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A TWO STAGE APPROACH FOR THE SEISMIC ANALYSIS OF BASE-ISOLATED STRUCTURES

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Abstract. *Base Isolation System (BIS) is a very effective strategy for reducing the effects of earthquakes on building structures. The classical Response Spectrum Method (RSM) continues to be the most common approach for the design of base-isolated buildings. This paper offers a new strategy of seismic analysis and design for such structures in conjunction with the RSM. The main advantages of the proposed approach are: first, reduced computational effort with respect to an exact complex-valued modal analysis, which is obtained through a two-stage transformation of coordinates, both involving real-valued eigenproblems; second, accurate representation of the damping, which is pursued by consistently defining different viscous damping ratios for the modes of vibration of the coupled dynamic system made of superstructure and BIS; third, ease of use, since a convenient reinterpretation of the combination coefficients leads to a novel Damping-Adjusted Combination (DAC) rule, in which just a single response spectrum is required for the reference value of the viscous damping ratio.*

Keywords: BIS (Base Isolation System); CQC (Complete Quadratic Combination) rule; DCF (Damping Correction Factor); random vibration theory; RSM (Response Spectrum Method).

1 INTRODUCTION

Different types of Base Isolation System (BIS) have been developed over the years in the attempt of mitigating seismic effects on building structures, lowering structural and non-structural damages and reducing the risk for the occupants [1,2].

The main result of a BIS is to increase the fundamental period of vibration of the isolated structure, which is permitted by large deformations at the isolation level. This effect is contrasted either by providing isolators with high inherent damping, or by coupling them with additional fluid and/or metallic dampers. This combination of large period of vibration and high damping capabilities tremendously reduces the earthquake-induced forces in the superstructure, which in turn can be designed to remain in the elastic range, even in the case of strong ground motions. As a consequence, less energy is dissipated in the superstructure. It follows that, from a mathematical point of view, a seismically isolated structure is a non-classically damped system [3-6], with most of the energy being dissipated at the isolators' level, although this point is often overlooked in practice [7-10].

The classical Response Spectrum Method (RSM) continues to be the most common approach for the design of base-isolated buildings. This technique requires the following steps: (i) solve an eigenproblem for the undamped structure, i.e. frequencies, participation factors and deformed shapes for the first modes of vibration; (ii) select the appropriate viscous damping ratio for each mode; (iii) take from the pertinent design spectrum the maximum value for each modal response; (iv) combine the modal maxima to get the sought design response of the structure.

The last two steps introduce the main sources of inaccuracy in the practical application of the RSM. Indeed, most seismic codes furnish the response spectrum just for a reference value of the viscous damping ratio, which is typically $\zeta_0 = 0.05$, and thus quite different from the usual values assumed for BIS and superstructure. This circumstance motivates the use of a Damping Correction Factor (DCF), for which various expressions have been proposed [11-13]. It has been recently shown that such expressions can be quite different from each other [14], and they are not effective for seismically isolated structure [15,16].

In parallel, the combination rules available in the literature are not fully adequate. The so-called CQC (Complete Quadratic Combination) rule [17] is generally adopted by practitioners. This formula combines the ordinates of the given response spectrum by using the cross-correlation coefficients of the modal oscillators [18,19]. It has been recognized that the conventional expressions of CQC are unsuitable for analyzing non-classically damped structures, and hence alternative techniques have been proposed [20-22]. Unfortunately, the additional computational burden associated with the solution of complex-valued eigenproblems continues to limit the practical applications of these methods.

It follows that a new combination rule would be required for the analysis and design of base-isolated structures. To be effective in practice, and overcome the main shortcomings of existing techniques, an improved RSM should embed the following features: first, reduced computational effort; second, accurate representation of the damping; third, ease of use, requiring a single response spectrum for the reference value of the viscous damping ratio. These practical needs constitute the motivation of the present study.

The proposed DAC rule, which is alternative to the classical CQC rule and incorporates the features highlighted above, can be viewed as a special variant of the method originally developed by Falsone and Muscolino [23,24] for non-classically damped structures, and recently improved by Muscolino and Palmeri [25] for the seismic analysis of primary structures with light secondary appendages. The numerical results included in this contribution demonstrate the accuracy of the proposed method of analysis.

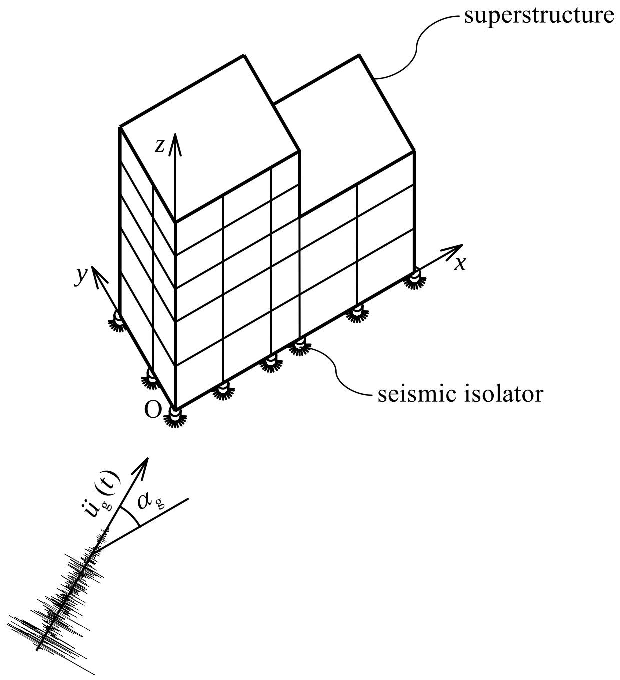

Figure 1. 3D sketch of a base-isolated building subjected to ground acceleration $\ddot{u}_{\mathrm{g}}(t)$ with angle of attack α_{g}.

2 EQUATIONS OF MOTION OF COUPLED BIS-SUPERSTUCTURE

Let us consider the n-floor base-isolated building depicted in Figure 1, subjected to the horizontal ground acceleration $\ddot{u}_{\mathrm{g}}(t)$ in the generic line of action defined by the angle α_{g}. Assuming diaphragmatic constraint for slabs and masses lumped at the storey's level, the building possesses $3(n+1)$ Degrees of Freedom (DoFs). The isolation level has three DoFs, listed in the array $\mathbf{u}_{\mathrm{b}}(t) = \left\{u_{\mathrm{x}}^{(\mathrm{b})}(t) \quad u_{\mathrm{y}}^{(\mathrm{b})}(t) \quad u_{\varphi}^{(\mathrm{b})}(t)\right\}^{\mathrm{T}}$, the superscript T denoting the transpose operator; these DoFs can be respectively defined as horizontal displacements along x and y directions of the origin O and rotation of the base about the vertical axis z, all taken with respect to the ground. The superstructure has $3n$ DoFs, listed in the n-dimensional block array $\mathbf{u}_{\mathrm{s}}^{\mathrm{T}}(t) = \left\{\mathbf{u}_1^{\mathrm{T}}(t) \mid \mathbf{u}_2^{\mathrm{T}}(t) \mid \cdots \mid \mathbf{u}_n^{\mathrm{T}}(t)\right\}^{\mathrm{T}}$, where the i-th three-dimensional array $\mathbf{u}_i(t) = \left\{u_{\mathrm{x}}^{(i)}(t) \quad u_{\mathrm{y}}^{(i)}(t) \quad u_{\varphi}^{(i)}(t)\right\}^{\mathrm{T}}$ collects origin's displacements and storey's rotation for the i-th level, with $i = 1, \cdots, n$, all taken with respect to the isolation level. Since BIS (Base Isolation System) and superstructure are characterised by a high contrast of mechanical properties, the substructure approach is resorted to. Accordingly, BIS and superstructure are handled as two interconnected substructures, and the equations ruling the seismic motion of the coupled dynamic system can be posed in the form:

