COMPDYN 2011
3rd ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis, V. Plevris (eds.)
Corfu, Greece, 25–28 May 2011

COMPUTATIONAL EFFICIENCY OF PROGRESSIVE INCREMENTAL DYNAMIC ANALYSIS

Marko Brozovič[1] and Matjaž Dolšek[1]

[1] University of Ljubljana, Faculty of Civil and Geodetic Engineering
Jamova 2, 1000 Ljubljana, Slovenia
{mbrozovi, mdolsek}@ikpir.fgg.uni-lj.si

Keywords: Incremental dynamic analysis, Precedence list, Ground motion selection, Computational efficiency, PBEE toolbox.

Abstract. *Progressive incremental dynamic analysis involves precedence list of ground motion records aiming of selecting the most representative ground motions from a set of ground motions. The precedence list of ground motion records provides the advantage of a simple mathematical model, which is not computationally demanding, and it is defined as an optimization problem, which can be solved with sufficient accuracy by means of a simple procedure. Therefore, the seismic response parameters are computed progressively, starting from the first ground motion record in the precedence list. After an acceptable tolerance is achieved, the analysis can be terminated. Such approach facilitates practical application of incremental dynamic analysis (IDA), especially, if the number of ground motions in a set of ground motions is large, and its computational efficiency is higher than ordinary IDA. In the paper the precedence list of ground motion records is determined for three reinforced concrete frame structures in order to evaluate computational efficiency of the progressive IDA. The results indicate that 16th, 50th and 84th fractiles of seismic response parameters of frame structures can be determined with sufficient accuracy if about 12 to 15 records out of 30 are used in the analysis, this indicating that progressive IDA requires much less computational time than IDA.*

1 INTRODUCTION

Incremental dynamic analysis (IDA) [1] is widely used parametric analysis method for seismic performance assessment of structures. It enables direct evaluation of record-to-record variability in structural response through a set of ground motion records. If the set of ground motions is large, IDA becomes computationally demanding. In order to accommodate its practical application, precedence list of ground motions has been introduced [2, 3], aiming at selecting the most representative ground motion records for IDA analysis.

In progressive IDA the "selected" IDA curves are computed progressively from the first ground motion in the precedence list till the acceptable tolerance is attained. Therefore seismic response parameters are estimated by using the nonlinear dynamic analysis, which is performed for a limited number of records from a list. Such approach can significantly reduce computational efforts and consequently facilitate the use of nonlinear dynamic analysis for practical applications.

Since precedence list of ground motion records is based on dynamic response of equivalent SDOF model the computational efficiency of the progressive IDA varies from structure to structure and depends also on the set of ground motion records used in analysis. The objective of this study is to assess the computational efficiency of progressive IDA for different reinforced concrete frames, which were modeled by using the PBEE toolbox [4] in conjunction with OpenSees [5]. The toolbox was developed in Matlab [6] and provides functions for the rapid determination of simplified nonlinear structural models, performance assessment by employing different methods and the post-processing of analysis results. Since the PBEE toolbox is focused on simple nonlinear models, material nonlinearity is modeled only with the moment-rotation relationship in the plastic hinges of the columns and beams.

2 SUMMARY OF PROGRESSIVE INCREMENTAL DYNAMIC ANALYSIS

Progressive IDA was recently proposed [3] aiming at minimizing the computational time, which is required for prediction of seismic response parameters in the case if they are estimated with nonlinear dynamic analysis. For that reason a precedence list of ground motion records was introduced. The determination of such a list is an optimization problem and can be sufficiently solved by minimizing the defined fitness function, which is determined based on response of a simple model, e.g. the single-degree-of-freedom (SDOF) model.

The methodology was originally proposed for prediction of median seismic response (i.e. median IDA curve) [2]. However its use was extended for prediction of 16^{th}, 50^{th} and 84^{th} fractile response [3]. In progressive IDA the so-called "selected" IDA curves are computed progressively starting from the first ground motion record in the precedence list. When the required tolerance in the prediction of seismic response is achieved, the analysis can be terminated. The benefit of progressive IDA, in comparison to the IDA, is the reduction of the computational effort. However, determination of the precedence list of ground motion records also requires some computational time, firstly, for IDA analysis of the simple model (e.g. SDOF system), which is needed in order to determine a precedence list, and secondly, for the optimization of the precedence list of ground motion records. Computational time needed for determination of precedence list is usually shorter than time needed for one IDA analysis of a structure.

