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INFLUENCE OF TYPE OF WAVE AND ANGLE OF INCIDENCE ON THE SEISMIC RESPONSE OF PILE FOUNDATIONS AND PILE SUPPORTED STRUCTURES

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Abstract. *The dynamic response of pile foundations and pile supported structures has been extensively studied during the last four decades. Even though, there exists yet the need of achieving a better understanding on different aspects of the problem, such as the response of inclined piles, the influence of the presence of nearby structures, the importance of various kinds of non-linearities, or the influence of the different parameters of the soil-foundation-structure system on the seismic response of the superstructure and on the internal efforts arising along the piles.*

The type of seismic waves impinging on the site, and the angle of incidence of the waves, are important parameters of the seismic excitation. However, vertical incidence of waves is usually assumed in the analysis of the seismic response of piles, and only a small percentage of the large amount of works related to this topic take the angle of incidence into account.

In this work, the influence of the type of wave and its angle of incidence on the seismic response of pile foundations and pile supported structures is investigated using a direct approach. To this end, a frequency-domain boundary element - finite element formulation is used, being the Boundary Element Method used to model the soil as a homogeneous, isotropic, viscoelastic, semi-infinite region; and the Finite Element Method used to model both piles (as Euler-Bernoulli beams) and superstructure (formed by horizontal rigid slabs and extensible vertical elastic piers). The code is able to model the incidence of Rayleigh waves, and also P, SH and SV body waves with a general angle of incidence.

The formulation is briefly presented at the beginning of the paper. Some validation results, in terms of kinematic interaction factors of pile foundations, are presented. Then, different results in terms of internal efforts in piles and inter-storey drift in the superstructure are presented for different types of waves. It is shown that the angle of incidence has a great influence on the structural response, especially in the case of the SV wave, where the critical angle (at which there is a change in the nature of the reflected waves, and whose value depends exclusively on the Poisson's ratio) plays a very important role, as the seismic response of the structure increases greatly around such angle. It is also shown that, in general, the vertical incidence is not the most unfavourable situation.

1 INTRODUCTION

The study presented herein is integrated on a research line focused on the development of numerical models used to determine the dynamic response of structures of different typology. The dynamic response of both deep foundations and piled structures is a topic deeply studied. However, improvements in the comprehension of some aspects of the problem, such as the influence of the direction of propagation of the waves defining the excitation, are still needed.

The numerical methods used to solve the equations of the problem are both the Boundary Element Method (BEM) and the Finite Element Method (FEM). The former allows the treatment of infinite or semi-infinite regions since it implicitly verifies the radiation conditions. Therefore, the BEM is used to model the soil as a homogeneous, isotropic, viscoelastic, semi-infinite region. On the other hand, the FEM is used to model both piles (as Euler-Bernoulli beams) and superstructure (formed by horizontal rigid slabs and extensible vertical elastic columns). The dynamic interaction between the different regions is rigorously formulated using equilibrium and compatibility conditions, leading to a system of equations including unknowns on displacements and tractions on the boundaries of the regions. The system loads are seismic waves of different nature (P, SH and SV waves) with a generic angle of incidence impinging the structural system.

2 MAIN OBJECTIVES

This work investigates the influence of the type of the incident wave and its angle of incidence on the dynamic response of pile foundations and piled buildings in terms of internal efforts in piles and inter-storey drift amplitudes in the superstructure. It also looks into the influence of parameters such as the slenderness of the building on the magnitudes mentioned above. To this end, volumetric P, SH and SV waves propagating through a homogeneous, semi-infinite domain with a generic angle of incidence are taken into account.

3 PROBLEM DEFINITION

3.1 General aspects

In this work, a frecuency-domain boundary element - finite element formulation [1] is used, being the Boundary Element Method used to model the soil as a homogeneous, isotropic, viscoelastic semi-infinite region; and the Finite Element Method used to model piles (as Euler-Bernoulli beams) and superstructure (formed by horizontal rigid slabs and extensible vertical columns). A more detailed description of both the numerical aspects of the Boundary Element Method and its use on dynamical problems can be found in [2], while a proper description of the Finite Element Method can be seen in [3].

Once this process is done, the system of equations arising from applying the Boundary Element Method to the soil is coupled with that one coming from the use of the Finite Element Method to model the equations of motion of beams, columns and piles (as seen in [1]), leading to a single system of equations describing the behaviour of the entire problem.

The loads taken into account in the problem are seismic loads. Total displacements and tractions due to the seismic action can be found as the superposition of those caused by the so-called incident field (representing the original waves, coming from a far source) and the scattered field (representing the one produced by the reflection and refraction phenomena). This way, a matrix equation can be written for every domain Ω (see [1, 4]) as:

$$H^{ss}\ u^{s} - G^{ss}\ p^{s} - \sum_{j=1}^{n_p} G^{sp_j}\ q^{s_j} + \sum_{j=1}^{n_p} \delta_j\ \Psi^{sj}\ F_{p_j} = H^{ss}\ u_I^{s} - G^{ss}\ p_I^{s} \tag{1}$$

being H^{ss}, G^{ss} and G^{sp_j} the influence coefficients, u^s and p^s the displacements and tractions of the total field, n_p the number of piles on the domain, q^{s_j} the tractions on piles due to the soil, δ_j a parameter taking a unitary value if the jth load line contains the tip of a floating pile and zero otherwise, Ψ^{sj} a three-component vector representing the contribution of the axial force F_{p_j} on the tip of the jth load line and u_I^s and p_I^s the displacements and tractions of the reflected field. The expressions of the latter two values can be found for a generic angle of incidence in [5] (in terms of displacements and tractions) or in [6] (in terms of Lame's potentials).

3.2 Incident field equations

Let $\boldsymbol{s}$ be a vector defining the direction of propagation of a certain wave and $\boldsymbol{d}$ a vector defining the direction of the particle displacements. These directions are perpendicular in S waves and coincident in P waves. It should also be taken into account that when a wave reaches the free surface of the halfspace, a reflection phenomenon occurs, leading to the propagation of a number of new waves depending on the nature of the incident one. Thus, when an SH wave reaches the free surface, the reflected wave is a single SH wave. If the incident wave is a P or an SV one, then after the process of reflection there appears both a P and an SV waves (see figure 1).

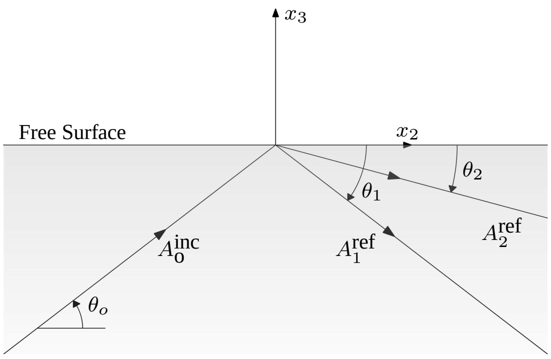

Figure 1: Incident and reflected waves. Incident field

The displacements on the ith direction (x_i, $i = 1, 2, 3$) can be written as:

$$u_i = \sum_{j=1}^{n} d_i^j\ A_j\ \mathrm{e}^{-\mathrm{i}\, k_j\ \left(\mathbf{s}^{(j)}\cdot \mathbf{r}\right)} \tag{2}$$

being u_i the component on the ith direction of the total displacement, n the total number of waves of the problem (i.e., the incident plus the reflected waves), d_i^j the component on the ith direction of the vector containing the direction cosines of the displacements produced by the jth wave, A_j the amplitude of the jth wave, k_j the wave number of the jth wave (defined as the ratio between the excitation frequency ω and the velocity of propagation of the wave) and $\mathbf{s}^{(j)} \cdot \mathbf{r}$ the dot product of the vector defining the direction of propagation of the jth wave and the vector containing the coordinates of the point where the displacements are calculated.

Once the expressions of the field of displacements are obtained for a point, it is easy to determine the small strain tensor using the compatibility equations. In addition, and provided that the soil is a linear, elastic, homogeneous, isotropic solid, the stress tensor can be obtained using the Hooke's Law.

The boundary conditions of the problem are free surface conditions. Thus, σ_{33} and σ_{23} should be zero in points located on the surface (i.e., those with zero value of their x_3 coordinate). As a consequence of the application of boundary conditions, the angle θ_o of incidence of a wave is equal to the angle θ_1 of the reflected wave of the same kind of the incident one.