$$\hat{\mathbf{M}} \cdot \ddot{\mathbf{u}}(t) + \hat{\mathbf{C}} \cdot \dot{\mathbf{u}}(t) + \hat{\mathbf{K}} \cdot \mathbf{u}(t) = \hat{\mathbf{G}} \cdot \boldsymbol{\tau}_{\mathrm{b}}(\alpha_{\mathrm{g}})\, \ddot{u}_{\mathrm{g}}(t)\,, \tag{1}$$

where the dot symbol $(\cdot)$ means matrix product; $\mathbf{u}(t) = \left\{\mathbf{u}_{\mathrm{s}}^{\mathrm{T}}(t) \mid \mathbf{u}_{\mathrm{b}}^{\mathrm{T}}(t)\right\}^{\mathrm{T}}$ is the super-array of the DoFs of the base-isolated building; $\boldsymbol{\tau}_{\mathrm{b}}(\alpha_{\mathrm{g}}) = \left\{\cos(\alpha_{\mathrm{g}}) \quad \sin(\alpha_{\mathrm{g}}) \quad 0\right\}^{\mathrm{T}}$ is the three-dimensional array listing the influence coefficients of the ground shaking; the matrices of mass $\hat{\mathbf{M}}$, viscous damping $\hat{\mathbf{C}}$ and elastic stiffness $\hat{\mathbf{K}}$ are those of the whole structure, and can be assembled from the corresponding matrices of superstructure and isolation level:

$$\hat{\mathbf{M}} = \left[\begin{array}{c|c} \mathbf{M}_\mathrm{s} & \mathbf{M}_\mathrm{s}\cdot\mathbf{R}_\mathrm{sb} \\ \hline \mathbf{R}_\mathrm{sb}^\mathrm{T}\cdot\mathbf{M}_\mathrm{s} & \mathbf{m}_\mathrm{r} \end{array}\right];\quad \hat{\mathbf{C}} = \left[\begin{array}{c|c} \mathbf{C}_\mathrm{s} & \mathbf{O}_{3n,3} \\ \hline \mathbf{O}_{3,3n} & \mathbf{c}_\mathrm{b} \end{array}\right];\quad \hat{\mathbf{K}} = \left[\begin{array}{c|c} \mathbf{K}_\mathrm{s} & \mathbf{O}_{3n,3} \\ \hline \mathbf{O}_{3,3n} & \mathbf{k}_\mathrm{b} \end{array}\right];\quad \hat{\mathbf{G}} = -\left[\begin{array}{c} \mathbf{M}_\mathrm{s}\,\mathbf{R}_\mathrm{sb} \\ \hline \mathbf{m}_\mathrm{r} \end{array}\right], \tag{2}$$

in which the symbol $\mathbf{O}_{p,q}$ stands for a zero matrix having p rows and q columns, while $\mathbf{R}_\mathrm{sb} = \left[\mathbf{I}_3 \mid \mathbf{I}_3 \mid \cdots \mid \mathbf{I}_3\right]^\mathrm{T}$ is a Boolean matrix of dimensions $3n\times 3$, the symbol $\mathbf{I}_p$ being the identity matrix of size p; $\mathbf{M}_\mathrm{s}$, $\mathbf{C}_\mathrm{s}$ and $\mathbf{K}_\mathrm{s}$ in Eqs. (2) are the $(3n)$-dimensional symmetric matrices of mass, damping and stiffness of the superstructure, assumed to be fixed to its own base; $\mathbf{c}_\mathrm{b}$ and $\mathbf{k}_\mathrm{b}$ in Eqs. (2) are the three-dimensional symmetric matrices collecting the effective values of viscous damping and elastic stiffness for the BIS, respectively; finally, $\mathbf{m}_\mathrm{r}$ is the three-dimensional symmetric mass matrix of the whole structure assumed as a rigid body moving on top of the seismic isolators:

$$\mathbf{m}_\mathrm{r} = \mathbf{m}_\mathrm{b} + \mathbf{R}_\mathrm{sb}^\mathrm{T}\cdot\mathbf{M}_\mathrm{s}\cdot\mathbf{R}_\mathrm{sb}\,, \tag{3}$$

in which $\mathbf{m}_\mathrm{b}$ is the three-dimensional mass matrix of the isolated base. One can easily verify that even though the mass, damping and stiffness matrices of two substructures separately taken satisfy the well-known condition due to Caughey-O'Kelly [26] the coupled BIS-superstructure assembly is in general a non-classically damped dynamic system.

3 TWO-STAGE TRANSFORMATION OF COORDINATES FOR THE MODAL EQUATIONS OF MOTION

The main difficulty in the direct use of Eq. (1) lies in the non classical nature of the energy dissipation for the whole structural system, which does not allow diagonalising together the matrices of mass, damping and stiffness in the real space. To overcome this drawback, a convenient two-stage transformation of coordinates is introduced in this section.

3.1 Stage 1

Two independent transformations of coordinates are initially operated on the base-isolated buildings, i.e. on superstructure and BIS, which can be written in compact form:

$$\left\{\begin{array}{c} \mathbf{u}_\mathrm{s}(t) \\ \hline \mathbf{u}_\mathrm{b}(t) \end{array}\right\} = \left[\begin{array}{c|c} \mathbf{\Phi}_\mathrm{s} & \mathbf{O}_{3n,3} \\ \hline \mathbf{O}_{3,m} & \mathbf{\Phi}_\mathrm{b} \end{array}\right]\cdot\left\{\begin{array}{c} \mathbf{q}_\mathrm{s}(t) \\ \hline \mathbf{q}_\mathrm{b}(t) \end{array}\right\}, \tag{4}$$

where the arrays $\mathbf{q}_\mathrm{s}(t) = \left\{q_{\mathrm{s},1}(t) \ \cdots \ q_{\mathrm{s},m}(t)\right\}^\mathrm{T}$ and $\mathbf{q}_\mathrm{b}(t) = \left\{q_{\mathrm{b},1}(t) \ \ q_{\mathrm{b},2}(t) \ \ q_{\mathrm{b},3}(t)\right\}^\mathrm{T}$ list the first $m \le 3n$ modal coordinates of the base-fixed superstructure and the three modal coordinates of the BIS, respectively; and where the $3n\times m$ tall matrix $\mathbf{\Phi}_\mathrm{s}$ and the 3×3 square matrix $\mathbf{\Phi}_\mathrm{b}$ are the associated real-valued modal matrices of the two subsystems, which can be evaluated along with the corresponding spectral matrices $\mathbf{\Omega}_\mathrm{s} = \mathrm{diag}\left\{\omega_{\mathrm{s},1},\cdots,\omega_{\mathrm{s},m}\right\}$ and $\mathbf{\Omega}_\mathrm{b} = \mathrm{diag}\left\{\omega_{\mathrm{b},1},\omega_{\mathrm{b},2},\omega_{\mathrm{b},3}\right\}$ as solution of two decoupled real-valued eigenproblems:

$$\mathbf{K}_\mathrm{s}^{-1}\cdot\mathbf{M}_\mathrm{s}\cdot\mathbf{\Phi}_\mathrm{s} = \mathbf{\Phi}_\mathrm{s}\cdot\mathbf{\Omega}_\mathrm{s}^{-2}\,;\quad \mathbf{k}_\mathrm{b}^{-1}\cdot\mathbf{m}_\mathrm{r}\cdot\mathbf{\Phi}_\mathrm{b} = \mathbf{\Phi}_\mathrm{b}\cdot\mathbf{\Omega}_\mathrm{b}^{-2}\,, \tag{5}$$

with the respective ortho-normalization conditions

$$\mathbf{\Phi}_\mathrm{s}^\mathrm{T}\cdot\mathbf{M}_\mathrm{s}\cdot\mathbf{\Phi}_\mathrm{s} = \mathbf{I}_m\,;\quad \mathbf{\Phi}_\mathrm{b}^\mathrm{T}\cdot\mathbf{m}_\mathrm{r}\cdot\mathbf{\Phi}_\mathrm{b} = \mathbf{I}_3\,. \tag{6}$$

