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COMBINATION COEFFICIENTS FOR YIELDING STRUCTURES UNDER TRI-DIRECTIONAL EARTHQUAKE EXCITATIONS

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Abstract. *Seismic regulations and guidelines for buildings and bridges prescribe simplified combination rules to obtain the maximum structural response under multi-directional earthquake effects. An unfavorable internal force usually develops under the combined effects of an earthquake motion. This study uses the spectrum intensity concept to investigate the tri-directional effects of earthquakes on structures. For this purpose, a set of recent and past thirteen earthquakes (M>6) are selected to predict tri-directional effects. Inelastic velocity response spectra for these earthquakes are numerically obtained and plotted for damping ratios of* ξ*=0.05 and 0.20, representing a wide range of damped and heavily damped (e.g. base isolated) structures. Spectrum intensities for both orthogonal and vertical directions and for the resultant direction are calculated using a computer program developed for this purpose. Unfavorable response is then calculated by equating the resultant spectrum intensity to principle direction's intensity plus the other direction's contribution as a percentage of the principle component, or equating the resultant spectrum intensity to principle direction's intensity plus a percentage of the other direction's contribution, or vice versa. The results obtained are strongly earthquake-dependent. Based on the proposed analysis way, well-known building regulations are reviewed and evaluated by emphasizing the prescribed combination rules. Numerical results show that coefficients for the tri-directional contribution varies largely in the range between 0.01~0.68 for the selected force reduction factors of* μ*=2 and 8, revealing that in some cases the code defined combination values may yield unconservative seismic designs.*

1 INTRODUCTION

Since analysis of structures under all possible angles of earthquake excitations is complex and time consuming, simplified combination rules have been developed for practical design purposes. Elastic forces and displacements are calculated based on the combination coefficients proposed by existing building codes. A structural design without considering the orthogonal earthquake effects may generally result in insufficient member dimensions, as an unfavorable internal force distribution in the structural elements would usually develop under the combined effects (bi-directional or tri-directional) of an earthquake strong ground motion.

As a common approach, the square- root-of-sum-of-squares (SRSS) procedure is based on the assumption that the actions on an element affected by earthquake excitations in two and three directions are combined. With the help of elastic velocity response spectra, Sesigur, Celik, and Cili [1,2] proposed a way of analysis for the bi-directional and tri-directional effects of earthquakes using the characteristics of the selected earthquakes. The 30% and 40% rules are simplified approximations to the SRSS and the CQC methods. Many current design codes [3 to 7] for buildings and bridges require that members should be designed for 100 percent of the seismic forces in one direction plus 30 percent of the seismic forces in the perpendicular direction (the 30% rule). ATC-32 [6] requires the 40 percent rule to be used in the design of bridges under bi-directional effects. In order to investigate the bi-directional effects of inelastic response of single degree of freedom (SDOF) structures, Sesigur, Celik, and Cili [8] presented some design recommendations.

For yielding/inelastic structures, the present study essentially aims to extend the bi-directional analysis way developed in [8] to tri-directional earthquake effects. As before, inelastic velocity response spectrum and the spectrum intensity concept (SI_H) corresponding to inelastic velocity response spectrum is used. Finally, numerical values for the combination coefficients are proposed and compared with code-defined values.

2 METHOD OF ANALYSIS

Since the velocity response spectrum is a powerful tool for estimating the damage potential of structures in the medium and long period range, the analysis method called as the spectrum intensity concept as proposed in [8] will be followed here for inelastic structures. The inelastic velocity response spectrum is obtained following the standard procedure given by Newmark [9]. The average acceleration method is used for nonlinear response analysis of SDOF system, taking γ=1/2 and β=1/4. The time step Δt is chosen as 0.02 to detect accurately the transitions from unloading to loading branches or around sharp corners of the force-deformation curves. Force-deformation relation is considered as a cyclic ideal elasto-plastic curve. Inelastic analysis is carried out by the force reduction factors of μ=2 and 8, representing significantly rigid (e.g. masonry) and ductile structures (e.g. steel or RC).

Housner introduced a measure of ground motion intensity which defines the integral of the velocity response spectrum over a period range from T_1 to T_2 as the spectrum intensity (1) where SI, ξ, T, and S_v are intensity, damping ratio, fundamental period of the structure and the ordinate of the velocity spectrum respectively. T_1 and T_2 are proposed as 0.10~0.50sec and 2.50~5.00sec. This integral can be evaluated for any desired damping ratio (note that Housner recommended using ξ=0.20); however, to clarify the impact of damping ratio on the evaluation of this integral, damping ratios of ξ= 0.05 and 0.20 are chosen.

$$SI = \int_{T_1}^{T_2} S_V(\xi, T, t)\,dT \tag{1}$$

For near-fault ground motions (L ≤ 15 km, T_0 (fundamental period of soil) ≈ 0.20sec, near the epicenter, stiff soil conditions and minimum focal depth is H=30km), one component of the spectrum intensity might be negligible, and the other could be significant, (e.g. the 10.18.1989 Loma Prieta and the 01.17.1994 Northridge earthquakes). For the far-field motions (L=40~50km, stiff soil conditions, T_0 ≈ 0.05~0.50sec and 2.50~6.00sec), if the horizontal components of the intensities are close to each other then both components must be taken into account, (e.g. the 05.18.1940 El-Centro earthquake). For the earthquakes with a long period, the unfavorable conditions occur in the resultant direction, (e.g. the 07.06.1964 Mexico-City earthquake). Table 1 presents the data of thirteen recorded accelerograms used in the present study. Three earthquakes that greatly affected populated areas in Turkey are also chosen.