Another aspect of interest is that the incidence of an SV wave with a smaller angle than a so-called critical one (directly proportional to the ratio between the velocity of S and P waves on the medium) causes the reflection of both an SV wave and an additional wave. This wave is a P wave in the complex field but represents a surface wave with motions along two perpendicular directions out of phase in the real field. This fact remarkably influences the dynamic response of structures submitted to these types of waves.

4 VALIDATION RESULTS

Once the problem has been briefly explained, it is time to validate the formulation and its computational implementation. To this end, some selected results, in terms of kinematic interaction factors on displacements and rotations at pile caps of different pile foundations are presented.

Some selected results taken from [7] are used to validate the formulation. In these numerical examples, the soil internal damping is $\beta = 0.05$, the ratio between the material modulae is $E_p/E_s = 10^2$ or $E_p/E_s = 10^3$, the ratio between densities is $\rho_p/\rho_s = 1.5$, the piles aspect ratio is $L/d = 20$, and the Poisson's ratios are $\nu_s = 1/3$ (for the soil) and $\nu_p = 0.25$ (for the piles). Results are presented for single piles and groups of 3×3 and 4×4 piles submitted to SH, SV and P waves whose direction of propagation is contained in the x_2x_3 plane.

Figure 2 shows the results obtained using the BEM-FEM model compared with those obtained by Kaynia and Novak using a discrete layer matrix approach. Results of kinematic interaction factors on displacements are presented in terms of the modulus of the horizontal displacement at the pile head ($|u|$) with respect to the modulus of the corresponding free field motion ($|u_{ff}|$). Alternatively, the results of kinematic interaction factors on rotations are presented as the ratio between the modulus of the rotation measured at the pile cap times the pile diameter ($|\phi|\ d$) and the modulus of the corresponding free field motion. All the results are plotted against the dimensionless frequency, defined as the ratio between the excitation frequency times the pile diameter and the soil shear-wave velocity (i.e., $a_o = \omega d/c_s$). It can be seen that the agreement between the results is very good, with differences reaching 5 or 6 per cent.

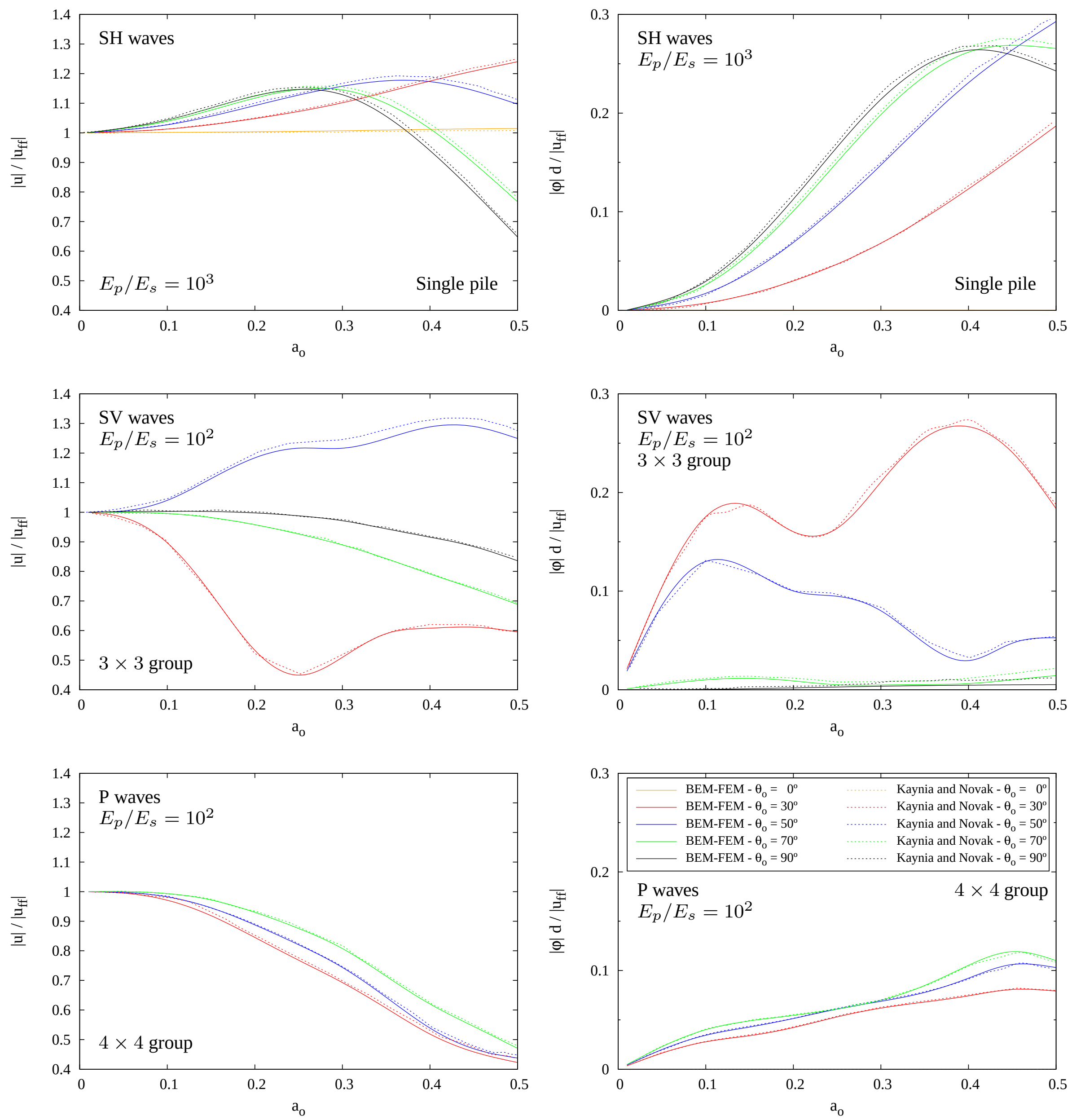

Figure 2: Kinematic interaction factors on displacements (left) and rotations (right) of some different foundations. Comparison with Kaynia and Novak [7]

5 INFLUENCE OF THE TYPE OF WAVE AND ANGLE OF INCIDENCE ON THE SEISMIC RESPONSE OF PILE FOUNDATIONS AND PILE SUPPORTED STRUCTURES

5.1 Problem definition

The problem studied from now on is sketched in figure 3. The main objective of this example is illustrating the influence of the type of wave and its angle of incidence on the seismic response of pile foundations and pile supported structures. The behaviour of a piled structure is studied. To this end, the superstructure is depicted with a single rigid slab supported by massless flexible columns. This system can represent both a single degree of freedom system (like a one-storey shear building) or an equivalent system defining the behaviour of a multimodal structure according to a specific mode of vibration.

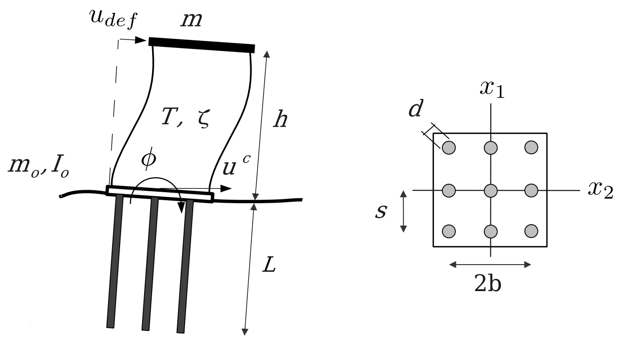

Figure 3: Problem definition

The dynamic behaviour of the structure can be defined by its rigid base fundamental period T, the height h of the resultant of the inertia forces for the first mode, the mass m participating on the first mode and the corresponding structural damping ζ. The horizontal stiffness of the structure is $\mathcal{K} = 4\pi^2 m/T^2$, with a hysteretic damping given by a complex stiffness of the type $k = \mathcal{K}(1 + 2\mathrm{i}\zeta)$. The structure is founded on a square 3×3 pile group embedded in a viscoelastic halfspace. Pile groups are defined by the length L and diameter d of the piles, the distance s between adjacent piles, the pile cap mass m_o and its moment of inertia I_o with respect to a horizontal axis going through its center of gravity and by a parameter b measuring half of the width of the foundation. Thus, the movement of the system can be expressed by three degrees of freedom. These degrees of freedom represent horizontal and rocking movements at the rigid pile cap and inter-storey drift in the superstructure.