Upon substitution of Eqs. (4) into Eq.(1), after some simple algebra, the equations of motion take the form:

$$\bar{\mathbf{m}} \cdot \ddot{\mathbf{q}}(t) + \bar{\mathbf{c}} \cdot \dot{\mathbf{q}}(t) + \bar{\mathbf{k}} \cdot \mathbf{q}(t) = \bar{\mathbf{g}}(\alpha_{\mathrm{g}})\, \ddot{u}_g(t)\,, \tag{7}$$

in which $\mathbf{q}(t) = \left\{ \mathbf{q}_{\mathrm{s}}^{\mathrm{T}}(t) \;\vdots\; \mathbf{q}_{\mathrm{b}}^{\mathrm{T}}(t) \right\}^{\mathrm{T}}$ is $(m+3)$-dimensional array collecting the modal coordinates of both superstructure and BIS. The other quantities appearing in Eq. (7) are so defined:

$$\begin{gathered}
\bar{\mathbf{m}} = \left[\begin{array}{c|c} \mathbf{I}_m & \boldsymbol{\Phi}_{\mathrm{s}}^{\mathrm{T}} \cdot \mathbf{M}_{\mathrm{s}} \cdot \mathbf{R}_{\mathrm{sb}} \cdot \boldsymbol{\Phi}_{\mathrm{b}} \\ \hline \boldsymbol{\Phi}_{\mathrm{b}}^{\mathrm{T}} \cdot \mathbf{R}_{\mathrm{sb}}^{\mathrm{T}} \cdot \mathbf{M}_{\mathrm{s}} \cdot \boldsymbol{\Phi}_{\mathrm{s}} & \mathbf{I}_3 \end{array} \right]; \\
\bar{\mathbf{c}} = \left[\begin{array}{c|c} 2\boldsymbol{\zeta}_{\mathrm{s}} \boldsymbol{\Omega}_{\mathrm{s}} & \mathbf{O}_{m,3} \\ \hline \mathbf{O}_{3,m} & 2\boldsymbol{\zeta}_{\mathrm{b}} \boldsymbol{\Omega}_{\mathrm{b}} \end{array} \right]; \quad \bar{\mathbf{k}} = \left[\begin{array}{c|c} \boldsymbol{\Omega}_{\mathrm{s}}^2 & \mathbf{O}_{m,3} \\ \hline \mathbf{O}_{3,m} & \boldsymbol{\Omega}_{\mathrm{b}}^2 \end{array} \right]; \quad \bar{\mathbf{g}}(\alpha_{\mathrm{g}}) = -\left[\begin{array}{c} \boldsymbol{\Phi}_{\mathrm{s}}^{\mathrm{T}} \mathbf{M}_{\mathrm{s}} \mathbf{R}_{\mathrm{sb}} \\ \hline \boldsymbol{\Phi}_{\mathrm{b}}^{\mathrm{T}} \mathbf{m}_{\mathrm{r}} \end{array} \right] \cdot \boldsymbol{\tau}_{\mathrm{b}}(\alpha_{\mathrm{g}})\,.
\end{gathered} \tag{8}$$

3.2 Stage 2

The reduced matrices $\bar{\mathbf{m}}$, $\bar{\mathbf{c}}$ and $\bar{\mathbf{k}}$ specified in Eq.(8), as the corresponding matrices of Eqs.(1) , do not satisfy the Caughey-O'Kelly condition [26]. Hence, the practical difficulty persists that the conventional formulation of the RSM is not applicable. This drawback can be overcome by using a further transformation of coordinates:

$$\mathbf{q}(t) = \tilde{\boldsymbol{\Phi}} \cdot \tilde{\boldsymbol{\gamma}}(\alpha_{\mathrm{g}}) \cdot \tilde{\boldsymbol{\theta}}(t)\,, \tag{9}$$

where $\tilde{\boldsymbol{\theta}}(t) = \left\{ \tilde{\theta}_1(t) \;\; \cdots \;\; \tilde{\theta}_{m+3}(t) \right\}^{\mathrm{T}}$ is the $(m+3)$-dimensional array of the modal coordinates obtained by combining superstructure and BIS; the square transformation matrix $\tilde{\boldsymbol{\Phi}}$, along with the diagonal spectral matrix $\tilde{\boldsymbol{\Omega}} = \operatorname{diag}\left\{ \tilde{\omega}_1, \cdots, \tilde{\omega}_{m+3} \right\}$, can be obtained as solution of the new real-valued eigenproblem:

$$\bar{\mathbf{k}}^{-1} \cdot \bar{\mathbf{m}} \cdot \tilde{\boldsymbol{\Phi}} = \tilde{\boldsymbol{\Phi}} \cdot \tilde{\boldsymbol{\Omega}}^{-2}\,, \tag{10}$$

with the ortho-normalization condition $\tilde{\boldsymbol{\Phi}}^{\mathrm{T}} \cdot \bar{\mathbf{m}} \cdot \tilde{\boldsymbol{\Phi}} = \mathbf{I}_{m+3}$; $\tilde{\boldsymbol{\gamma}}(\alpha_{\mathrm{g}}) = \operatorname{diag}\left\{ \tilde{\gamma}_1(\alpha_{\mathrm{g}}), \cdots, \tilde{\gamma}_{m+3}(\alpha_{\mathrm{g}}) \right\}$ is the diagonal matrix collecting the $(m+3)$ modal participation coefficients $\tilde{\gamma}_i(\alpha_{\mathrm{g}}) = \left\{ \tilde{\boldsymbol{\Phi}} \right\}_i^{\mathrm{T}} \cdot \bar{\mathbf{g}}(\alpha_{\mathrm{g}})$, $\left\{ \tilde{\boldsymbol{\Phi}} \right\}_i$ being the i-th column of the matrix $\tilde{\boldsymbol{\Phi}}$; and where the over-tilde denotes all the quantities associated with the proposed stage-2 transformation of coordinates. In doing so, the reduced equations of motion in the modal space take the alternative form:

$$\ddot{\tilde{\boldsymbol{\theta}}}(t) + \tilde{\boldsymbol{\Xi}} \cdot \dot{\tilde{\boldsymbol{\theta}}}(t) + \tilde{\boldsymbol{\Omega}}^2 \cdot \tilde{\boldsymbol{\theta}}(t) = \mathbf{1}_{m+3}\, \ddot{u}_g(t)\,, \tag{11}$$

in which the symbol $\mathbf{1}_s$ stands for a unit array of size s , while $\tilde{\boldsymbol{\Xi}} = \tilde{\boldsymbol{\Phi}}^{\mathrm{T}} \cdot \bar{\mathbf{c}} \cdot \tilde{\boldsymbol{\Phi}}$ is the matrix of viscous damping in the transformed modal space. Given the non-classical nature of the energy dissipation in base-isolated buildings, this matrix is generally sparse, but with off-diagonal terms much less than the diagonal ones. Hence, from an engineering point of view, the modal coupling in terms of damping forces can be neglected, so that the time evolution of i-th modal coordinate $\tilde{\theta}_i(t)$ is ruled by:

$$\ddot{\tilde{\theta}}_i(t) + 2\,\tilde{\zeta}_i \tilde{\omega}_i\, \dot{\tilde{\theta}}_i(t) + \tilde{\omega}_i^2\, \tilde{\theta}_i(t) = \ddot{u}_g(t)\,. \tag{12}$$

where $\tilde{\zeta}_i = \tilde{\Xi}_{ii} / 2\tilde{\omega}_i$. In this equation $\tilde{\Xi}_{ii}$ and $\tilde{\omega}_i = \tilde{\Omega}_{ii}$ are the elements on principal diagonal of the matrices $\tilde{\boldsymbol{\Xi}}$ and $\tilde{\boldsymbol{\Omega}}$, respectively. It is worth emphasizing that the additional computational burden required by the proposed two-stage transformation of coordinates is very low. Indeed, in real applications the size of the second eigenproblem (Eq. (10)) is much less than the size of the eigenproblem required for a conventional modal analysis, being very often $m+3 \ll 3(n+1)$.