Event	Date	Station	M	a_{max} (T)	a_{max} (L)	a_{max} (V)
El Centro	05.19.1940	El Centro	Ms=7.2	306.9	210.7	201.3
Parkfield	06.27.1966	Cholame	ML=6.1	433.2	360.0	135.5
Tokachi-Oki	05.16.1968	Hachinoe Harbour	M=7.9	229.6	180.2	114.2
San Fernando	02.09.1971	Pacoima Dam	Ms=6.6	1202.6	1137.5	685.3
Miyagi Ken Oki	06.12.1978	Tohoku University	M=7.4	258.1	203.4	152.8
Tabas	09.16.1978	Tabas	Ms=7.4	819.9	835.6	675.4
Loma Prieta	10.18.1989	Capitola	Ms=7.1	559.0	595.7	878.6
Erzincan	03.13.1992	Erzincan	M=6.9	505.5	486.1	243.0
Northridge	01.17.1994	Sylmar-Olive View	Ms=6.7	592.6	826.8	525.0
Kobe	01.16.1995	Takarazu	M=6.9	680.3	680.4	425.1
Kocaeli	08.17.1999	Yarimca	Mw=7.2	322.2	230.2	241.1
Chi-Chi	09.20.1999	CHY028-N	Ms=7.6	805.9	590.4	330.9
Duzce	11.12.1999	Bolu	Mw=7.2	713.8	806.8	198.7

Table 1. Characteristics of the selected earthquakes

In some cases, the third component (i.e. the vertical component) of an earthquake could be of great importance. For example, large span cantilevered or other structures or structures having beams or cantilevers that support heavy vertical (gravity) column loading, and structures with irregularities in elevation might be critical under the effect of vertical component of an earthquake.

As explained in [8], under tri-directional earthquake excitations and under elastic conditions, the unfavorable response of a structure could be calculated in two alternative ways: The first way is to equate the resultant spectrum intensity to principle direction's intensity plus the other direction's (i.e. the other horizontal and vertical directions) equal contributions (α) as a percentage of the principle component. The second way would be to equate the resultant spectrum intensity to principle direction's intensity plus other components' unequal contributions (i.e. λ_1 for the other horizontal component and λ_2 for the vertical component) as a percentage of the principle direction's intensity. α, λ_1, and λ_2 have numerically been calculated for each of the selected earthquake data. These coefficients can then be implemented in the combination rules to obtain the maximum response parameters. Apparently, numerical values are expected to be earthquake-dependent.

This analysis procedure will be generalized here for both tri-directional earthquake excitations and yielding/inelastic structures. A single degree of freedom (SDOF) system subjected to tri-directional strong ground motions and the evaluation of velocity spectrum intensities are illustrated in Figure 1.

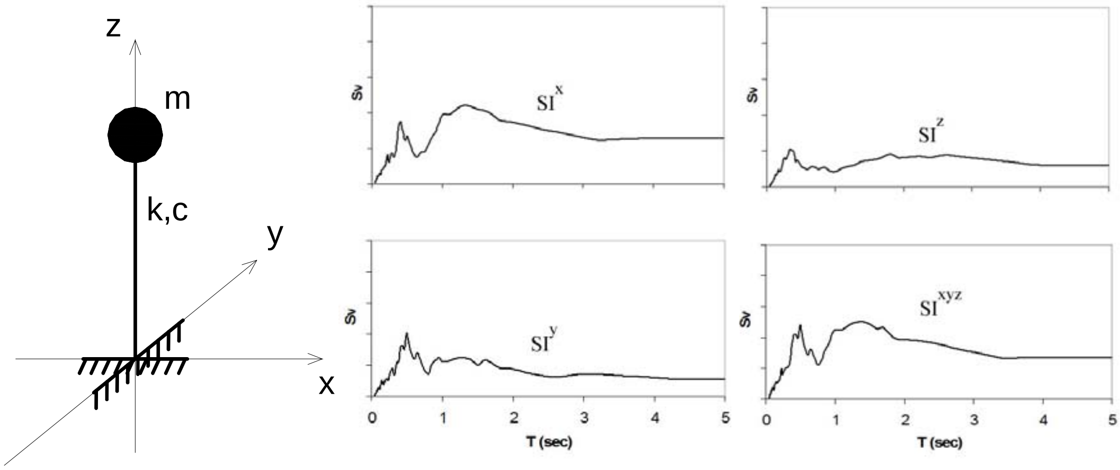

Figure 1. Single degree of freedom system under tri-directional earthquake excitation and spectrum intensities for each component and resultant

Here, x indicates the earthquake direction that produces maximum internal forces in the system. Spectrum intensities for both orthogonal and vertical directions as well as for the resultant (i.e. the unfavorable) direction are defined as SI^x , SI^y , SI^z and SI^{xyz}, respectively. Depending on these numerically obtained values, the above mentioned coefficients can then be calculated using the following equations:

$$\alpha=0.5(SI^{xyz}/SI^x - 1) \tag{2}$$

$$\lambda_1= (SI^{xyz} / SI^x) - (1+\lambda_2) \tag{3}$$

$$\lambda_2= (SI^{xyz} / SI^x) - (1+\lambda_1) \tag{4}$$

For a given ground motion, the resultant inelastic velocity response ($\dot{x}_r(t)$) of a SDOF system can be calculated using the square root of the squares of each orthogonal component's contributions ($\dot{x}_1(t), \dot{x}_2(t), \dot{x}_3(t)$) as follows:

$$\dot{x}_r(t) = \sqrt{\left|\dot{x}_1(t)\right|^2 + \left|\dot{x}_2(t)\right|^2 + \left|\dot{x}_3(t)\right|^2} \tag{5}$$

The maximum values of the above equation for a given damping ratio and natural vibration period give the ordinates of the resultant velocity spectra. This is shown in (6).

$$(S_v)_r = \left|\dot{x}_r(t)\right|_{\max} \tag{6}$$

3 COMPUTER PROGRAM

To carry out the numerical computations, a special computer program that accepts strong ground motion data for the three components of the selected earthquakes, damping ratios, natural vibration periods, and mass as input, was coded. A flow-chart of the program is given in Figure 2. Both elastic and inelastic spectrum intensities are then calculated for the three orthogonal and resultant directions. Inelastic velocity response spectra for the selected earthquakes are numerically obtained and plotted for damping ratios of ξ=0.05 and 0.20, representing a wide range of damped and heavily damped like base isolated buildings or buildings with viscous or other type of dampers.

