pressure on the wall is obtained using a mathematical procedure. In this study, the effects of both horizontal and vertical components of earthquake are taken into account using a pseudo-dynamic approach. The effect of phase differences in both shear and primary waves travelling through the backfill due to seismic excitation are also considered by entering the finite shear and primary wave velocities in the analysis. The results are provided in tabular and graphical non-dimensional forms and compared with pseudo-static method to highlight the realistic non-linearity of the seismic pressure distribution.*

1 INTRODUCTION

Estimation of the seismic earth pressure is an important topic of research for safe design of retainings wall in the seismic areas. It is common in practice that the seismic accelerations in both horizontal and vertical directions are considered in terms of equivalent static forces, the so called pseudo-static accelerations. Using the pseudo-static approach, several researchers have developed various methods to determine the seismic earth pressure on rigid retaining walls. These have been started from pioneering work performed by Okabe (1926) and Mononobe & Matsuo (1929), commonly known as Mononobe-Okabe (MO) method. The MO method was modified and simplified by Seed and Whitman. The MO method is attractive due to its simplicity since engineers are familiar with the Coulomb method. The MO method basically employs only force equilibrium and thus is not capable of determining other information such as the location of the resultant force or variation of earth pressure distribution along the wall. Moreover, the pseudo static approaches, do not, in essence, incorporate time dependent effect of applied earthquake load and effect of shear and primary waves.

Steedman and Zeng (1990) considered a harmonic sinusoidal term for horizontal seismic acceleration and studied the effect of phase change and finite shear wave velocity in analysis of retaining walls. They found that the finite shear wave velocity does not have significant influence on the magnitude of the total thrust on the wall. However, it has significant effect on the pressure distribution.

Choudhury and Nimbalkar (2006) have incorporated the effect of vertical seismic acceleration due to vertically propagating primary waves through the backfill soil. They have studied the effect of various parameters such as wall friction angle (δ) and soil friction angle (φ) on lateral earth pressures.

In earlier pseudo dynamic approaches, the seismic earth pressure distribution was obtained by differentiating the total active thrust. The static earth pressure and the dynamic earth pressure acting on the retaining wall could be obtained separately. In this manner, the static earth pressure distribution becomes a linear function of depth. However many experimental results show that the distribution of static active earth pressure on the face of a rough wall depends on the mode of wall movement and the distribution of active earth pressure on a rigid wall is nonlinear (Tsagareli, 1965; Sherif and Fang, 1984). Therefore, assuming linear earth pressure distribution on walls in static condition in these methods is questionable.

Recently, two methods were developed to present mathematical solutions for computing the distribution of lateral earth pressures in static (Wang, 2000) and seismic (Ghazavi and Safarzadeh, 2003) conditions. However, these solutions are crude and need further improvement.

The present study develops a new method for determination of magnitude, distribution and the height of point of application of total active thrust on rigid retaining walls in seismic condition. It also incorporates the effects of both horizontal and vertical seismic excitations. Moreover, the effects of seismic load on lateral earth pressure are investigated using pseudo-dynamic approach.

2 BASIC EQUATION OF ANALYTICAL METHOD:

There are some assumptions required to determine the seismic active earth pressure. As shown in Fig. 1, a fixed base vertical rigid retaining wall with H height is considered. The backfill is assumed to be homogenous, dry, and cohesionless. The failure surface is also assumed to be planar (Fig. 2). Considering analytical model for propagating waves through the backfill soil in the horizontal and vertical directions, the effect of horizontal and vertical seismic acceleration are investigated.

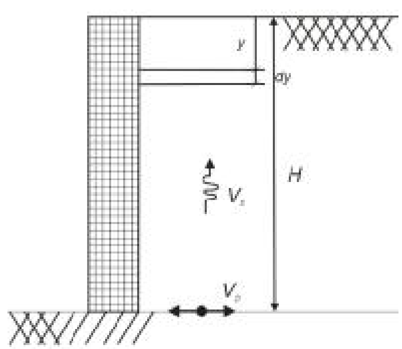

Figure 1. Model for Wave Propagation in Backfill Soil

Fig. 2 shows a failure plane inclined at an angle α with respect to the horizontal direction. An element with a thickness of dy at depth y from the ground surface is considered.