Remarkably, only two approximations have been introduced so far, namely:

i) modal truncation for the superstructure, since only the first m modes of vibration are retained for the base-fixed superstructure;

ii) modal decoupling, since the off-diagonal terms in modal matrix of viscous damping are neglected.

Both approximations are acceptable in the vast majority of practical design situations.

4 CONVENTIONAL RESPONSE SPECTRUM METHOD

Before proceeding with the formulation of the proposed Response Spectrum Method (RSM), let us summarize the conventional implementation of the RSM as formulated in EC8 [27], for base-isolated buildings, which requires the following steps:

1. Select the number $\hat{m}$ of vibrational modes to be retained for the superstructure-BIS coupled dynamic system (in practice, $\hat{m} \ll 3(n+1)$);

2. Compute modal matrix $\hat{\mathbf{\Phi}}$, whose columns are the real-valued modal shapes of the superstructure-BIS combined system, and the associated spectral matrix $\hat{\mathbf{\Omega}} = \operatorname{diag}\{\hat{\omega}_1, \cdots, \hat{\omega}_{\hat{m}}\}$, which are solution of the real-valued eigenproblem:

$$\hat{\mathbf{K}}^{-1} \cdot \hat{\mathbf{M}} \cdot \hat{\mathbf{\Phi}} = \hat{\mathbf{\Phi}} \cdot \hat{\mathbf{\Omega}}^{-2} \,, \tag{13}$$

with the normalization condition $\hat{\mathbf{\Phi}}^{\mathrm{T}} \cdot \hat{\mathbf{M}} \cdot \hat{\mathbf{\Phi}} = \mathbf{I}_{\hat{m}}$, where $\hat{\mathbf{M}}$ and $\hat{\mathbf{K}}$ are the block matrices of mass and stiffness introduced in Eq. (2).

3. Define the equivalent viscous damping ratio ζ_{b} for the BIS and ζ_{s} for the superstructure.

4. For the generic structural response of interest, $y(t)$, define the set of influence coefficients $\hat{e}_i$ of the $\hat{m}$ modal coordinates $\hat{q}_i(t)$, with $i = 1, \cdots, \hat{m}$, so that:

$$y(t) \cong \hat{y}(t) = \sum_{i=1}^{\hat{m}} \hat{e}_i \, \hat{q}_i(t) \,, \tag{14}$$

where the hat means the use of conventional approach. Notice that in the influence coefficients $\hat{e}_i$.

5. Evaluate the design value of $\hat{y}(t)$ according to the Complete Quadratic Combination (CQC) rule (Wilson et al. 1981):

$$\hat{Y} = \sqrt{\sum_{i=1}^{\hat{m}} \sum_{k=1}^{\hat{m}} \hat{\rho}(i,k) \, \hat{e}_i \, \hat{g}_i(\alpha_{\mathrm{g}}) \, \hat{e}_k \, \hat{g}_k(\alpha_{\mathrm{g}}) \frac{A_{\mathrm{e}}\left(\hat{T}_i, \hat{\zeta}_i\right)}{\hat{\omega}_i^2} \frac{A_{\mathrm{e}}\left(\hat{T}_k, \hat{\zeta}_k\right)}{\hat{\omega}_k^2}} \,, \tag{15}$$

where $\hat{g}_i(\alpha_{\mathrm{g}}) = \left\{\hat{\mathbf{\Phi}}\right\}_i^{\mathrm{T}} \cdot \hat{\mathbf{G}} \cdot \boldsymbol{\tau}_{\mathrm{b}}(\alpha_{\mathrm{g}})$ is the i-th participation factor; $\hat{\rho}(i,k)$ is the correlation coefficient between i-th and k-th modes of vibration, usually computed under the assumption that the seismic acceleration is a zero-mean stationary Gaussian white noise [17-19]; and $A_{\mathrm{e}}\left(\hat{T}_i, \hat{\zeta}_i\right)$ is the i-th ordinate of the elastic response spectrum in terms of pseudo-acceleration. This quantity depends in turn on periods of vibration $\hat{T}_i = 2\pi/\hat{\omega}_i$ and viscous damping ratios $\hat{\zeta}_i$ of the $\hat{m}$ modes of vibration. It is generally assumed that the modal viscous damping ratios for the BIS-superstructure combined system take the values $\hat{\zeta}_i = \zeta_{\mathrm{b}}$ for $i \le 3$ and $\hat{\zeta}_i = \zeta_{\mathrm{s}}$ for $i \ge 4$. Indeed, the first three modes of vibrations are those associated with large deformations of the seismic isolators, and hence the energy dissipation of the

BIS dictates the damping, while higher modes of vibration mainly involves the deformation of the superstructure, which dissipates less energy.

It may be useful to remember that Eq. (15) is consistent with the random vibration theory, so that the sought design value $\hat{Y}$ is defined as the expected extreme value of the structural response $\hat{y}(t)$ during the seismic motion, that is:

$$\hat{Y} = \mathrm{E}\left\langle \max\left|\hat{y}(t)\right|\right\rangle = \mathrm{PF}\left\langle \hat{y}(t)\right\rangle \mathrm{SD}\left\langle \hat{y}(t)\right\rangle , \tag{16}$$

where the symbols $\mathrm{E}\langle\cdot\rangle$, $\mathrm{PF}\langle\cdot\rangle$ and $\mathrm{SD}\langle\cdot\rangle$ denote Expectation (E) operator, dimensionless Peak Factor (PF) and Standard Deviation (SD), respectively; and where the notation $|x|$ stands for the modulus of the scalar *x*. In the conventional CQC rule it is also made the additional simplifying assumption that PFs of structural response $\hat{y}(t)$ and modal coordinates $\hat{q}_i(t)$ are approximately equal.

5 PROPOSED DAMPING-ADJUSTED COMBINATION RULE

Aim of this section is to then formulate a Damping-Adjusted Combination (DAC) rule in which a new set of combination and correction coefficients, consistently based on the random vibration theory, enables one to make use of a single response spectrum in combining the modal maxima. The particularization of the proposed DAC rule in the case of white noise excitation, leading to handy closed-form expressions, is presented in the next section. The equivalence of dimensionless PF for structural response and PFs for the contributing modal responses (i.e. one of the main assumptions of CQC rule) is also removed.