A similar way that was followed in the computation of spectrum intensities can be used in obtaining the maximum internal forces (E_{xyz}) in a structural member under tri-directional effects. This can be evaluated by the internal forces developed in the principal direction (E_x) plus the other direction's (both the other horizontal and vertical) contributions (λ_1, λ_2). For this purpose, α, λ_1, and λ_2 obtained through the spectrum intensity concept can be used:

$$E_{xyz} = E_x + \lambda_1 E_x + \lambda_2 E_x \tag{7a}$$

$$E_{xyz} = \lambda_1 E_y + E_y + \lambda_2 E_y \tag{7b}$$

These equations and coefficients are actually defined in Eurocode 8 [5] to take into account the tri-directional earthquake effects on structures. λ_1 and λ_2 are proposed as 0.30. On the other hand, for design purposes, the equal contributions of the other components can also be taken into account as given in (7c). Therefore, α coefficient is of special interest to evaluate the combination rules in structural codes.

$$E_{xyz} = E_x + 2\alpha E_x \tag{7c}$$

For the selected earthquake ground motions (a total of thirteen), inelastic velocity response spectra for each of the orthogonal, vertical directions and for the resultant direction are shown in Figure 3, following the procedure summarized above. Also, Figure 4 shows hysteretic curves of the elasto-plastic SDOF systems for selected earthquakes. α, λ_1, and λ_2 are numerically calculated for each of the selected earthquakes and the results are also summarized in Table 2. If λ_1 is considered as a value of 0.30 (as given in many codes), λ_2 is generally obtained as negative values that can be interpreted as the %30 rule is sufficient on the basis of spectrum intensity concept followed in this work. Therefore, the contribution of the vertical component may be neglected in most cases.

4 MULTI-COMPONENT COMBINATION RULES IN CURRENT CODES

Most design codes that address the multi-component ground motion problem, specify that the contributions to a response quantity from the orthogonal components of seismic input be combined either by the square root of sum of squares (SRSS) rule or by a percentage rule. The 30% rule is a linear approximation of the combined response. The 40% rule is also a linear approximation recommended for the analysis of nuclear and bridge structures in the ATC-32. The SRSS and 30% rules are prescribed in Eurocode 8.

The unfavorable earthquake loads are produced by the following combinations.

$$E_x,\ \lambda E_y,\ \mu E_z$$
$$\lambda E_x,\ E_y,\ \mu E_z$$
$$\lambda E_x,\ \lambda E_y,\ E_z$$

Here, E_x, E_y and E_z are internal forces developed during an earthquake loading. For practical design purposes, the λ and μ coefficients are chosen as 0.30 and 0.20 respectively. The UBC97 requires the use of either the SRSS rule or the 30% rule, but only for structures having certain types of irregularities. The current (Caltrans) bridge design specifications require the 30% rule for all structures.