5.2 Parameters of the problem

Mechanical and geometric properties of soil and foundation are defined by the soil damping ratio $\beta = 0.05$, the Young's modulus ratio $E_p/E_s = 10^2$, the pile-soil densities ratio $\rho_s/\rho_p = 0.7$, the slenderness ratio of the piles $L/d = 15$, the soil Poisson's ratio $\nu_s = 0.4$ and the separation between adjacent piles $s/d = 5$.

The parameters used to depict the dynamic behaviour of the superstructure are the aspect ratios $h/b = 2$ and 4, the ratio between the stiffnesses of structure and soil $h/(T\ c_s) = 0.3$, being c_s the shear waves velocity, and the structural damping $\zeta = 0.05$. It is also of interest to know the moment of inertia of the foundation, taken as the 5% of the mh^2 factor, the structure-soil mass ratio $m/(4\ \rho_s\ b^2\ h) = 0.2$ and the foundation-structure mass ratio $m_o/m = 0.25$. The values chosen for these three last parameters are considered to be representative for typical constructions.

5.3 Variation of horizontal free-field motion with the angle of incidence

As the following results will be adimensionalized with the horizontal free-field motion at ground surface, it is interesting to study its evolution with the angle of incidence. For this purpose, it is worth noting that the horizontal free-field motion at ground surface is constant with the angle of incidence when the incident wave is an SH one. On the other hand, the variation of such a magnitude divided by the corresponding amplitude is shown on figure 4 for SV or P incident waves. Note the sharp variation of the values around the critical angle on SV waves ($\theta_{cr} = 52.24^o$ for $\nu_s = 0.2$, $\theta_{cr} = 57.69^o$ for $\nu_s = 0.3$ and $\theta_{cr} = 65.91^o$ for $\nu_s = 0.4$) and the smooth but marked change in the case of P waves.

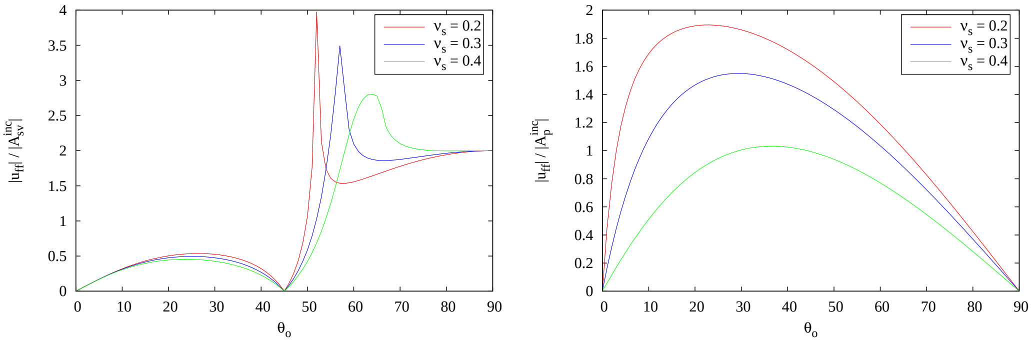

Figure 4: Variation of horizontal free-field motion at ground surface with the angle of incidence and the Poisson's ratio of the soil. Incident SV and P waves

5.4 Lateral deflection of the building

In this case, deflection amplitude can be defined in terms of variables shown on figure 3 as:

$$u_{def} = u - u_c - \phi\, h \tag{3}$$

being u the absolute displacement of the vibrating mass. This way, this deflection relates the shear forces F on the base of the structure with the stiffness k of the system ($F = k\, u_{def}$).

The first set of results is shown on figures 5 and 6. They represent the evolution of the inter-storey drift amplitude with the dimensionless frequency for SH, SV and P incident waves. Results for five different angles of incidence (30, 50, 60, 70 and 90 degrees) are shown. As expected, the deflection shows a maximum around the fundamental frequency of the structure taking SSI into account. For SH, but specially of P waves, the angle of incidence has little influence on the deflection. On the contrary, it shows a very strong influence for the SV wave. Note that u_{def} is normalized by u_{ff}, which for $\theta_o = 30^o$ and 50 has significantly smaller values than for the rest of angles. This variation of u_{ff} with θ_o (shown above in figure 4) is in part responsible for the great increment of the deflection for $\theta_o = 30^o$. In addition, rotations at pile cap are substantially greater for $\theta_o = 30^o$ than for the rest of angles, and since u_{def} depends on such a value (see equation 3), the observed values are justified.

5.5 Internal efforts in piles

This last set of results represents axial, shear forces and bending moments at the pile head of the four representative piles. Two different aspect ratios are considered ($h/b = 2$ and 4) and results are normalized by the corresponding stiffness of a Euler-Bernoulli beam with the properties of the pile.

The results shown on figures from 7 to 15 have a general trend of increasing efforts with the aspect ratio. Like on deflection drift amplitudes, these results also have a peak value around the fundamental frequency of the building taking SSI into account.

It is worth noting that the axial forces arising from the incidence of an SH wave are null on the central piles of the group. This is because the vertical displacement of the center of gravity of the pile cap is also null, rotating the pile cap around an axis passing through the central piles of the group. Therefore, central piles are unloaded.