Let $y(t)$ be the time history of a generic structural quantity of interest for the purposes of designing a base-isolated building. Without loss of generality, this structural response can be expressed as linear combination of the DoFs of the system:

$$y(t) = \mathbf{d}_\mathrm{s}^\mathrm{T} \cdot \mathbf{u}_\mathrm{s}(t) + \mathbf{d}_\mathrm{b}^\mathrm{T} \cdot \mathbf{u}_\mathrm{b}(t) , \tag{17}$$

where the arrays $\mathbf{d}_\mathrm{s} = \left\{ d_{\mathrm{s},1} \quad \cdots \quad d_{\mathrm{s},3n} \right\}^\mathrm{T}$ and $\mathbf{d_b} = \left\{ d_{\mathrm{b},1} \quad d_{\mathrm{b},2} \quad d_{\mathrm{b},3} \right\}^\mathrm{T}$ collect the influence coefficients of superstructure and isolation level, respectively. Once the suggested two-stage transformation of coordinates is adopted, the generic structural response $y(t)$ can be expressed by a linear combination of the modal responses:

$$y(t) \cong \tilde{y}(t) = \tilde{\boldsymbol{\varepsilon}}^T \, \tilde{\boldsymbol{\theta}}(t) = \sum_{i=1}^{m+3} \tilde{\varepsilon}_i \, \tilde{\gamma}_i(\alpha_\mathrm{g}) \, \tilde{\theta}_i(t) , \tag{18}$$

in which $\tilde{\boldsymbol{\varepsilon}} = \left\{ \tilde{\varepsilon}_1 \quad \cdots \quad \tilde{\varepsilon}_{m+3} \right\}^\mathrm{T}$ is the array listing the influence coefficients of the $m+3$ modal coordinates for structural response under consideration, and can be assembled as:

$$\tilde{\boldsymbol{\varepsilon}} = \tilde{\boldsymbol{\Phi}}^\mathrm{T} \cdot \left\{ \begin{array}{c} \boldsymbol{\Phi}_\mathrm{s}^\mathrm{T} \cdot \mathbf{d}_\mathrm{s} \\ \hdashline \boldsymbol{\Phi}_\mathrm{b}^\mathrm{T} \cdot \mathbf{d}_\mathrm{b} \end{array} \right\}^\mathrm{T} . \tag{19}$$

By assuming that structural response $\tilde{y}(t)$ and modal responses $\tilde{\theta}_i(t)$, with $i = 1, \cdots, (m+3)$, are zero-mean Gaussian random processes, the standard deviation $\mathrm{SD}\left\langle \tilde{y}(t) \right\rangle$ can be expressed in the form:

$$\mathrm{SD}\left\langle \tilde{y}(t) \right\rangle = \sqrt{\sum_{i=1}^{m+3}\sum_{k=1}^{m+3} \tilde{\rho}(i,k)\, \tilde{\varepsilon}_i\, \tilde{\gamma}_i(\alpha_{\mathrm{g}})\, \tilde{\sigma}_i\, \tilde{\varepsilon}_k\, \tilde{\gamma}_k(\alpha_{\mathrm{g}})\, \tilde{\sigma}_k} \,, \tag{20}$$

where $\tilde{\rho}(i,k)$ is the following correlation coefficient:

$$\tilde{\rho}(i,k) = \tilde{\sigma}_{i,k} \big/ \left(\tilde{\sigma}_i\, \tilde{\sigma}_k \right), \tag{21}$$

in which $\tilde{\sigma}_i$ and $\tilde{\sigma}_{i,k}$ stand for Standard Deviation (SD) of the i-th stationary modal response $\tilde{\theta}_i(t)$ and steady-state covariance between $\tilde{\theta}_i(t)$ and $\tilde{\theta}_k(t)$, respectively:

$$\tilde{\sigma}_i = \mathrm{SD}\left\langle \tilde{\theta}_i(t) \right\rangle = \sqrt{\mathrm{E}\left\langle \tilde{\theta}_i(t)^2 \right\rangle}\,; \tag{22}$$

$$\tilde{\sigma}_{i,k} = \mathrm{E}\left\langle \tilde{\theta}_i(t)\, \tilde{\theta}_k(t) \right\rangle. \tag{23}$$

Following now the same strategy recently proposed by Muscolino and Palmeri [25] for the seismic analysis of light secondary substructures, let us rewrite the modal correlation coefficient in the equivalent form:

$$\tilde{\rho}(i,k) = \frac{\tilde{\sigma}_{i,k}}{\tilde{\sigma}_i\, \tilde{\sigma}_k} = \tilde{r}(i,k) \frac{\tilde{\sigma}_i^{(0)}\, \tilde{\sigma}_k^{(0)}}{\tilde{\sigma}_i\, \tilde{\sigma}_k}\,, \tag{24}$$

in which the novel combination coefficient $\tilde{r}(i,k)$ is so defined:

$$\tilde{r}(i,k) = \frac{\mathrm{E}\left\langle \tilde{\theta}_i(t)\, \tilde{\theta}_k(t) \right\rangle}{\tilde{\sigma}_i^{(0)}\, \tilde{\sigma}_k^{(0)}}\,, \tag{25}$$

where the denominator $\tilde{\sigma}_i^{(0)} = \mathrm{SD}\left\langle \tilde{\theta}_i^{(0)}(t) \right\rangle$ denotes the standard deviation of the stationary seismic response of a dummy SDoF oscillator associated with the i-th mode of vibration of the base-isolated building. This dummy oscillator has unit mass, a reference value of the viscous damping ratio, ζ_0, for which the elastic response spectrum in known, and undamped period of vibration $\tilde{T}_i = 2\pi / \tilde{\omega}_i$. Therefore, the dynamic response of this dummy oscillator is ruled by:

$$\ddot{\tilde{\theta}}_i^{(0)}(t) + 2\,\zeta_0 \tilde{\omega}_i\, \dot{\tilde{\theta}}_i^{(0)}(t) + \tilde{\omega}_i^2\, \tilde{\theta}_i^{(0)}(t) = \ddot{u}_g(t)\,. \tag{26}$$

Interestingly, Eq. (26) can be simply derived from Eq. (12) by substituting the viscous damping ratio (i.e., the reference value ζ_0 instead of the actual modal value $\tilde{\zeta}_i$).
Once the new set of combination coefficients has been introduced, substitution of Eq. (24) into Eq.(20), and the result into Eq. (16), gives:

$$\tilde{Y} = \mathrm{PF}\left\langle \tilde{y}(t) \right\rangle \sqrt{\sum_{i=1}^{m+3}\sum_{k=1}^{m+3} \tilde{r}(i,k)\, \tilde{\varepsilon}_i\, \tilde{\gamma}_i(\alpha_{\mathrm{g}})\, \tilde{\varepsilon}_k\, \tilde{\gamma}_k(\alpha_{\mathrm{g}})\, \tilde{\sigma}_i^{(0)}\, \tilde{\sigma}_k^{(0)}}\,. \tag{27}$$

This formula can be further manipulated by multiplying and dividing each term in the double summation by the PF of the corresponding dummy oscillator, and by introducing the dimensionless correction coefficient $\tilde{\chi}_i^{(0)}$ so defined:

$$\tilde{\chi}_i^{(0)} = \mathrm{PF}\left\langle \tilde{y}(t) \right\rangle \big/ \mathrm{PF}\left\langle \tilde{\theta}_i^{(0)}(t) \right\rangle. \tag{28}$$

Indeed, according to Eq. (16) and to the conventional CQC rule, the expected extreme value of the dynamic response of a dummy oscillator can be assumed to be equal to the maximum displacement given by the pertinent elastic response spectrum:

$$\left\{\mathrm{PF}\left\langle\tilde{\theta}_i^{(0)}(t)\right\rangle\tilde{\sigma}_i^{(0)}\right\}=\mathrm{E}\left\langle\max\left|\tilde{\theta}_i^{(0)}(t)\right|\right\rangle=\frac{A_e\left(\tilde{T}_i,\zeta_0\right)}{\tilde{\omega}_i^2}. \tag{29}$$