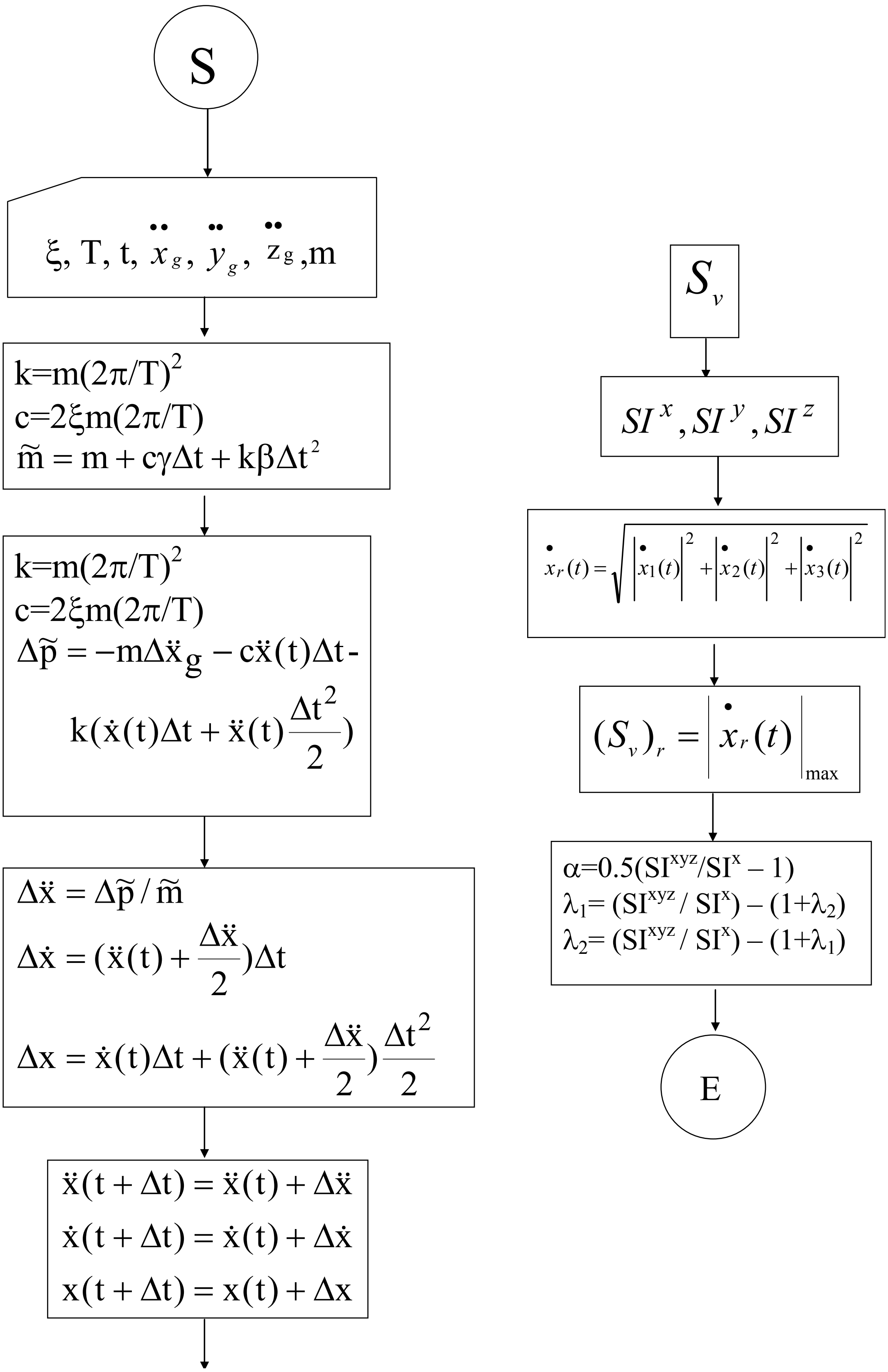

Figure 2. Flow-chart of the computer program

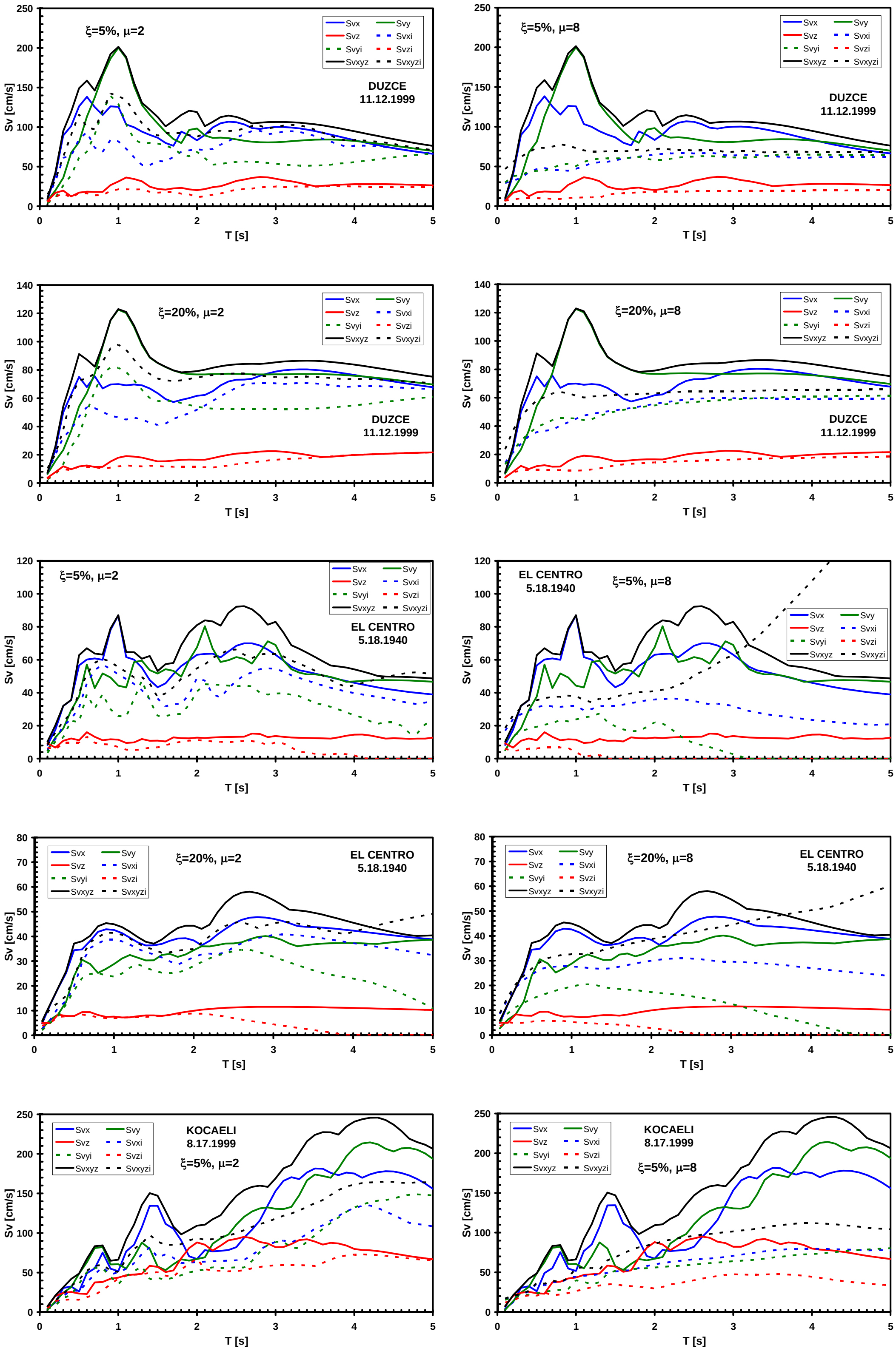

Figure 3. Elastic and Inelastic velocity response spectra for each orthogonal and resultant directions