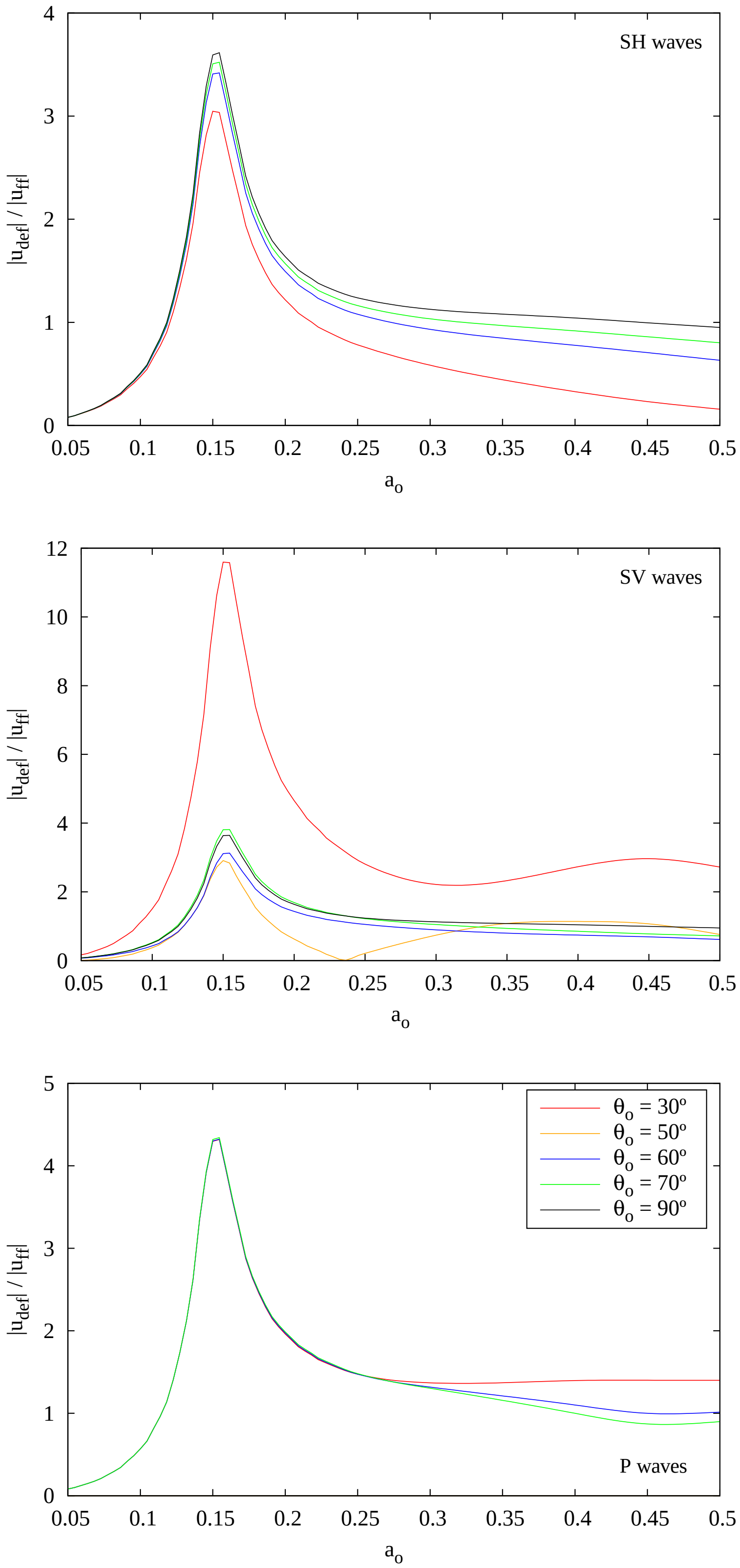

Figure 5: Inter-storey drift amplitudes. Aspect ratio $h/b = 2$

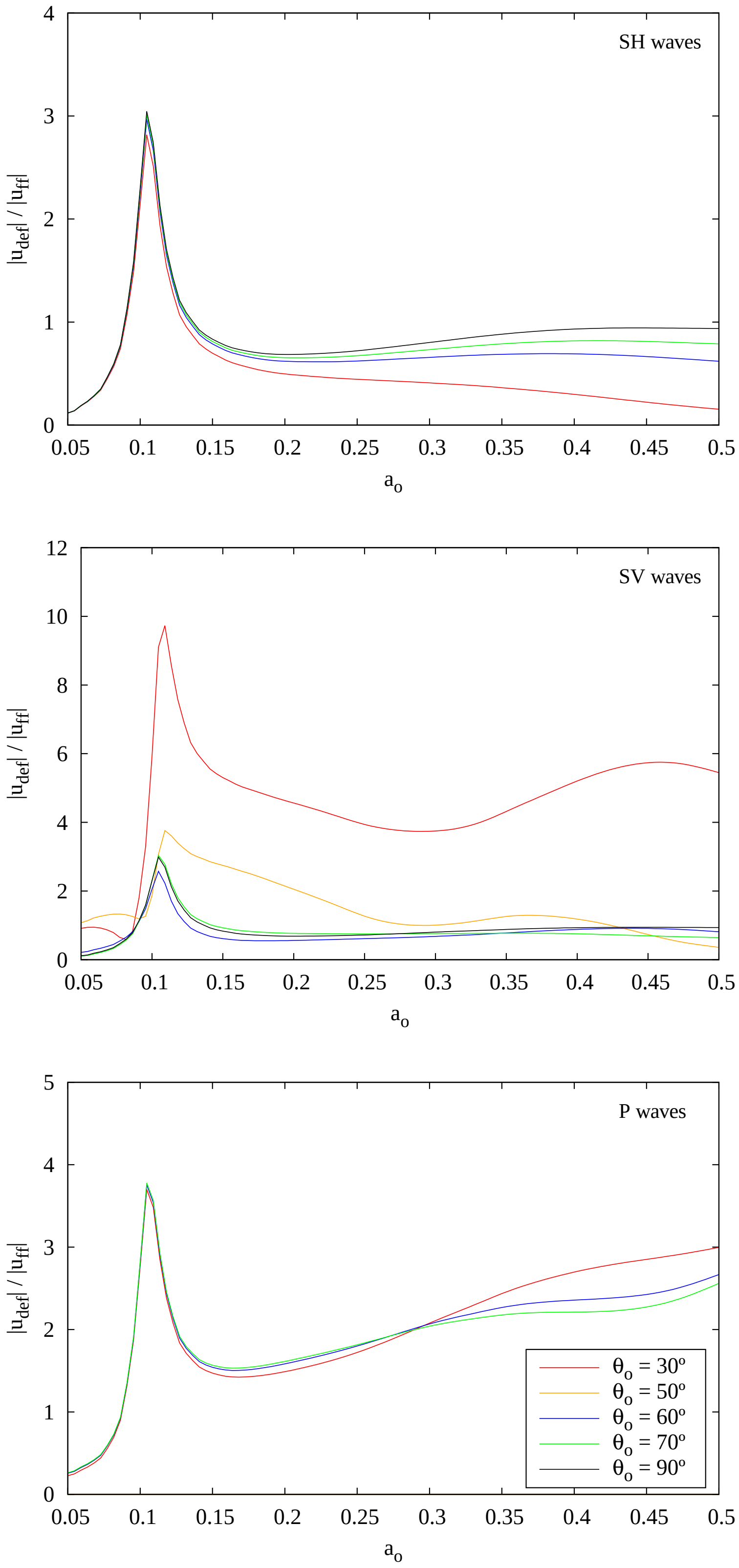

Figure 6: Inter-storey drift amplitudes. Aspect ratio $h/b = 4$

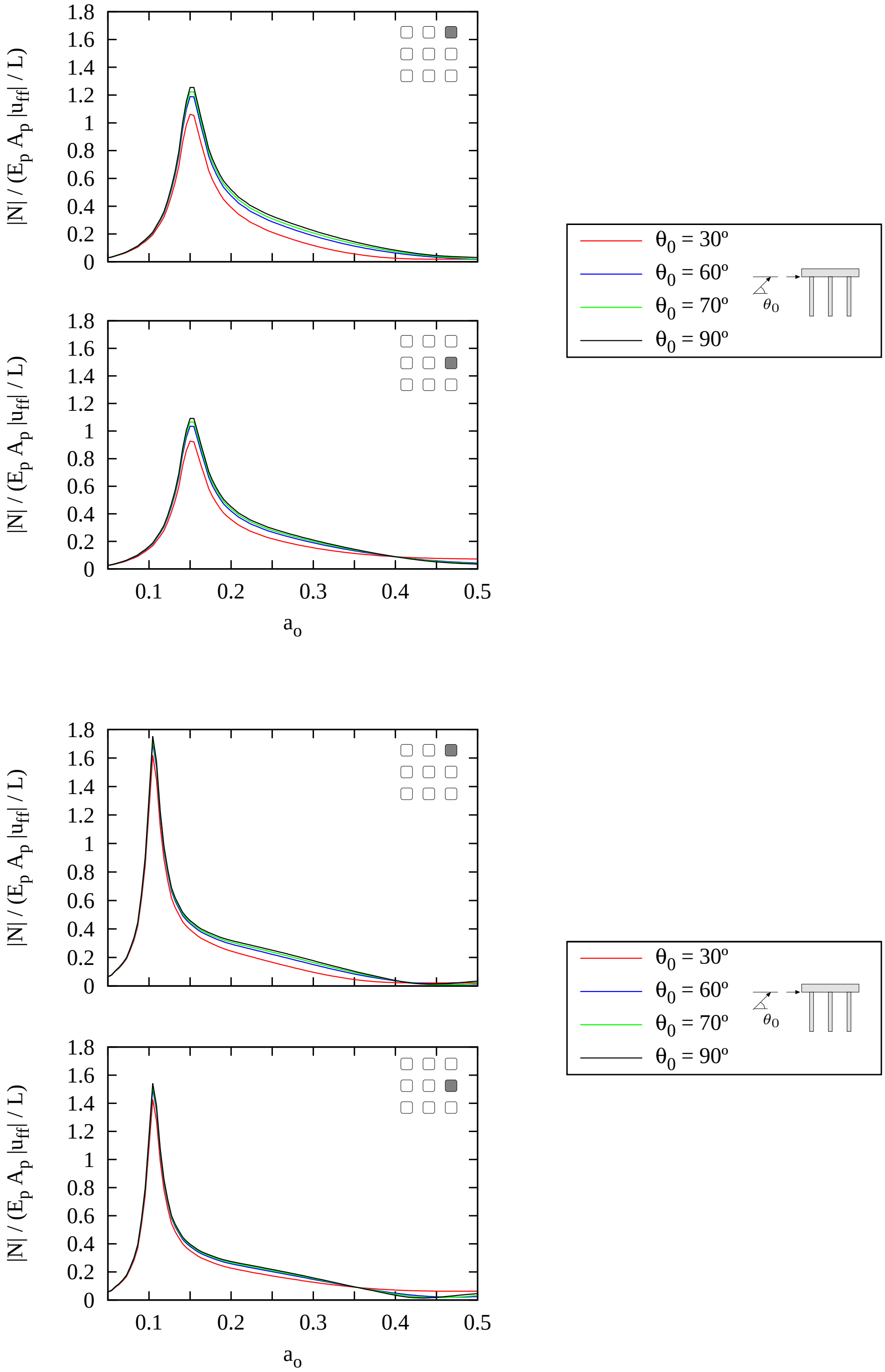

Figure 7: Axial forces at pile heads. Incident SH waves. Aspect ratios $h/b = 2$ (up) and $h/b = 4$ (down). Analysed pile of the group indicated by the sketch (central right and right top piles)