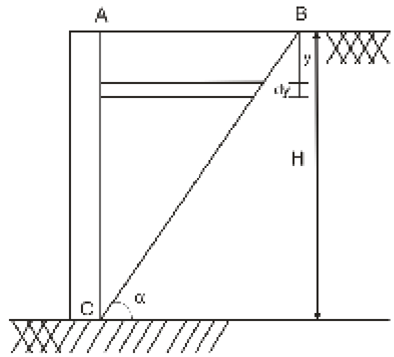

Figure 2. Soil Failed Wedge and Element of Soil Backfill

If the base of the wall is subjected to harmonic horizontal seismic acceleration ($a_h = k_h.g$) and harmonic vertical seismic acceleration ($a_v = k_v.g$), the accelerations at any depth z and time t from the wall top can be expressed as:

$$a_h(z,t) = a_h\, sin(\omega(t - \frac{H-y}{V_s}) \tag{1}$$

$$a_v(z,t) = a_v\, sin(\omega(t - \frac{H-y}{V_p}) \tag{2}$$

where H = height of the wall, V_P , V_s = wave velocities in the horizontal and vertical directions, respectively, and a_h , a_v = amplitude of acceleration in the horizontal and vertical directions, respectively. The horizontal and vertical seismic accelerations acting on the soil wedge as described in Eqs. (1) and (2) are not constants but dependent on time and phase difference in shear and primary waves propagating vertically through the backfill. This is normally proposed in pseudo-dynamic analysis of retaining walls.

3 DETERMINATION OF LATERAL EARTH PRESSURE

In the following sections the lateral static and seismic earth pressure determine separately then the total active earth pressure is obtained by add these two separate parts.

3.1 Determination of static lateral earth pressure

Consider as element of soil wedge with a thickness of $\mathbf{dy}$ at depth y from the ground surface (Fig.2). The forces exerted on this element are due to the vertical pressure $\boldsymbol{\sigma_y}$ on the top of the element, the vertical reaction $\boldsymbol{\sigma_y + d\sigma_y}$ on the bottom of the element, the normal reaction $\boldsymbol{\sigma_x}$ of the retaining wall, the shear $\boldsymbol{\tau_x}$ between the backfill and the back of the retaining wall, the normal reaction $\boldsymbol{\sigma_r}$ of the rest of the soil, the shear stress $\boldsymbol{\tau_r}$ between the sliding wedge and the rest of the backfill. Fig. 3 shows the stresses and forces exerted on an arbitrary element of the backfill soil.

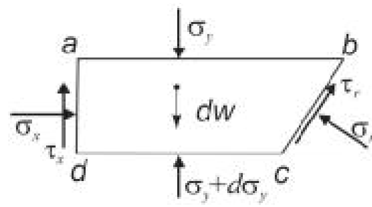

Figure 3. Forces and Stresses on an Element of Backfill

The equilibrium of horizontal forces on the element (Fig. 3) results in:

$$\sigma_x + \tau_r \cot \alpha - \sigma_r = 0 \tag{3}$$

Considering cohesionless backfill and assuming a full mobilization of shear forces along the failure plane gives:

$$\tau_x = \sigma_x \tan \delta \tag{4}$$

$$\tau_r = \sigma_r \tan \varphi \tag{5}$$

The horizontal earth pressure acting on the wall (σ_x)is equal to:

$$\sigma_x = K\sigma_y \tag{6}$$

where K is the lateral pressure coefficient.