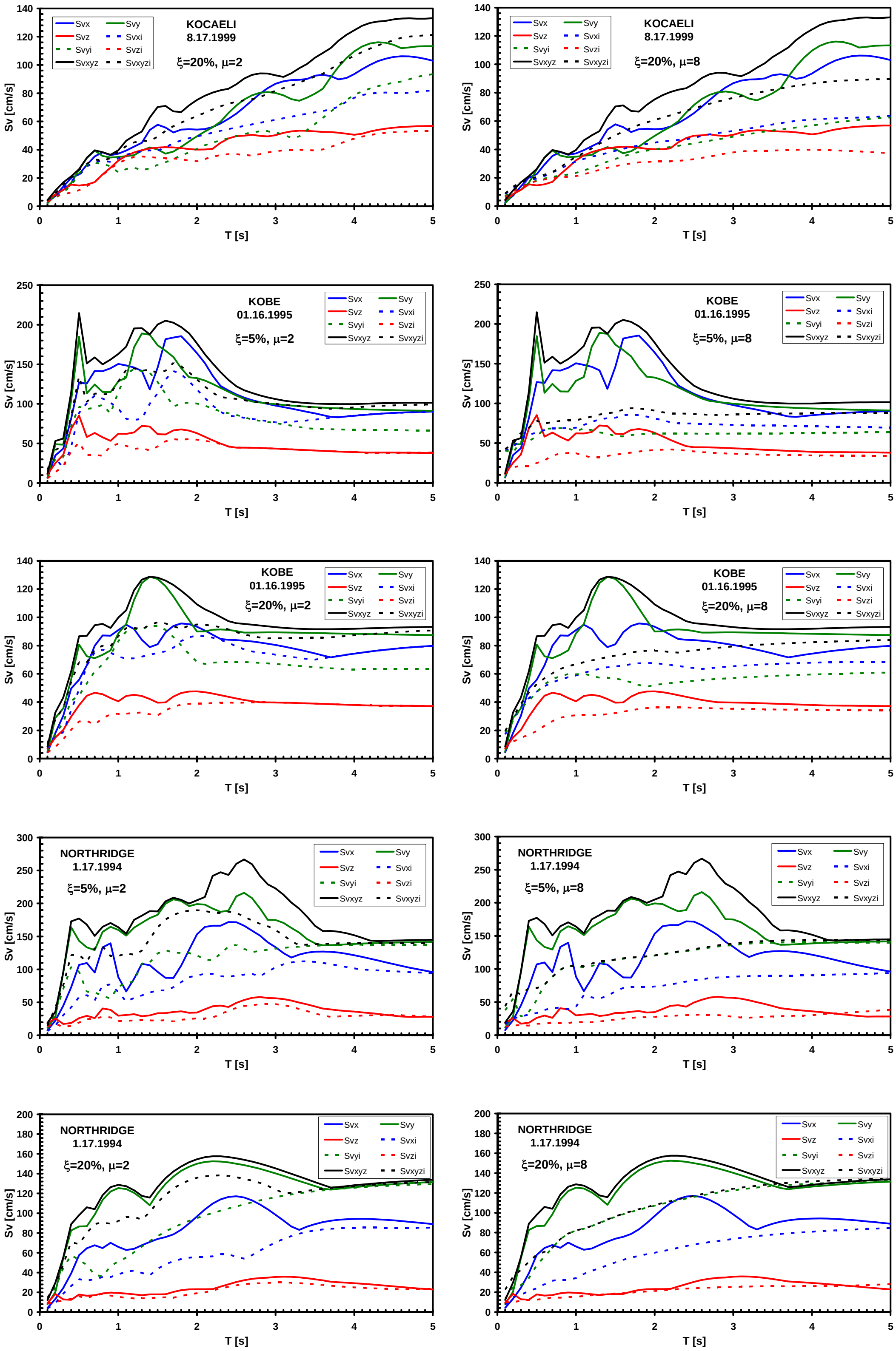

Figure 3. Elastic and Inelastic velocity response spectra for each orthogonal and resultant directions (continued)