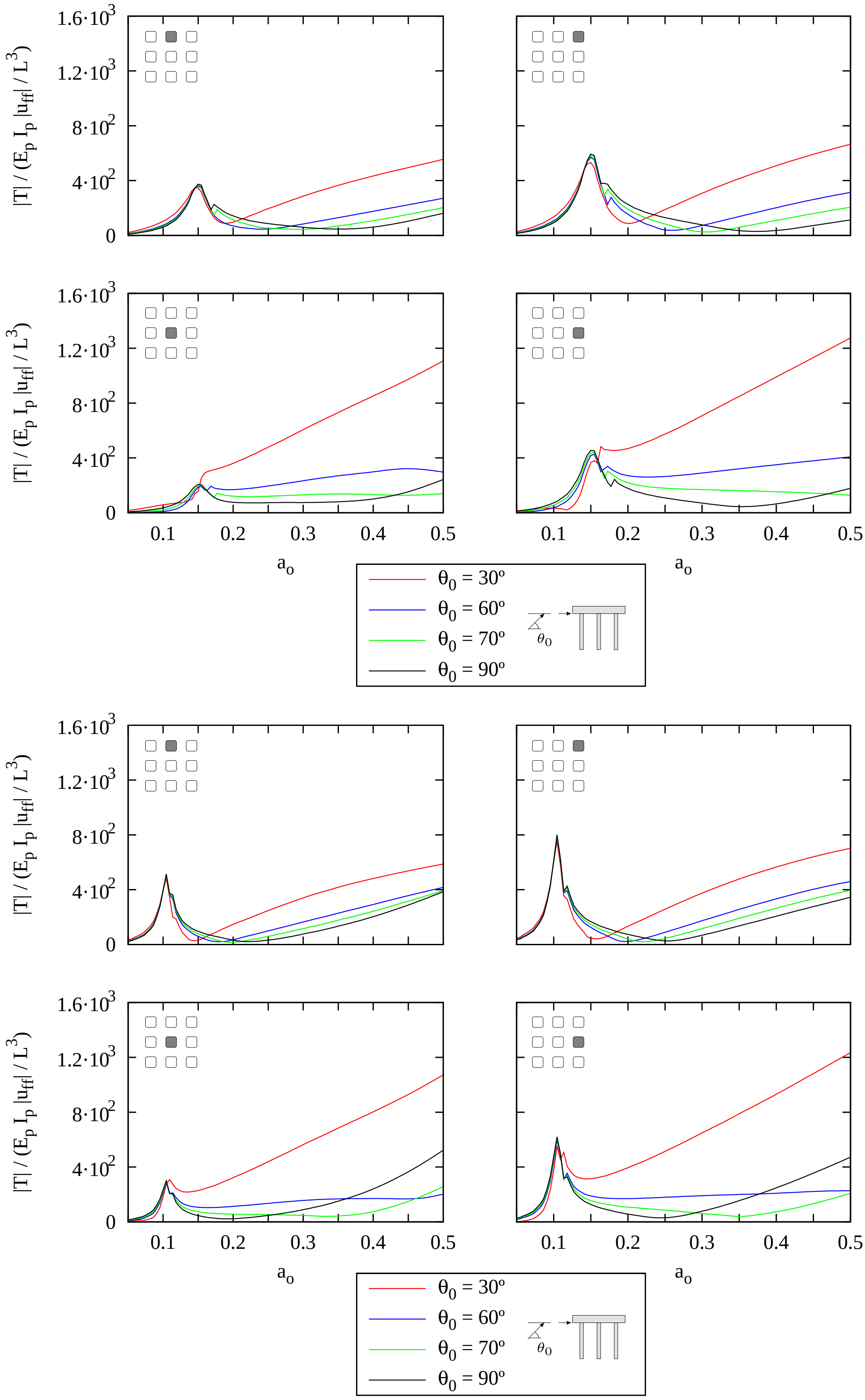

Figure 8: Shear forces at pile heads. Incident SH waves. Aspect ratios $h/b = 2$ (up) and $h/b = 4$ (down). Analysed pile of the group indicated by the sketch (central, central top, central right and right top piles)

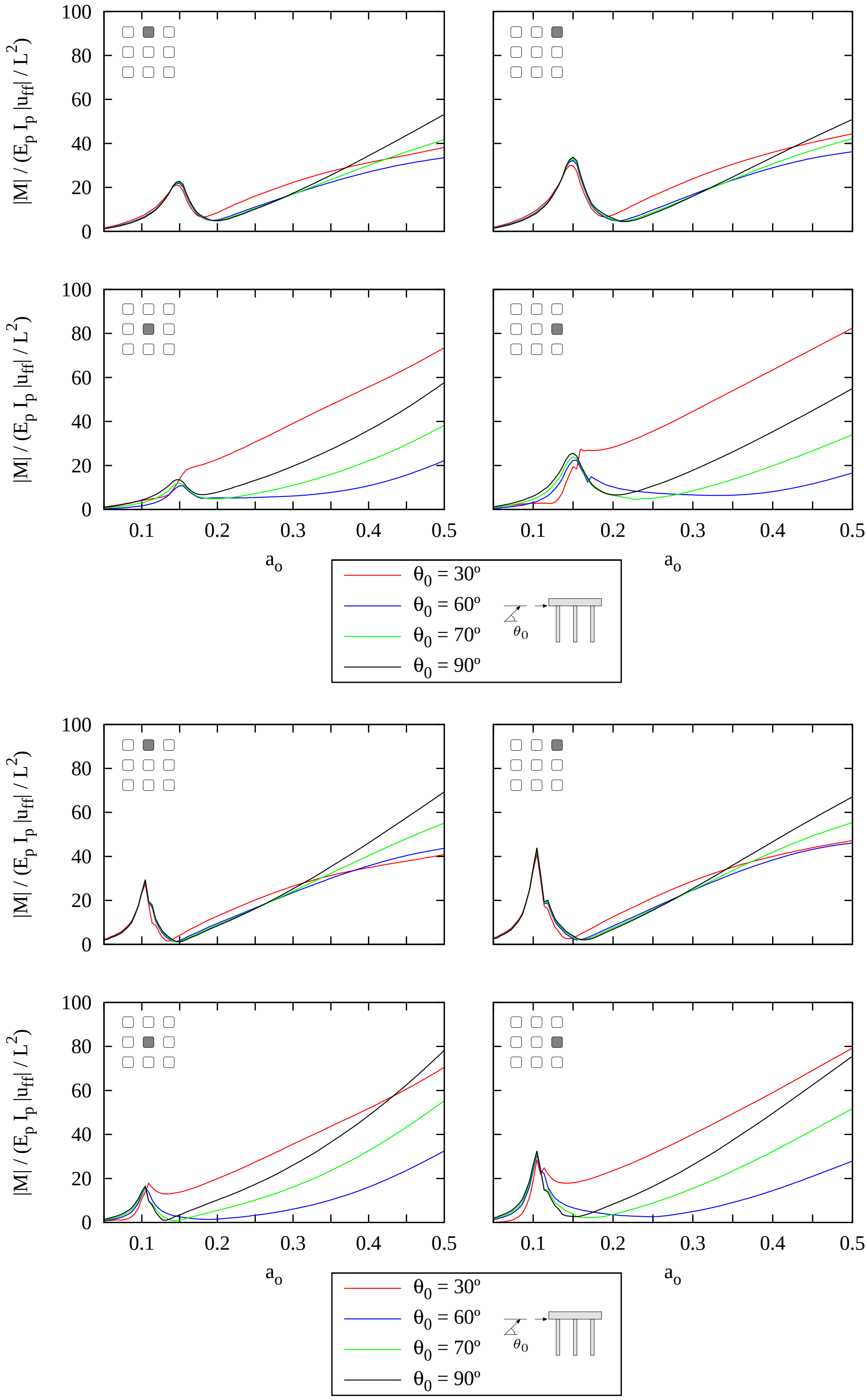

Figure 9: Bending moments at pile heads. Incident SH waves. Aspect ratios $h/b = 2$ (up) and $h/b = 4$ (down). Analysed pile of the group indicated by the sketch (central, central top, central right and right top piles)