Therefore, Eq. (27) gives:

$$\tilde{Y}=\sqrt{\sum_{i=1}^{m+3}\sum_{k=1}^{m+3}\tilde{r}(i,k)\frac{\tilde{\varepsilon}_i\,\tilde{\gamma}_i(\alpha_\mathrm{g})\,\tilde{\chi}_i^{(0)}}{\tilde{\omega}_i^2}\frac{\tilde{\varepsilon}_k\,\tilde{\gamma}_k(\alpha_\mathrm{g})\,\tilde{\chi}_k^{(0)}}{\tilde{\omega}_k^2}A_e\left(\tilde{T}_i,\zeta_0\right)A_e\left(\tilde{T}_k,\zeta_0\right)}, \tag{30}$$

which is formally similar to the CQC rule of Eq. (15). However, there are three fundamental novelties in the proposed formula:

1. The combination coefficient $\tilde{r}(i,k)$, defined by Eq.(25), is no more the correlation coefficient between i-th and k-th modal oscillators of the base-isolated building, and requires the evaluation in the frequency domain of the following quantities:

$$\tilde{\sigma}_i^{(0)}=2\int_0^{+\infty}\left|\tilde{H}_i^{(0)}(\omega)\right|^2 S_g(\omega)\,d\omega\,; \tag{31}$$

$$\mathrm{E}\left\langle\tilde{\theta}_i(t),\tilde{\theta}_k(t)\right\rangle=2\int_0^{+\infty}\tilde{H}_i^*(\omega)\,\tilde{H}_k(\omega)\,S_g(\omega)\,d\omega\,. \tag{32}$$

in which $S_\mathrm{g}(\omega)$ is the Power Spectral Density (PSD) function simulating the energy distribution of the design ground shaking and:

$$\tilde{H}_i^{(0)}(\omega)=\left[\left(\tilde{\omega}_i^2-\omega^2\right)+j\left(2\,\zeta_0\,\tilde{\omega}_i\,\omega\right)\right]^{-1}; \tag{33}$$

$$\tilde{H}_i(\omega)=\left[\left(\tilde{\omega}_i^2-\omega^2\right)+j\left(2\,\tilde{\zeta}_i\,\tilde{\omega}_i\,\omega\right)\right]^{-1}. \tag{34}$$

2. The correction coefficient $\tilde{\chi}_i^{(0)}$ is introduced with respect to the i-th dummy oscillator (Eq.(28)), so that the hypothesis of equivalence between dimensionless PFs for structural response and contributing modal responses is removed;

3. Most importantly, only the elastic response spectrum for the reference value ζ_0 of the viscous damping ratio is required. It follows that the use of a semi-empirical DCF is avoided, and the effects of different viscous damping ratios in the actual modal oscillators are consistently taken into account by the novel combination ($\tilde{r}$) and correction ($\tilde{\chi}^{(0)}$) coefficients introduced in the double summation of Eq.(30). For this reason the formula has been termed Damping-Adjusted Combination (DAC) rule.

It is worth mentioning that all the quantities appearing in the proposed DAC rule of Eq. (30) can be easily evaluated in practice, so that the computational effort is just slightly higher with respect to the traditional CQC rule. Indeed, the modal parameters ($\tilde{\varepsilon}_i$, $\tilde{\gamma}_i$, $\tilde{\omega}_i$ and $\tilde{T}_i$) simply require the application of the two-stage transformations of coordinates presented in the third section, while the novel coefficients $\tilde{r}$ and $\tilde{\chi}_i^{(0)}$ introduced for the DAC rule just depend on the first spectral moments of both actual modal oscillators and dummy oscillators, which in turn can be easily computed by using traditional techniques of the random vibration theory.

6 CLOSED FORM EXPRESSION UNDER WHITE NOISE ASSUMPTION

The traditional CQC rule is usually applied under the simplified assumption that the ground acceleration is a stationary white noise, i.e. with energy uniformly distributed over the frequencies. The same assumption is considered in this section with the purpose of deriving simple closed-form expressions for the new coefficients introduced in the proposed Damping-Adjusted Combination (DAC) rule, in so reducing the computational burden. Indeed, when the Power Spectral Density (PSD) function of the input is constant, it is possible to evaluate the exact solution of the following integrals appearing in previous expressions. In particular by assuming that $S_g(\omega)=1$ the new combination coefficients of Eq. (25) becomes:

$$\tilde{r}(i,k)=\frac{\zeta_0}{\sqrt{\tilde{\zeta}_i\,\tilde{\zeta}_k}}\,\tilde{\rho}(i,k)\,, \tag{35}$$

where $\tilde{\rho}(i,k)$ is the correlation coefficient between i-th and k-th modes of vibration evaluated assuming the seismic excitation as a white noise process [18,19]:

$$\tilde{\rho}(i,k)=\frac{8}{\tilde{C}_{i,k}}\sqrt{\tilde{\zeta}_i\,\tilde{\zeta}_k}\;\tilde{\omega}_i\,\tilde{\omega}_k\sqrt{\tilde{\omega}_i\,\tilde{\omega}_k}\left(\tilde{\zeta}_i\,\tilde{\omega}_i+\tilde{\zeta}_k\,\tilde{\omega}_k\right), \tag{36}$$

and

$$\tilde{C}_{i,k}=\tilde{B}_{i,k}^2+4\,\tilde{A}_{i,k}\,\tilde{\zeta}_i\,\tilde{\zeta}_k\,\tilde{\omega}_i\,\tilde{\omega}_k+4\left(\tilde{\zeta}_i^2+\tilde{\zeta}_k^2\right)\tilde{\omega}_i^2\,\tilde{\omega}_k^2\ , \tag{37}$$

$$\tilde{A}_{i,k}=\tilde{\omega}_i^2+\tilde{\omega}_k^2\ ;\quad \tilde{B}_{i,k}=\tilde{\omega}_i^2-\tilde{\omega}_k^2\ . \tag{38}$$

Under the white noise approximation, an additional simplification arises in the computation of the coefficient $\tilde{\chi}_i^{(0)}$ defined in Eq. (28) by introducing the Vanmarcke's PF [28,29], for the structural response of interest $\mathrm{PF}\langle\tilde{y}(t)\rangle$ and for the dummy responses $\mathrm{PF}\langle\tilde{\theta}_i^{(0)}(t)\rangle$:

$$\mathrm{PF}\langle\tilde{y}(t)\rangle=\sqrt{2\ln\left\{2.89\,N^+\langle\tilde{y}(t)\rangle\left[1-\exp\left(-1.77\,q\langle\tilde{y}(t)\rangle^{1.20}\sqrt{\ln\left(2.89\,N^+\langle x(t)\rangle\right)}\right)\right]\right\}}\,; \tag{39}$$

$$\mathrm{PF}\langle\tilde{\theta}_i^{(0)}(t)\rangle=\sqrt{2\ln\left\{0.4601\,\tilde{\omega}_i\,T_{\mathrm{g}}\left[1-\exp\left(-0.3283\sqrt{\ln\left(0.4601\,\tilde{\omega}_i\,T_{\mathrm{g}}\right)}\right)\right]\right\}}\,. \tag{40}$$

Importantly, $\mathrm{PF}\langle\tilde{\theta}_i^{(0)}(t)\rangle$ depends only on the dimensionless quantity $\tilde{\omega}_i\,T_{\mathrm{g}}$, T_{g} being the duration of the strong-phase of the ground motion, while $\mathrm{PF}\langle\tilde{y}(t)\rangle$ depends on the expected number of upcrossings of the time axis, $N^+\langle\tilde{y}(t)\rangle$, and on the bandwidth parameter $q\langle\tilde{y}(t)\rangle$. These quantities can be evaluated as:

$$N^+\langle\tilde{y}(t)\rangle=\frac{1}{2\pi}\sqrt{\frac{\lambda_2\langle\tilde{y}(t)\rangle}{\lambda_0\langle\tilde{y}(t)\rangle}}\,T_{\mathrm{g}}\ ; \tag{41}$$

$$q\langle\tilde{y}(t)\rangle=\sqrt{1-\frac{\lambda_1\langle\tilde{y}(t)\rangle^2}{\lambda_0\langle\tilde{y}(t)\rangle\,\lambda_2\langle\tilde{y}(t)\rangle}}\ . \tag{42}$$