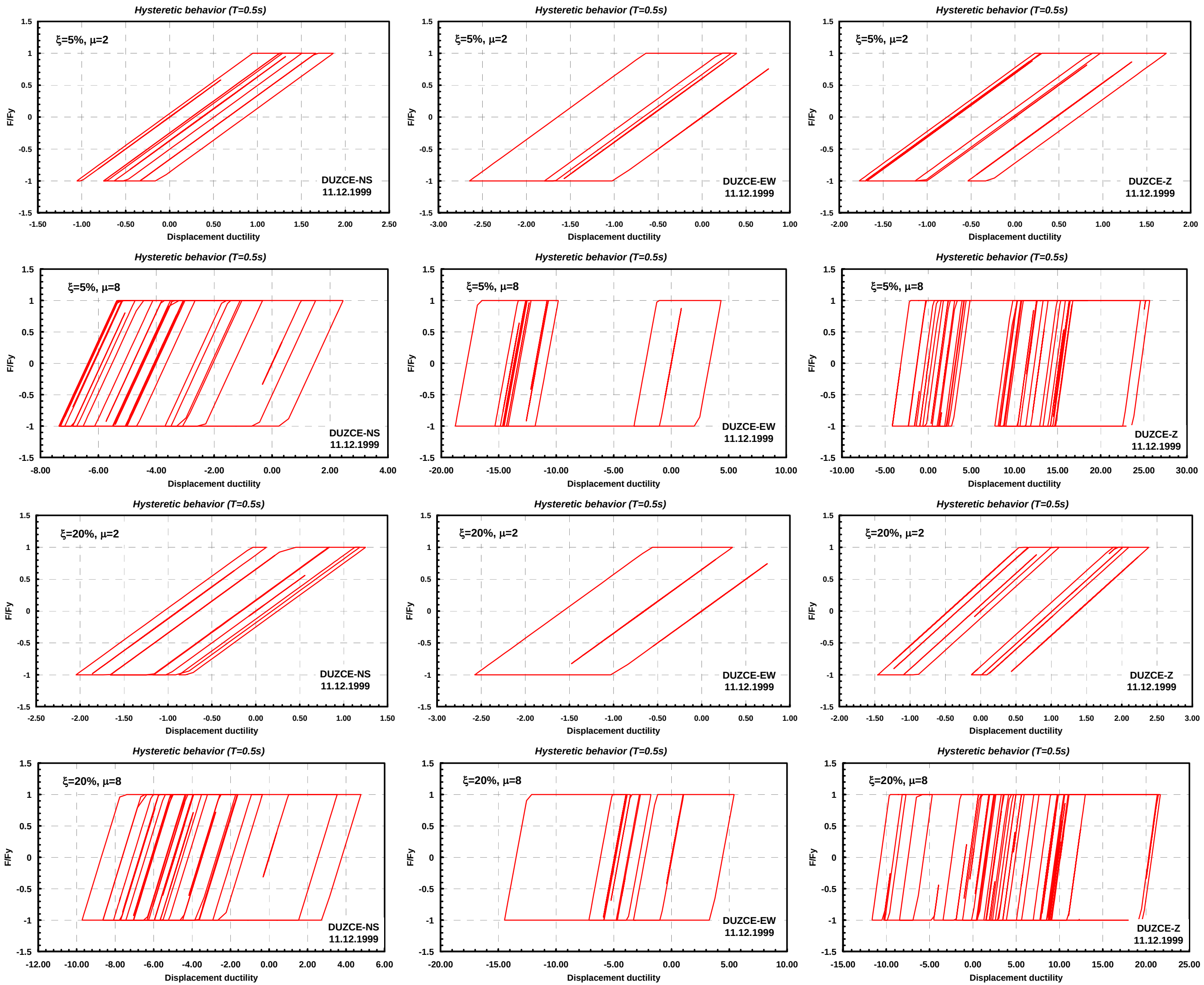

Figure 4. Hysteretic behavior of elasto-plastic SDOF systems for selected earthquakes

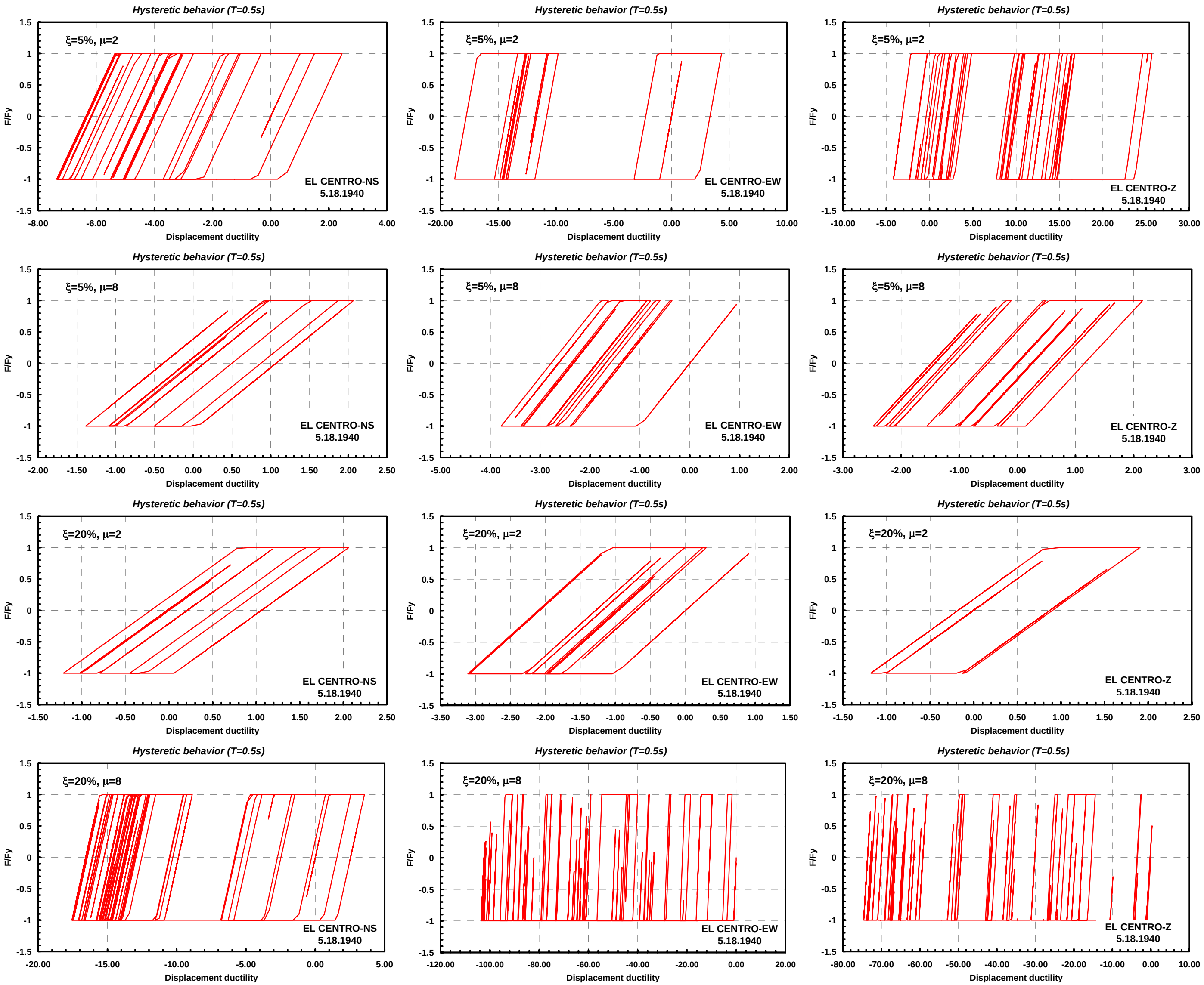

Figure 4. Hysteretic behavior of SDOF systems for selected earthquakes (continued)