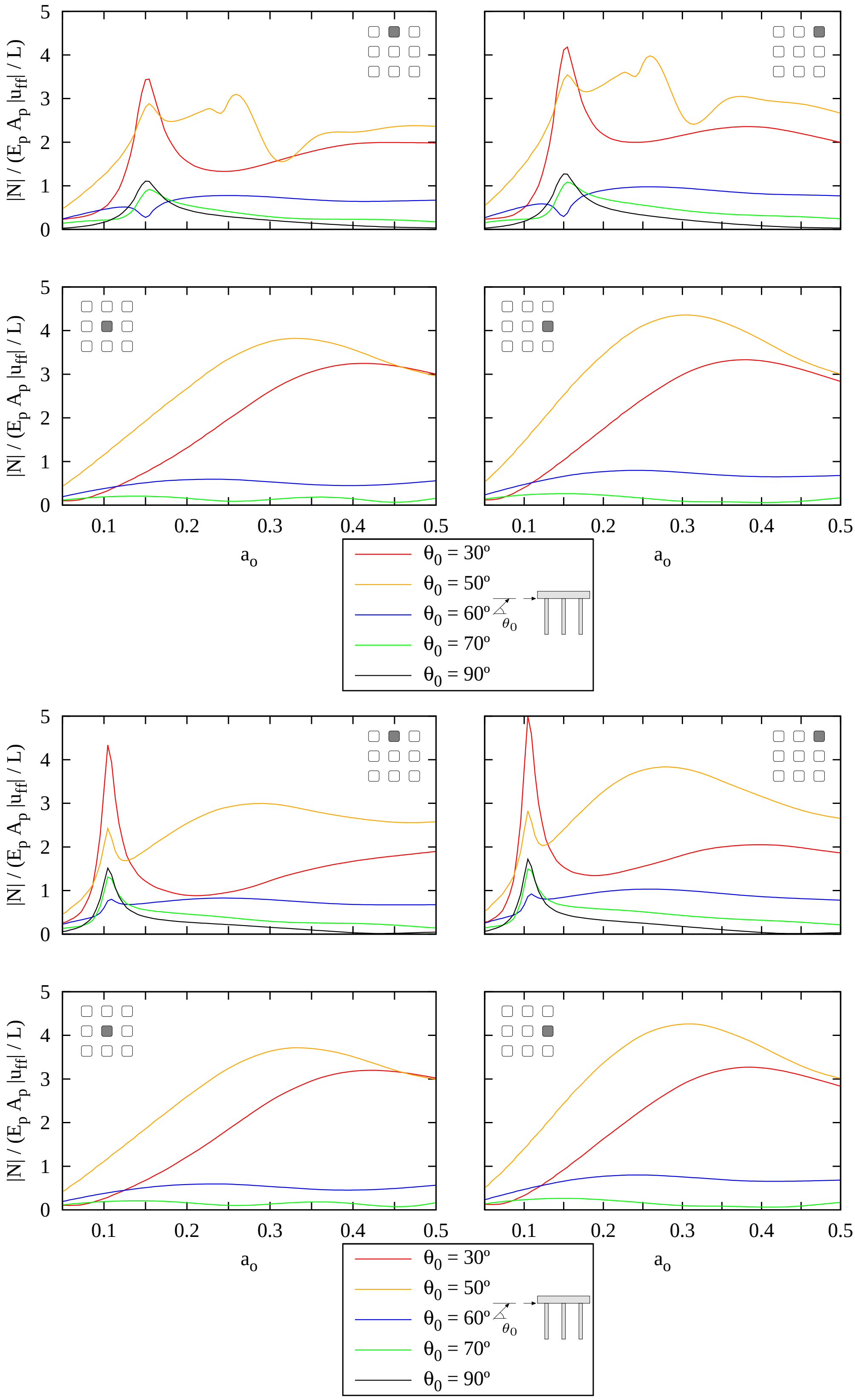

Figure 10: Axial forces at pile heads. Incident SV waves. Aspect ratios $h/b = 2$ (up) and $h/b = 4$ (down). Analysed pile of the group indicated by the sketch (central, central top, central right and right top piles)

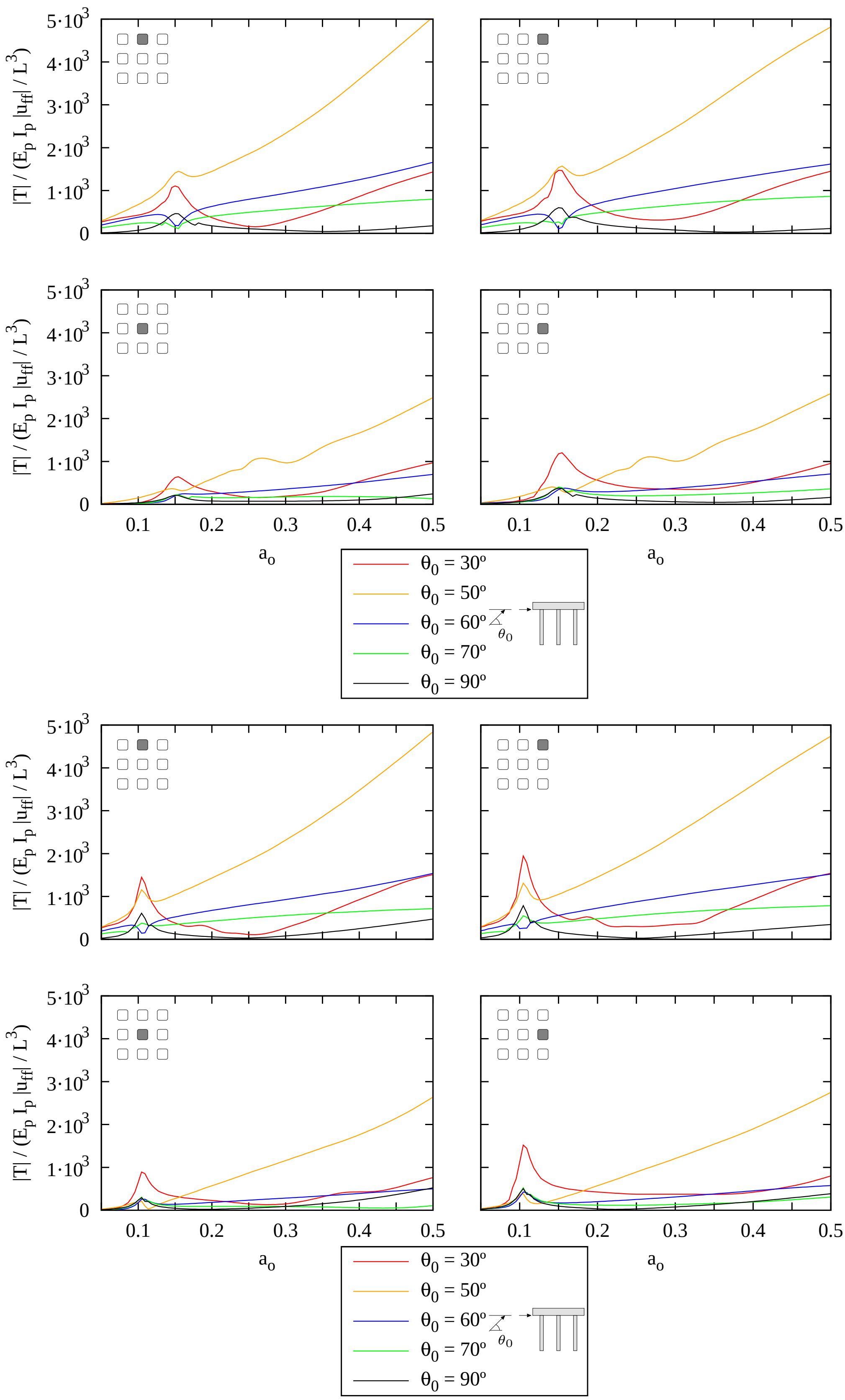

Figure 11: Shear forces at pile heads. Incident SV waves. Aspect ratios $h/b = 2$ (up) and $h/b = 4$ (down). Analysed pile of the group indicated by the sketch (central, central top, central right and right top piles)