Applying equilibrium condition to the vertical forces on the element (Fig. 3) results in:

$$\sigma_y(H-y)\cot\alpha + dW - (\sigma_y + d\sigma_y)(H-y-dy) \times \cot\alpha - \tau_x\, dy - \tau_r \frac{r}{\sin\alpha}\sin\alpha - \sigma_r \times \frac{dy}{\sin\alpha}\cos\alpha = 0 \tag{7}$$

With more simplifications, Eq. (7) may be converted to:

$$\frac{\sigma_y}{dy} = \gamma + \frac{1}{(H-y)}\left[\sigma_y - \sigma_r - (\tau_r + \tau_x)\tan\ \alpha\right] \tag{8}$$

$$\frac{\sigma_y}{dy} = [1-ak]\frac{\sigma_y}{H-y} + \gamma \tag{9}$$

where:

$$a = \frac{\cos(\alpha-\varphi-\delta)}{\sin(\alpha-\varphi)}\frac{\tan\alpha}{\cos\delta}$$

Eq. (9) is basic for computing the seismic lateral earth pressure on the wall and its solution is expressed as:

$$\sigma_y = \frac{A}{k}[H-y]^{aK-1} + \gamma\frac{H-y}{ak-2} \tag{10}$$

where:

$$A = \left(\frac{\gamma K H^{2-aK}}{2-aK}\right) \tag{11}$$

From Fig. 2, it is seen that the boundary condition is ($\sigma_{y=0} = 0$). Thus the following solution for the lateral earth pressure on the wall is obtained:

$$\sigma_{asx} = K \times H \left[\frac{1-\frac{y}{H}-\left(1-\frac{y}{H}\right)^{aK-1}\gamma}{ak-2}\right] \tag{12}$$

The lateral active earth pressure is obtained from:

$$\sigma_{as} = \sqrt{{\sigma_x}^2 + {\tau_x}^2} = \sqrt{{\sigma_x}^2 + {\sigma_x}^2\, tan^2\,\delta} = \frac{\sigma_{asx}}{cos\,\delta} \tag{13}$$

3.1.1 Determination of lateral earth pressure coefficient (K)

The lateral earth pressure coefficient, K, has a significant influence on the pressure distribution in seismic condition. Evaluation of this is a very important issue, which must be treated carefully. This paper proposed a method to determination of the lateral earth pressure coefficient, K. By using of moment equilibrium of each element of soil an attempt to determination of lateral earth pressure coefficient is made. The moment equilibrium of forces about point b in Fig. 3 gives:

$$\tau_x \times l \times dy - dW \times \frac{l}{2} + d\sigma_y \times \frac{l^2}{2} = 0 \tag{14}$$

where l is the length of upper side of element and given by:

$$l = \gamma \times l \times dy = cot\,\alpha\,(H-y) \tag{15}$$

Substituting Eq. (4) and (5) into Eq. (14) and omitting the differential terms of the second order gives:

$$\frac{d\sigma_y}{dy} = \gamma - \frac{2\,tan\,\varphi}{l} K\sigma_y \tag{16}$$

Equating Eq. (16) and Eq. (9) gives:

$$[1-ak]\frac{\sigma_y}{H-y} + \gamma = \gamma - \frac{2\,tan\,\varphi}{l} K\sigma_y \tag{17}$$

More simplification gives:

$$[1-ak] = -2\,tan\,\varphi\,\cot\alpha\,K \tag{18}$$

The lateral earth pressure coefficient is given by:

$$K = \frac{1}{(a - 2tan\,\varphi\,\tan\alpha)} \tag{19}$$

3.2 Determination of seismic lateral earth pressure

Similar to the pseudo-dynamic approach which considers finite shear wave velocity within the backfill material (Steedman and Zeng, 1990), it is also assumed that the backfill shear modulus (G) is constant with the depth. Only the phase and not the magnitude of accelerations are varying along the depth of the wall within backfill.

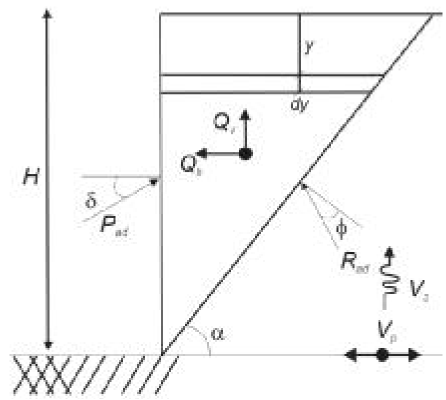

Figure 4. Analytical Model Considered for Computation of Seismic Part of Active Earth Pressure.