	ξ=0.05 μ=1				ξ=0.05 μ=2				ξ=0.05 μ=8			
	SI (cm/sec)	α	λ_1	λ_2	SI (cm/sec)	α	λ_1	λ_2	SI (cm/sec)	α	λ_1	λ_2
Elcentro	260.53	0.10	0.30	-	201.96	0.12	0.30	-	141.90	0.68	0.30	1.05
	250.64				151.85				50.19			
	60.98				30.69				6.15			
	311.96				248.77				333.57			
Parkfield	161.00	0.05	0.30	-	110.00	0.18	0.30	0.06	78.60		0.30	0.37
	133.67				100.05				82.22	0.30		
	47.63				39.12				34.20			
	175.87				149.68				131.52			
Hachinohe	109.88		0.30	-	90.72		0.30	-	70.43		0.30	0.17
	121.18	0.05			102.76	0.05			79.10	0.15		
	50.45				43.63				37.46			
	134.09				111.93				103.37			
San Fernando	702.00	0.08	0.30	-	463.00	0.23	0.30	0.17	283.00	0.51	0.30	0.71
	400.95				292.92				222.33			
	337.09				254.25				218.40			
	808.20				679.34				569.76			
Miyagi Ken Oki	341.51	0.05	0.30	-	346.57	0.09	0.30	-	627.18	0.14	0.30	-
	251.28				107.19				20.00			
	119.89				71.20				24.45			
	376.31				408.83				807.85			
Tabas	720.57		0.30	0.11	481.56		0.30	0.21	431.91		0.30	0.14
	845.36	0.10			578.54	0.13			507.67	0.11		
	324.35				250.05				189.11			
	1015.90				727.75				623.88			
Loma Prieta	990.00	0.02	0.30	-	672.00	0.05	0.30	-	473.00	0.02	0.30	-
	397.95				275.42				223.83			
	311.59				216.20				239.53			
	1032.35				741.44				495.55			
Erzincan	619.38	0.06	0.30	-	444.03	0.14	0.30	-	281.68		0.30	0.55
	483.64				397.03				322.29	0.31		
	158.94				105.90				64.12			
	692.51				564.53				520.66			
Northridge	570.00		0.30	0.23	412.00		0.30	0.43	359.00		0.30	0.37
	777.12	0.06			569.47	0.12			581.12	0.02		
	182.08				145.30				130.48			
	874.29				711.29				600.14			
Kobe	545.75		0.30	-	432.64	0.11	0.30	-	353.37	0.08	0.30	-
	552.44	0.08			411.03				302.45			
	242.05				206.86				170.00			
	637.73				526.64				412.95			
Kocaeli	575.00		0.30	0.00	395.00	0.16	0.30	0.03	301.00	0.18	0.30	0.06
	587.45	0.14			385.06				275.15			
	331.92				256.97				175.44			
	748.24				524.62				409.79			
Chi Chi	565.13		0.30	-	448.71		0.30	-	332.49		0.30	0.07
	611.64	0.08			451.53	0.13			335.62	0.18		
	216.04				148.68				145.88			
	709.30				567.17				456.80			
Duzce	452.00		0.30	-	368.00	0.12	0.30	-	286.00		0.30	-
	454.57	0.10			313.26				288.71	0.09		
	129.32				100.81				80.97			
	545.31				455.64				339.21			

Table 2. Computed spectrum intensities and combination coefficients (ξ=0.05)

	ξ=0.20 μ=1				ξ=0.20 μ=2				ξ=0.20 μ=8			
	SI (cm/sec)	α	λ1	λ2	SI (cm/sec)	α	λ1	λ2	SI (cm/sec)	α	λ1	λ2
Elcentro	193.92	0.05	0.30	-	164.60	0.08	0.30	-	132.90	0.26	0.30	0.22
	162.21				118.83				57.00			
	48.25				23.25				9.99			
	214.29				192.16				202.09			
Parkfield	113.00	0.10	0.30	-	83.50		0.30	0.24	68.20		0.30	0.49
	112.18				87.96	0.23			78.12	0.28		
	32.91				31.10				30.55			
	134.75				128.72				121.76			
Hachinoe	85.70		0.30	-	71.97		0.30	-	65.73		0.30	0.05
	94.87	0.04			85.58	0.04			74.32	0.10		
	37.70				35.07				34.17			
	101.98				93.16				88.78			
San	535.00	0.08	0.30	-	375.29	0.23	0.30	0.15	262.98	0.43	0.30	0.56
Fernando	315.74				245.69				211.14			
	266.89				211.58				199.35			
	619.36				544.92				489.69			
Miyagi Ken	242.82	0.03	0.30	-	239.47	0.06	0.30	-	297.62	0.07	0.30	-
Oki	175.59				70.26				21.43			
	77.31				48.51				24.18			
	258.29				266.35				341.81			
Tabas	515.58		0.30	-	385.62		0.30	0.02	370.30		0.30	0.08
	553.83	0.09			410.66	0.12			407.62	0.13		
	216.25				180.41				160.14			
	658.28				508.51				509.60			
Loma Prieta	630.00	0.02	0.30	-	473.00	0.03	0.30	-	379.00	0.02	0.30	-
	285.96				218.48				198.80			
	220.36				168.80				202.62			
	652.58				502.19				395.67			
Erzincan	450.62	0.06	0.30	-	325.41	0.23	0.30	0.16	246.77		0.30	0.46
	364.17				304.02				278.58	0.28		
	91.05				72.95				54.18			
	506.29				476.33				434.39			
Northridge	411.00		0.30	0.23	295.00		0.30	0.60	300.00		0.30	0.45
	608.67	0.02			469.32	0.10			513.49	0.01		
	122.86				105.68				106.61			
	630.79				561.51				525.73			
Kobe	379.44		0.30	-	351.21	0.09	0.30	-	306.26	0.09	0.30	-
	432.83	0.04			325.86				271.26			
	193.77				170.00				157.10			
	466.95				412.23				359.52			
Kocaeli	333.00	0.11	0.30	-	264.00	0.17	0.30	0.05	226.00	0.18	0.30	0.06
	330.40				245.46				206.17			
	208.90				176.39				154.31			
	409.56				355.48				308.03			
Chi Chi	447.78		0.30	-	370.00	0.10	0.30	-	309.00	0.16	0.30	0.03
	448.17	0.07			340.13				295.39			
	150.35				114.35				120.31			
	512.53				442.80				410.89			
Duzce	339.00		0.30	-	284.59	0.13	0.30	-	258.60		0.30	-
	373.32	0.05			266.25				261.00	0.09		
	88.17				73.14				70.14			
	409.83				360.26				305.53			

Table 3. Computed spectrum intensities and combination coefficients (ξ=0.20)

The Caltrans code does not specify the SRSS rule as an alternative. There are two alternative rules prescribed in IBC2003. One of the alternative rules combines the responses with two horizontal seismic components using the SRSS rule; the result is multiplied by the redundancy coefficient and added to the effect of the vertical component, which is written as a linear term in the design load combination. The other alternative rule combines the responses with two horizontal components using the 30% rule, and adds the effect of the vertical component in the same way. In the recent Turkish earthquake code of "Specification for buildings to be built in earthquake zones" the 30% rule is prescribed.