The symbol $\lambda_\ell\langle\tilde{y}(t)\rangle$ in Eqs. (41) and (42) is the spectral moments of order $\ell=0,1,2$ of the structural response $\tilde{y}(t)$, which under the white noise approximation can be evaluate in closed form as:

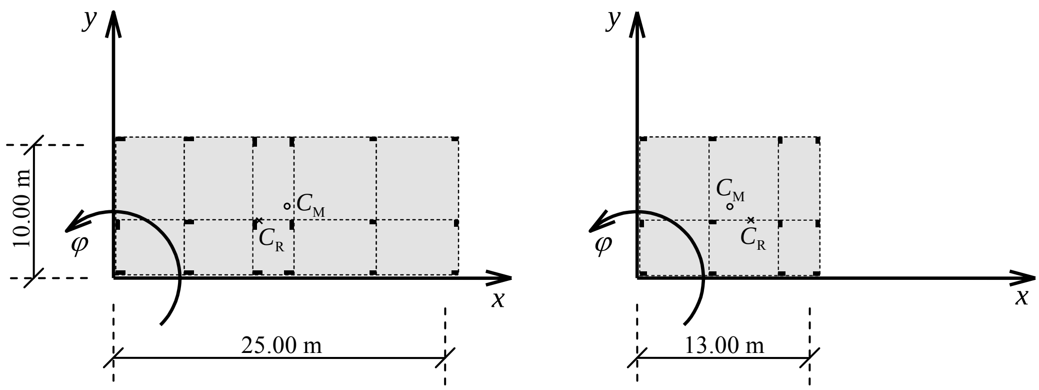

Figure 2. Bottom (left) and top (right) plan layouts of the base-isolated building considered in the numerical applications; circle (o) and cross (x) identify centre of mass (C_{M}) and centre of rigidity (C_{R}), respectively.

$$\lambda_0 \left\langle \tilde{y}(t) \right\rangle = \sum_{i=1}^{m+3} \sum_{k=i+1}^{m+3} \frac{4\pi\, \tilde{\varepsilon}_i\, \tilde{\varepsilon}_k}{\tilde{C}_{i,k}} \tilde{\gamma}_i\, \tilde{\gamma}_k \left(\tilde{\zeta}_i\, \tilde{\omega}_i + \tilde{\zeta}_k\, \tilde{\omega}_k \right); \tag{43}$$

$$\lambda_1 \left\langle \tilde{y}(t) \right\rangle = \sum_{i=1}^{m+3} \sum_{k=i+1}^{m+3} \frac{2\tilde{\varepsilon}_i\, \tilde{\varepsilon}_k}{\tilde{C}_{i,k}} \tilde{\gamma}_i\, \tilde{\gamma}_k \left[\left(\tilde{\zeta}_i\, \tilde{A}_{i,k} + 2\zeta_k\, \tilde{\omega}_i\, \tilde{\omega}_k \right) \tilde{D}_i + \left(\tilde{\zeta}_k\, \tilde{A}_{i,k} + 2\zeta_i\, \tilde{\omega}_i\, \tilde{\omega}_k \right) \tilde{D}_k - \tilde{B}_{i,k} \ln\left(\tilde{\omega}_i / \tilde{\omega}_k \right) \right]; \tag{44}$$

$$\lambda_2 \left\langle \tilde{y}(t) \right\rangle = \sum_{i=1}^{m+3} \sum_{k=i+1}^{m+3} \frac{4\pi\, \tilde{\varepsilon}_i\, \tilde{\varepsilon}_k}{\tilde{C}_{i,k}} \tilde{\gamma}_i\, \tilde{\gamma}_k\, \tilde{\omega}_i\, \tilde{\omega}_k \left(\tilde{\zeta}_i\, \tilde{\omega}_k + \tilde{\zeta}_k\, \tilde{\omega}_i \right), \tag{45}$$

in which:

$$\tilde{D}_i = \frac{1}{\sqrt{1-\tilde{\zeta}_i^2}} \arctan\left(\frac{\sqrt{1-\tilde{\zeta}_i^2}}{\tilde{\zeta}_i} \right). \tag{46}$$

7 NUMERICAL APPLICATIONS

7.1 Objective structure

Aimed at validating the proposed method of analysis and design, the seismic response of a representative 5-storey base-isolated building $(n = 5)$ has been investigated. A superstructure with irregular distributions of mass and stiffness in plan and elevation has been chosen (see Figure 1). The first three storeys have dimensions of $25\,\mathrm{m}$ by $10\,\mathrm{m}$, which reduce to $13\,\mathrm{m}$ by $10\,\mathrm{m}$ for the last two storeys. The floor layouts are sketched in Figure 2, where the position of centres

Table I. Exact ($\hat{T}_i$) and approximate ($\tilde{T}_i$) periods of vibration for the base-isolated building.

Mode i	$\hat{T}_i$ (s)	$\tilde{T}_i$ (s)	Inaccuracy (%)	Mode i	$\hat{T}_i$ (s)	$\tilde{T}_i$ (s)	Inaccuracy (%)
1	2.086	2.086	–0.002	5	0.244	0.243	–0.555
2	2.077	2.077	–0.005	6	0.214	0.208	–2.592
3	1.809	1.808	–0.011	7	0.143	0.141	–1.288

Table II. Analysis cases considered for validation purposes.

Case	Ground type	Duration of stationary part	Damping ratios		superstructure's modes
		T_g (s)	superstructure ζ_s	BIS ζ_b	m
(1)	A	15	0.02	0.12	8
(2)	A	30	0.02	0.012	8
(3)	C	30	0.02	0.012	8
(4)	C	30	0.01	0.18	8
(5)	C	30	0.01	0.18	13

of mass C_M (o) and rigidity C_R (x) is also shown. Their relative distance is $2.31\,\mathrm{m}$ for the first three storeys and $1.82\,\mathrm{m}$ for the last two storeys. The total mass of the base-isolated building is $M_{tot} = 1{,}106\,\mathrm{Mg}$ and the lateral stiffness of the BIS is $K_{lat}^{(b)} = 10{,}370\,\mathrm{kN/m}$, so that the nominal value of the isolation period is $T_{iso} = 2\pi\sqrt{M_{tot}/K_{lat}^{(b)}} = 2.05\,\mathrm{s}$.

7.2 Modal analysis

Aimed at investigating the accuracy of the proposed two-stage transformation of coordinates against the conventional modal analysis, exact ($\hat{T}_i = 2\pi/\hat{\omega}_i$) and approximate ($\tilde{T}_i = 2\pi/\tilde{\omega}_i$) periods of vibration for the first 8 modes of the undamped structure have been evaluated by using the full eigenproblem of Eq. (13) and the reduced one of Eq. (10), respectively. As shown in Table I, exact and approximate values are in excellent agreement, in so confirming the validity of the novel two-stage modal analysis for base-isolated buildings, which in turn is adopted for the proposed DAC rule.

7.3 Seismic analyses

Five methods of analysis have been applied and compared, namely: i) Response Spectrum Method (RSM), as formulated in EC8 [27]; ii) Monte Carlo Simulation (MCS), with 400 samples of artificially generated time histories of ground acceleration consistent with the response spectrum; iii) direct application of the random vibration theory with a consistent PSD function of the accelerograms; iv) proposed Damping-Adjusted Combination (DAC) rule; v) simplified version of the DAC rule under White Noise input (DAC-WN), as presented in Section 6. For each analysis, the angle of attack $\alpha_g = 90°$ has been used.