The mass of an element of wedge at depth z is

$$m = \frac{dW}{g} = \frac{\gamma}{g} \times l \times dz \quad (20)$$

The inertial force acting on a soil element is:

$$q_h = m.a_h(z,t) \quad (21)$$

Consider the shear wave velocity, $Vs = (\frac{G}{\rho})^{\frac{1}{2}}$, where, ρ is the density of the backfill material and primary wave velocity, $Vp = [\frac{G(2-2v)}{\rho(1-2v)}]^{1/2}$, where ν is the Poisson's ratio of the backfill are assumed to act within the soil media due to earthquake loading. For most geological materials, $\frac{Vp}{Vs} = 1.87$ (Das, 1993).

The period of lateral shaking, $T = \frac{2\pi}{\omega} = \frac{4H}{V_s}$, where ω is the angular frequency is considered in the analysis.

By substituting Eq. (1) into Eq. (22), the total horizontal inertial force acting within the failure zone can be expressed as:

$$Q_h(t) = \int_0^H m(z).a_h(z,t)dz = \frac{\lambda\gamma a_h}{4\pi^2 \mathrm{g} \tan\alpha}(2\pi H\cos\omega\xi + \lambda(sin\omega\xi - sin\omega t)) \quad (22)$$

where

$$\xi = t - \frac{H}{V_s}$$
$$\lambda = TV_s$$

By substituting Eq. (2) into Eq. (23), the total vertical inertial force acting within the failure zone can be expressed as:

$$Q_v(t) = \int_0^H m(z).a_v(z,t)dz = \frac{\eta\gamma a_v}{4\pi^2 g \tan\alpha}(2\pi H \cos\omega\psi + \eta(\sin\omega\psi - \sin\omega t)) \tag{23}$$

where

$$\psi = t - \frac{H}{V_p}$$

$$\eta = TV_p$$

The total seismic active thrust $P_{ad}(t)$ can be obtained by resolving the forces on the wedge and considering the equilibrium of the forces. Hence $P_{ad}(t)$ can be expressed as:

$$P_{ad} = \frac{Q_h\cos(\alpha-\varphi) - Q_v\sin(\alpha-\varphi)}{\cos(\alpha-\alpha+\varphi)} \tag{24}$$

The seismic portion of active earth pressure distribution can be obtained from:

$$\sigma_{ad} = \frac{\partial P_{ad}(z)}{\partial z} = \frac{\cos(\alpha-\varphi)k_h\gamma z}{\cos(\delta-\alpha+\varphi)\tan\alpha}\sin\omega\left(t-\frac{z}{V_s}\right) - \frac{\cos(\alpha-\varphi)k_v\gamma z}{\cos(\delta-\alpha+\varphi)\tan\alpha}\sin\omega\left(t-\frac{z}{V_p}\right) \tag{25}$$

4 DETERMINATION OF TOTAL LATERAL EARTH PRESSURE AND RESULTANT THRUST

The total lateral earth pressure is obtained by adding static and seismic portions of active earth pressure. Thus, the total earth pressure can be express as:

$$\sigma_{ae} = \sigma_s + \sigma_d \tag{26}$$

where

$$\sigma_s = \frac{\gamma H}{(a-2\tan\varphi\tan\alpha)\cos\delta}\left[\frac{1-\frac{y}{H}-\left(1-\frac{y}{H}\right)^{\frac{a}{(a-2\tan\varphi\tan\alpha)}-1}}{\frac{a}{(a-2\tan\varphi\tan\alpha)}-2}\right] \tag{27}$$

$$\sigma_d = \frac{\cos(\alpha-\varphi)\gamma y}{\cos(\delta-\alpha+\varphi)\tan\alpha}\left(k_h\sin\omega\left(t-\frac{y}{V_s}\right) - k_v\sin\omega\left(t-\frac{y}{V_p}\right)\right) \tag{28}$$

The total lateral active thrust is obtained by integrating static and seismic earth pressures. This gives:

$$P_{ae} = P_{as} + P_{ad} = \int_0^H \sigma_{ae} dy + \int_0^H \sigma_{ad} dy \tag{29}$$