To compare the numerical results obtained from this study, the variation of α coefficients with respect to selected damping ratios is further illustrated in Figures 5a,b. For the selected earthquake ground motions, it is observed that, for inelastic analysis, the maximum value of the coefficient α is changing wide range between 0.05~0.23 for μ=2, ξ=0.05 and 0.02~0.68 for μ=8, ξ=0.05, 0.03~0.23 for μ=2, ξ=0.20 and 0.01~0.43 for μ=8, ξ=0.20. To evaluate the contribution of the vertical component to the combination rule, λ_2 values are obtained whilst λ_1 values are taken as a value of 0.30. The results show that most of the λ_2 values are negative, hence if the orthogonal components are combined with a %30 rule, vertical component of the strong ground motion can be neglected.

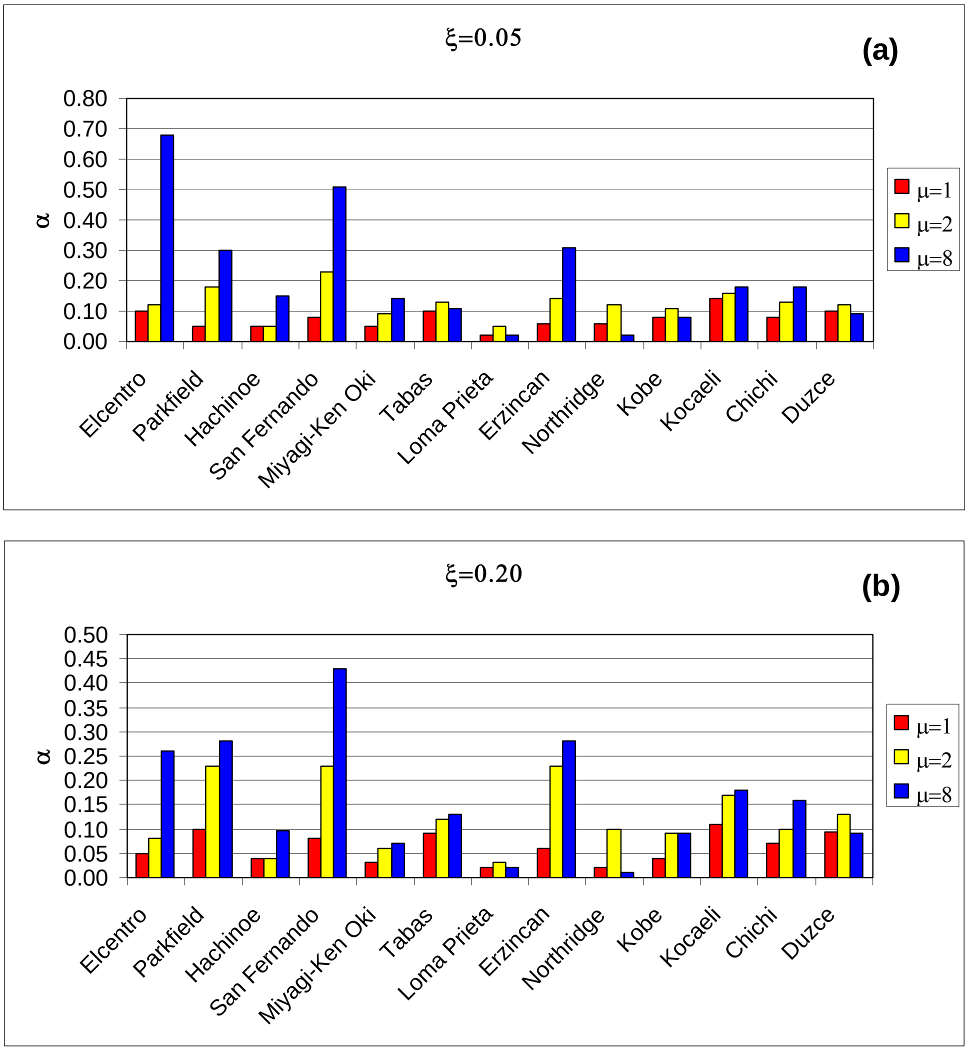

Figure 5. α coefficients for elastic (μ=1) and inelastic (μ=2 and μ=8) systems.

5 CONCLUSIONS

Among several methods of analyses (e.g. the CQC, 30%), the Housner's spectrum intensity concept was used to investigate tri-directional earthquake effects for inelastic response of structures. For this purpose, inelastic velocity response spectra were plotted for a set of thirteen ground motions for structures having damping ratios of ξ=0.05, 0.20, and first elastic vibration periods up to T=5 sec. Spectrum intensities for both orthogonal directions and for the resultant direction were obtained. Relevant coefficients commonly used in the percentage rules recommended in building codes were numerically calculated using the velocity spectrum intensities. Obtained numerical results show that the maximum value of the coefficient α is strongly earthquake dependent and varied between 0.05~0.23 for μ=2, ξ=0.05 and 0.02~0.68 for μ=8, ξ=0.05, 0.03~0.24 for μ=2, ξ=0.20 and 0.01~0.43 for μ=8, ξ=0.20, revealing that in some cases the code defined combination values may yield unconservative seismic designs.

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