In order to test the performances of the proposed approach in different design situations, five cases have been analysed, which differ in seismic input and/or damping properties and/or number of retained modes. As summarized in Table II, two different soil conditions have been considered, i.e. ground types A (rocks) and C (deep deposits of dense sands or stiff clay) as defined in EC8 [27], along with two durations of the stationary part of the accelerogram, i.e. $T_g = 15\,\mathrm{s}$ and $30\,\mathrm{s}$; two sets of viscous damping ratios have been used for superstructure and BIS, i.e. $\{\zeta_s = 0.02,\ \zeta_b = 0.12\}$ and $\{\zeta_s = 0.01,\ \zeta_b = 0.18\}$; either $m = 8$ or 13 modes of vibration have been retained for the superstructure.

Figure 3 depicts the profiles of inter-storey drifts along the epicentral direction of the ground shaking for the five design situations summarized in Table IV and for angle of attack $\alpha_g = 90°$, which is the weakest direction of the 3D frame. In the pictures, the mean value of the MCS with 400 samples is shown with a solid black line, and the corresponding interval of confidence (mean value $\pm$ standard deviation) is delimited by a pair of dashed thin lines; the prediction of the random vibration theory is shown with a dashed gray line, while the values consistent with

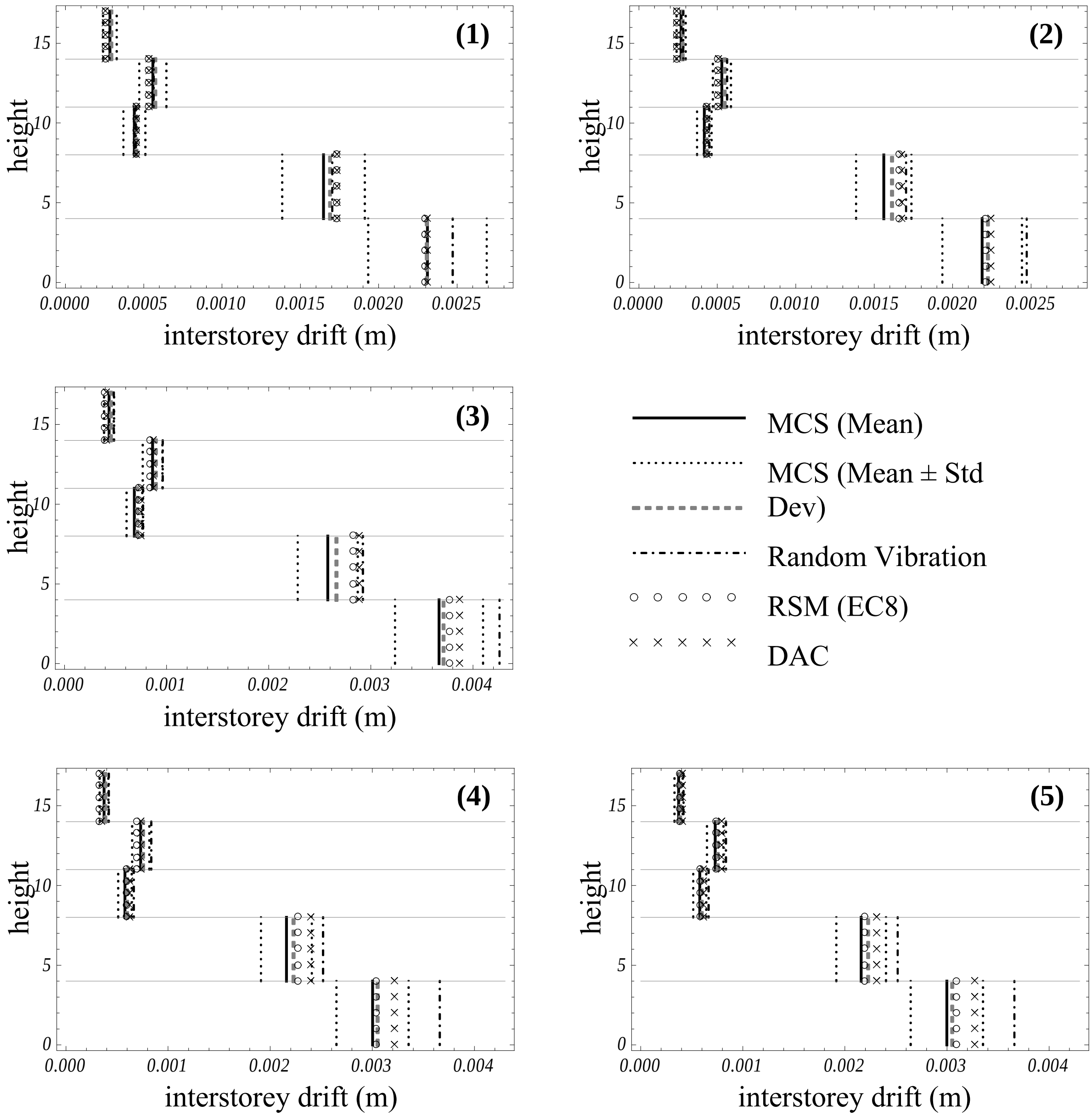

Figure 3. Profiles of superstructure's interstorey drifts for the five cases of Table IV when the agle of attack of the ground motion is $\alpha_g = 90°$ (weakest superstructure's direction).

RSM as formulated in EC8 are those with dot-dashed thin lines; DAC and DAC-WN results are reported with circles and crosses, respectively.

The inspection of these graphs reveals that:

- The proposed DAC and DAC-WN rules are always in good agreement with the results of MCS and random vibration theory.
- The results of the RSM (EC8) very often show large inaccuracies, falling most of the times outside the interval of confidence of the MCS.
- The larger is the stationary duration of the accelerogram, the smaller is the interval of confidence of the MCS (comparison of cases (1) and (2)), i.e. the more deterministic is the mean value of the generated response spectra.

- The predictions of the RSM (EC8) are less accurate when the viscous damping ratios ζ_s and ζ_b become smaller for superstructure and larger for BIS, respectively (comparison of cases (3) and (4)), i.e. when the discrepancy increases with respect to the reference value $\zeta_0 = 0.05$.(3)).
- The results' trend is not significantly affected by soil type (comparison of cases (2) and (3)) and number of modes retained in the analysis (comparison of cases (4) and (5)).

8 CONCLUSIONS

The two main sources of inaccuracy of the classical Response Spectrum Method (RSM) in the practical analysis and design of base-isolated buildings have been pointed out. First, the use of elastic response spectra for different values of the viscous damping ratio, although the seismic action is defined for just a single reference value; second, the use of combination rules not fully adequate for non-conventional structures. This is confirmed by the numerical results included in this paper, where the inaccuracy of the conventional RSM with DCF (Damping Correction Factor) and CQC (Complete Quadratic Combination) rule can be as large as 27%, which is unacceptable from an engineering point of view.
Aimed at overcoming these shortcomings, an improved RSM has been presented and validated. The proposed technique consists of a two-stage transformation of coordinates in parallel with a novel Damping-Adjusted Combination (DAC) rule. The following features have been embedded in the formulation: first, light computational effort, since the calculation of the exact complex-valued eigenproperties of base-isolated buildings is avoided; second, accurate representation of the damping for both superstructure and Base Isolation System (BIS); third, ease of use, requiring a single response spectrum for the reference value of the viscous damping ratio, i.e. the only spectrum which defines the seismic action. Numerical investigations for a realistic structure confirm the improved accuracy of the proposed method, leading toward more economical and/or dependable design of base-isolated buildings. The closed-form expressions derived under the white noise assumption for the ground acceleration allow reducing the computation burden in the proposed DAC rule, and hence they are particularly suitable for practical applications.

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