Substituting static and seismic thrust in Eq. (29) gives:

$$P_{ae} = \frac{1}{2}\gamma H^2 \frac{\sin(\alpha-\varphi)\cot\alpha}{\cos(\alpha-\varphi-\delta)} + \left(\frac{T}{2H\pi^2\tan\alpha}\right) \times \frac{(k_h V_s \cos(\alpha-\varphi) m_1 - k_v V_p \sin(\alpha-\varphi)\, m_2)}{\cos(\delta+\varphi-\alpha)} \tag{30}$$

where

$$m_1 = 2\pi\cos 2\pi\left(\frac{t}{T}-\frac{H}{TV_s}\right) + \frac{TV_s}{H}\left[\sin 2\pi\left(\frac{t}{T}-\frac{H}{TV_s}\right) - \sin 2\pi\frac{t}{T}\right]$$

$$m_2 = 2\pi\cos 2\pi\left(\frac{t}{T}-\frac{H}{TV_P}\right) + \frac{TV_P}{H}\left[\sin 2\pi\left(\frac{t}{T}-\frac{H}{TV_P}\right) - \sin 2\pi\frac{t}{T}\right]$$

The maximum value of P_{ae} is obtained by maximizing P_{ae} with respect to t and α

5 HEIGHT OF APPLICATION OF RESULTANT EARTH PRESSURE

The locations of the resultant of earth pressures on retaining wall in static and seismic conditions are determined using the Coulomb and MO methods for static and seismic conditions, respectively. In both methods, a linear distribution of earth pressures on the wall is assumed and thus the location of the total thrust is exact. In the MO method, Eq. (31) is normally used to obtain H_{ae} (Seed and Withman, 1970):

$$H_{ae} = \frac{P_a\left(\frac{H}{3}\right) + \Delta P_{ae}(0.6H)}{P_{ae}} \tag{31}$$

where $\Delta P_{ae} = P_{ae} - P_a$

For curvilinear distribution earth pressure presented in this paper, the height of the application of the resultant earth pressure can be determined using the following procedure. The resultant moment of the pressure about the wall bottom can be obtained from:

$$M_{ae} = \int_0^L (H-y)\sigma_{ae} dl \tag{32}$$

The height H_{ae} of application of the resultant pressure is:

$$H_{ae} = \frac{M_{ae}}{P_{ae}\cos\delta} \tag{33}$$

6 RESULTS AND DISCUSSION:

In this section, an example for a rigid retaining wall with a height of 5 m is considered. All necessary parameters are given in Fig. 5. As seen, the earth pressure distribution is non-linear and its maximum does not occur at the wall bottom.

Moreover the earth pressure decrease to zero at the bottom of wall in static condition but zero earth pressure does not occur at the bottom of wall in seismic condition.

For better comparison of result the distribution of earth pressure by assuming linear distribution of earth pressure in static condition is shown in Fig. 5. As observed, by making this assumption, the maximum earth pressure occurs at the wall bottom.