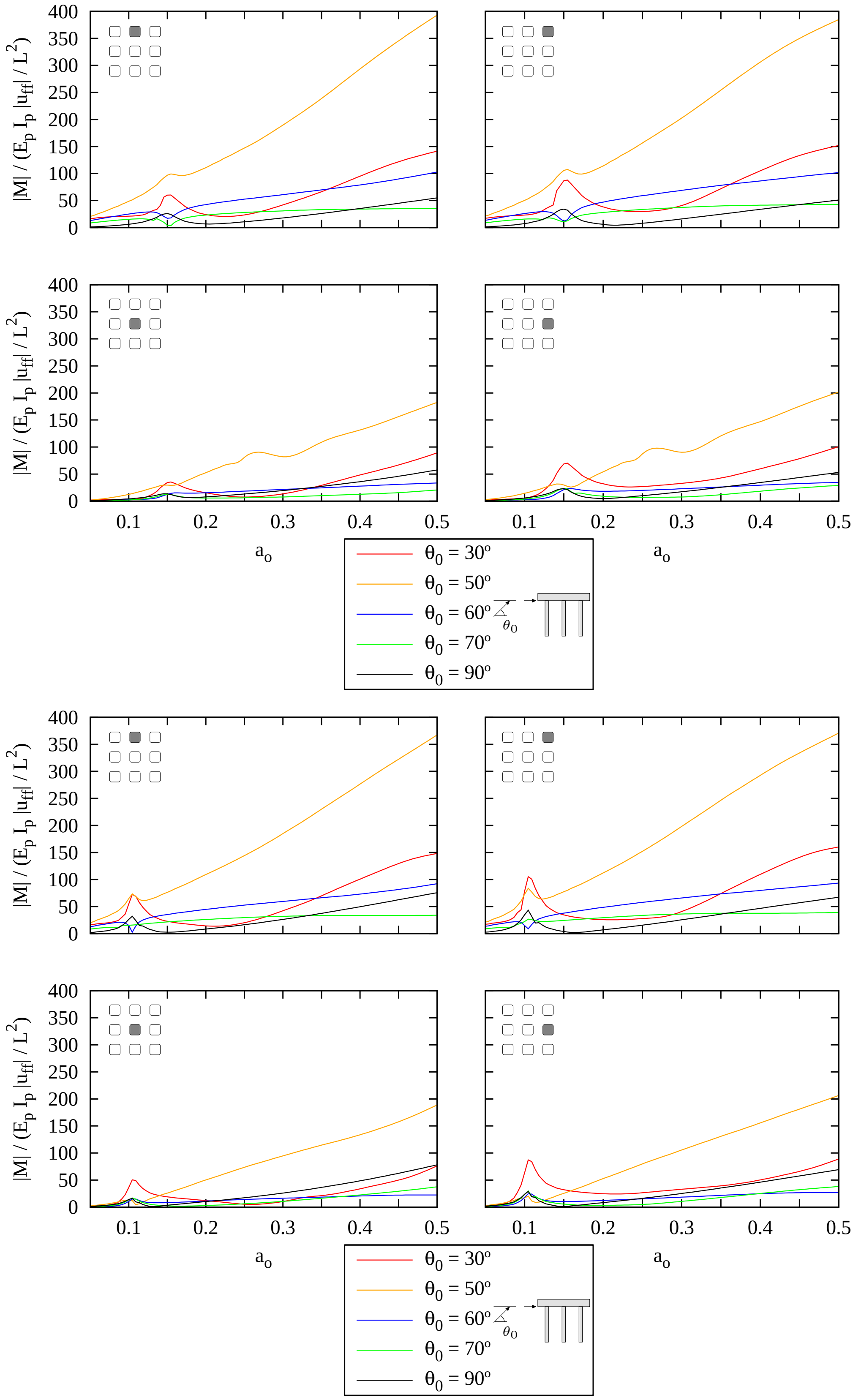

Figure 12: Bending moments at pile heads. Incident SV waves. Aspect ratios $h/b = 2$ (up) and $h/b = 4$ (down). Analysed pile of the group indicated by the sketch (central, central top, central right and right top piles)

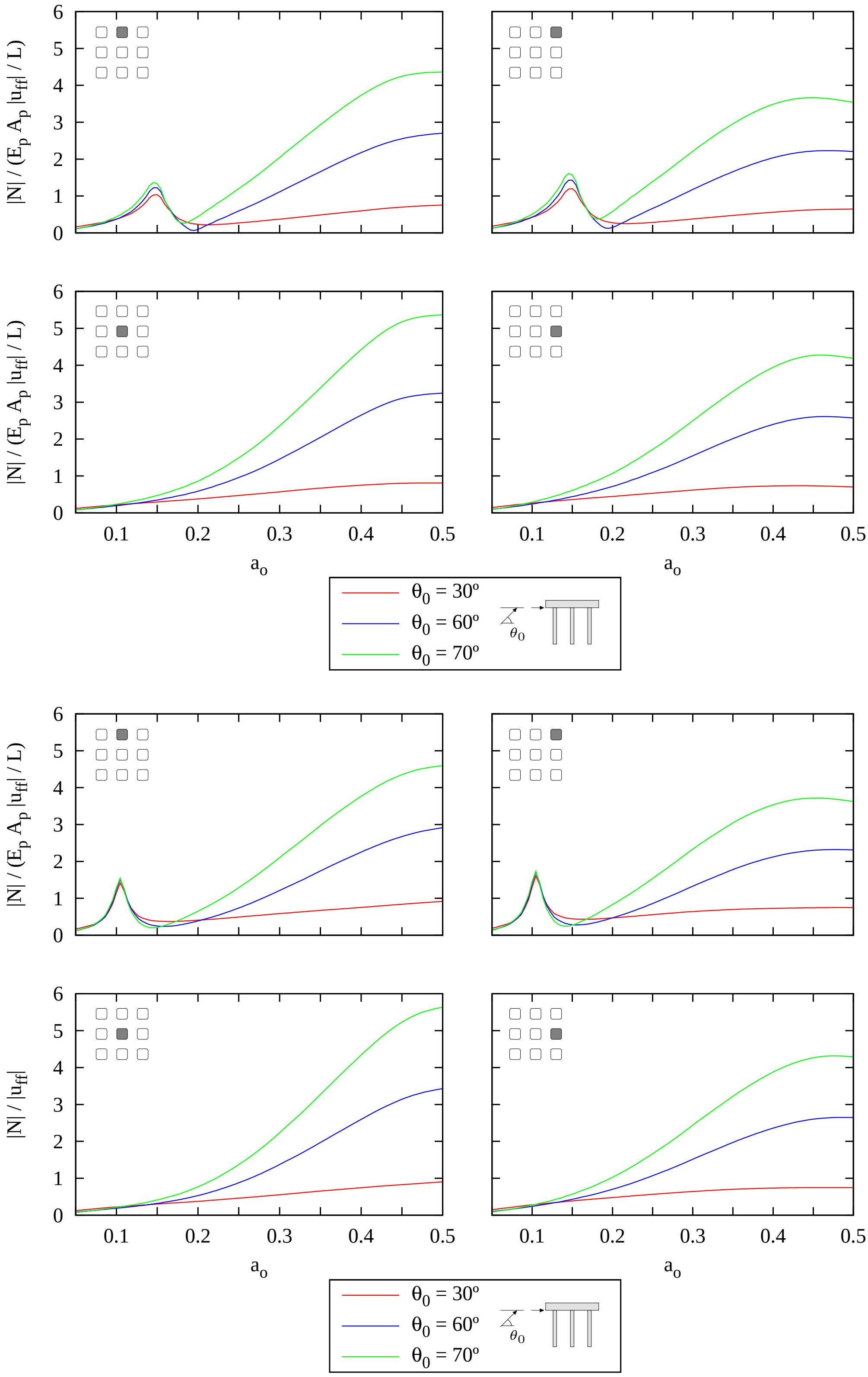

Figure 13: Axial forces at pile heads. Incident P waves. Aspect ratios $h/b = 2$ (up) and $h/b = 4$ (down). Analysed pile of the group indicated by the sketch (central, central top, central right and right top piles)