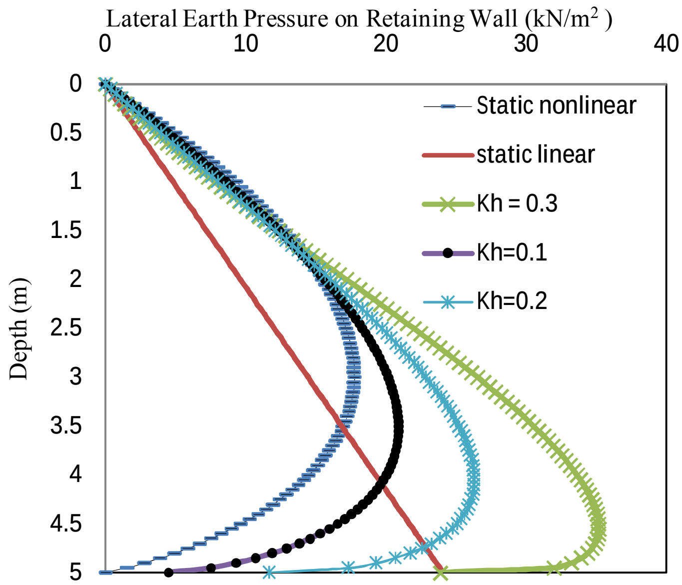

Figure 5. Distribution of Horizontal Earth Pressure on Retaining Wall for $\varphi = 33^{\circ}, \delta = 16^{\circ} k_v = 0, V_s = 100\ \frac{m}{s}, H = 5\ m, T = 0.2s, \frac{H}{\lambda} = 0.167, \frac{H}{\eta} = 0.09,\ \beta = 90^{\circ}, \gamma = 18\ \frac{kN}{m^3}$

As shown in Fig. 5, due to changes of lateral earth pressure with time and angle of slip surface (α), the distribution shown makes the total lateral thrust maximum. Hence the distribution of lateral earth pressure nonlinearly comes close to triangular. However, if the maximum overturning is taken into consideration, the distribution of earth pressure becomes close to parabolic variation (Fig. 6). Therefore, in the design of retaining wall, both distributions of lateral earth pressure should be taken into account.

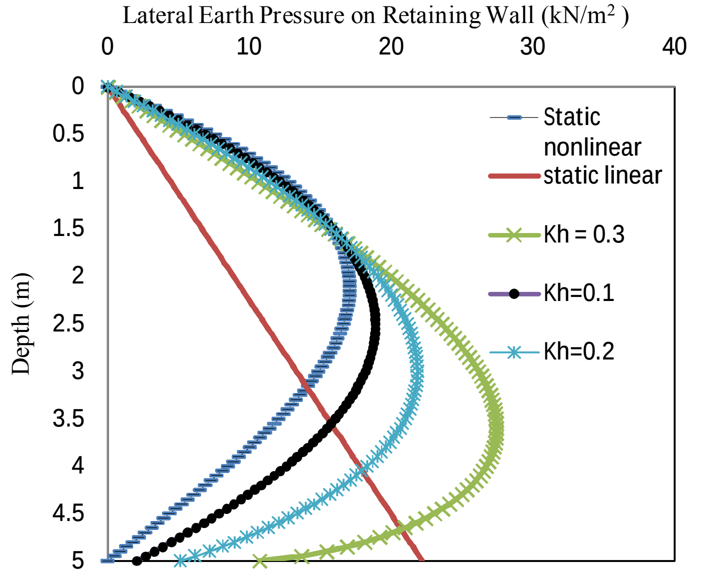

Figure 6. Distribution of Horizontal Earth Pressure on Retaining Wall for
$\varphi = 33^{\circ}, \delta = 16^{\circ} k_v = 0\,, V_s = 100\ \frac{m}{s}\,, H = 5\ m, T = 0.2s, \frac{H}{\lambda} = 0.167, \frac{H}{\eta} = 0.09,\ \beta = 90^{\circ}, \gamma = 18\ \frac{kN}{m^3}$

7 CONCLUSIONS:

This paper has proposed a pseudo-dynamic method by considering the time effect and phase change in shear and primary waves propagating in the backfill behind the rigid retaining wall. The seismic and static total thrust, the location of the thrust and distribution of lateral earth pressures due to static and seismic loading on rigid retaining walls have been presented using a closed form solution. The differential equation governing an arbitrary element at a given depth along the wall height has been derived and solved explicitly using appropriate boundary conditions. The seismic earth pressure distributions, the total thrust on the wall, and the location of the thrust have then been determined. The present study considers both static and seismic earth pressure distributions in a nonlinear manner. Parametric studies were performed to show how the earth pressure is distributed on a rigid retaining wall. The general superiority of the current solution is that the value of lateral earth pressure coefficient, K, is more accurate and lacks any simplifying assumptions. It has also been demonstrated that K is the most important parameter which influences the non-linear pressure distribution. It has also been found that due to changes of the lateral earth pressure with time and angle of slip surface (α), a certain distribution gives the maximum total lateral thrust. If the wall overturning is critical, a parabolic distribution of earth pressures offer the maximum moment. In general, in the design of retaining walls, both distributions of lateral earth pressures should be taken into account.

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