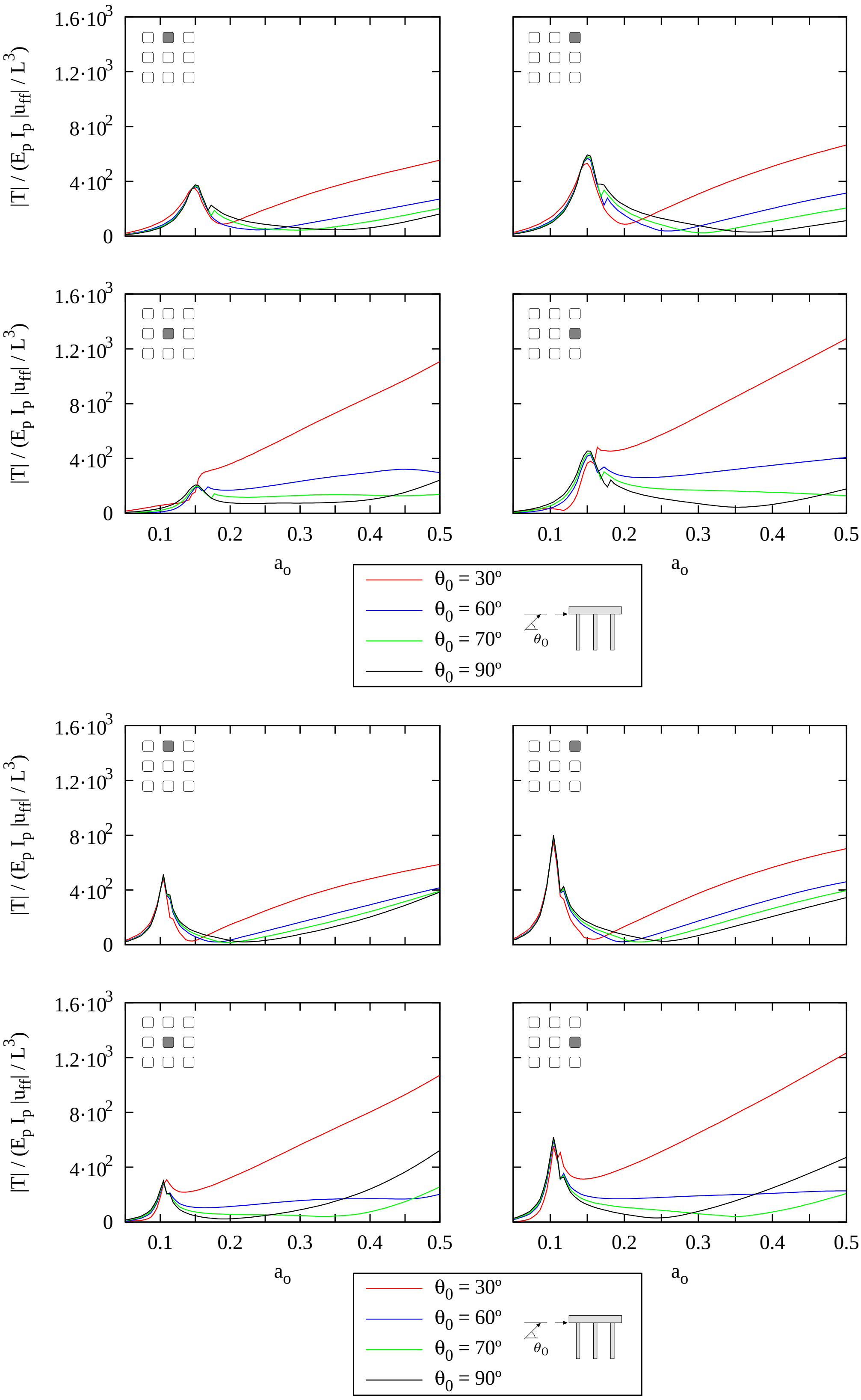

Figure 14: Shear forces at pile heads. Incident P waves. Aspect ratios $h/b = 2$ (up) and $h/b = 4$ (down). Analysed pile of the group indicated by the sketch (central, central top, central right and right top piles)

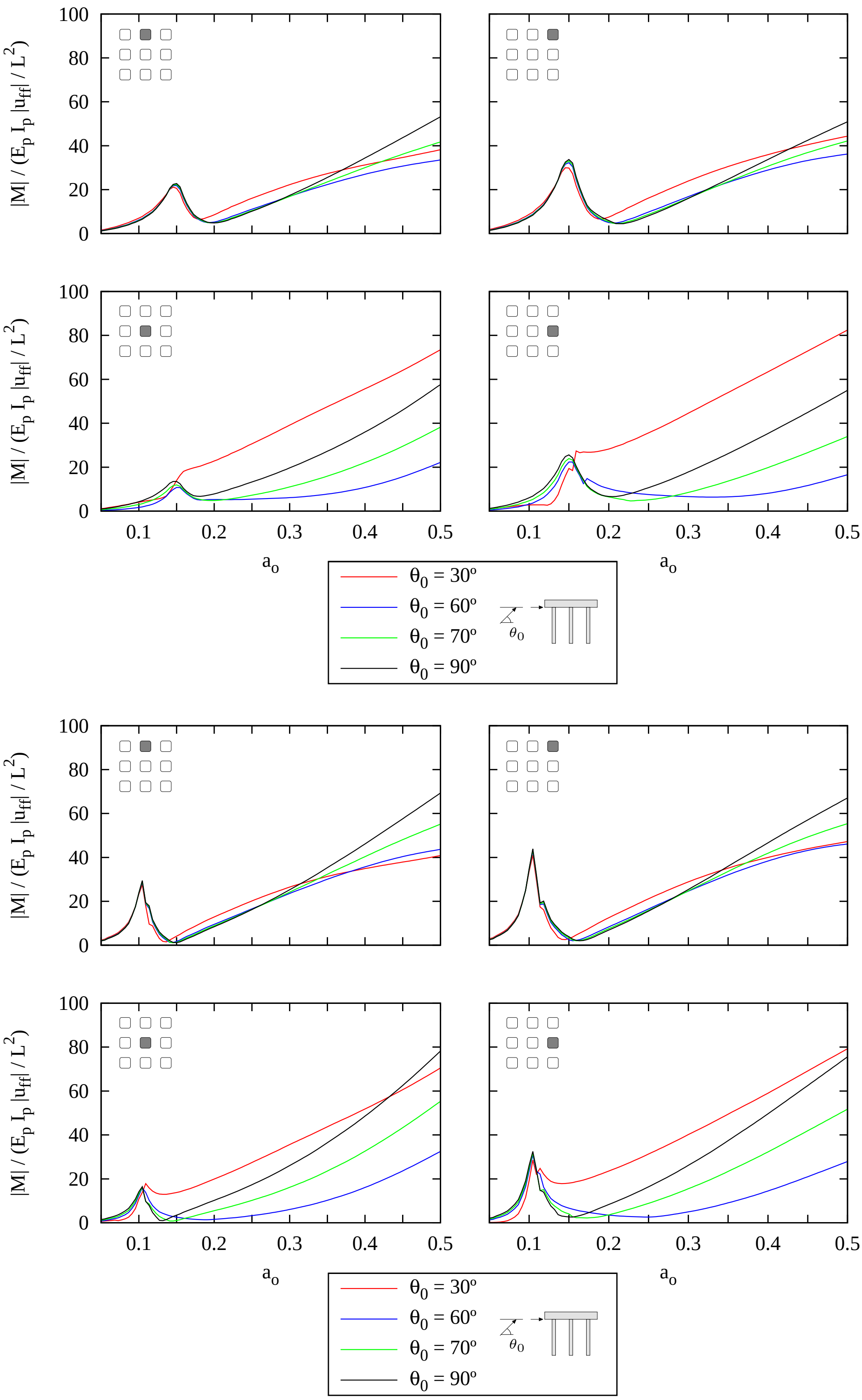

Figure 15: Bending moments at pile heads. Incident P waves. Aspect ratios $h/b = 2$ (up) and $h/b = 4$ (down). Analysed pile of the group indicated by the sketch (central, central top, central right and right top piles)

6 CONCLUSIONS

The influence of the type of wave and its angle of incidence on the dynamic response of pile foundations and superstructures has been studied throughout this work. To this end, the inter-storey drift amplitude and the internal efforts in piles of a piled building modelled as a single degree of freedom system are determined using a BEM-FEM approach. Results are presented in this work for three different incident volumetric waves (P, SH and SV) and two ratios between the height of the superstructure and half the width of the pile cap.

The angle of incidence has a strong influence on the behaviour of piled structures. This effect is particularly marked when the incident wave is an SV one. There exist great differences between the displacements and efforts arising from incident waves of the same amplitude but with angles slightly different from the critical angle. Then, the widely accepted hypothesis of vertical incidence of the waves does not have to necessarily be the most unfavorable situation from the point of view of both piled foundations or superstructures.

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