The input data for determination of the precedence list of ground motion records are IDA curves for the simple (e.g. SDOF) model. The next step is definition of the so called fitness function, which incorporates IDA curves of the simple model, and it is defined in a way that its value changes if the position of the ground motions identification numbers (IDs) in a list are rearranged. The precedence list is determined when the ground motions IDs are arranged in the order that the fitness function takes on a minimum value.

In general, the fitness functions can be defined for different purposes. However, for the precedence list of ground motions aiming at predicting the fractile IDA curves, it was found [2, 3], that a good measure for defining the fitness function is the area between the "original" fractile IDA curve and the "selected" fractile IDA curve, where the term "original" is used for the case if fractile IDA curve is obtained from all the IDA curves while the term "selected" is used if the fractile IDA curve is determined only for the first s ground motions from the precedence list. If this area is normalized with the area defined by the "original" fractile IDA curve then the dimensionless measure for the error between the two types of the fractile IDA curves can be defined as follows [3]

$$Error(s,f) = 100 \times \frac{\int_0^{EDP_{\max}(s,f)} \left|\Delta IM\left(s,f\right)\right| dEDP}{\int_0^{EDP_{\max,or}(f)} IM_{or}\left(f\right) dEDP} \tag{1}$$

where s is the number of selected subsets of three ground motions, IM and EDP are usual notations for intensity measure and engineering demand parameter, respectively, $IM_{or}(f)$ is the intensity measure of the "original" f-th fractile IDA curve, $EDP_{\max,or}(f)$ is the engineering demand parameter corresponding to the capacity point of the "original" f-th fractile IDA curve, $\Delta IM(s,f)$ is the difference in the IM corresponding to the "original" and "selected" f-th fractile IDA curve, and $EDP_{max}(s,f)$ is the maximum of the engineering demand parameters corresponding to the capacity point of the "selected" and "original" f-th fractile IDA curves. The error as defined in the Eq. (1) is not yet the fitness function since it depends only on the first s selected number of records from the precedence list and on the selected f-th fractile IDA curve. In order to define the fitness function the sum of $Error(s,f)$ over all values is required. In the case if the precedence list of ground motions is determined in order to predict the three fractile IDA curves it was found [3], that the best precedence list is obtained if the fitness function is defined in a way to give a preference to those ground motions, which IDA curves are close to the "original" fractile curves. Therefore, the fitness function Z is defined by sum of $Error(s,f)$ over the m number of subsets of ground motion records and over all f-th fractile IDA curves

$$Z = \frac{1}{m} \sum_{s=1}^{m} \sum_{f=1}^{3} Error\left(s,f\right) \tag{2}$$

Different techniques can be used to minimize the fitness function. The simplest possible way, which actually does not require optimization method, is gradual minimization of $Error(s,f)$. It was proven that gradual minimization of $Error(s,f)$ results in a value of the fitness function, which is close to the global minimum [3]. The first ground motion in the precedence list corresponds to the minimum value of errors (Eq. (1)) that are calculated for s=1, f=1 and for all n records in the given set of ground motion records. The second and third ground motion in the precedence list correspond to minimum error calculated for s=1, f=2 and f=3 for n-1 and n-2 records left to be placed in the precedence list, respectively. The following ground motion IDs in the precedence list are defined with repeating described procedure until all ground motion IDs are placed in the precedence list of ground motions.

Once the precedence list of ground motions is known, the difference between the fractile curve determined for s-1 and s subsets of ground motions can be obtained. This measure is called the tolerance function and is defined as follows [3]

$$Tolerance(s,f) = 100 \times \frac{\int_{0}^{Max\left[EDP_{\max}(s,f),EDP_{\max}(s-1,f)\right]} \left| IM(s,f) - IM(s-1,f) \right| dEDP}{\int_{0}^{EDP_{\max}(s-1,f)} IM(s-1,f)\, dEDP} \qquad (3)$$

where $IM(s,f)$ and $IM(s\text{-}1,f)$ are the values of the intensity measures for the f-th fractile IDA curves, which are determined, respectively, on the basis of the first s and s-1 subsets of ground motion records. The additional parameters introduced into Eq. (3) are the engineering demand parameters $EDP_{\max}(s,f)$ and $EDP_{\max}(s\text{-}1,f)$, which correspond to the capacity point of the f-th fractile curve determined from first s and s-1 subsets of ground motion records.

In progressive IDA "selected" IDA curves are computed progressively starting from the first ground motion in the precedence list as long as required tolerance in the prediction of seismic response is achieved.

3 EXAMPLE: SEISMIC PERFORMANCE ASSESSMENT OF DIFFERENT REINFORCED CONCRETE FRAMES USING PROGRESSIVE IDA

Computational efficiency of progressive incremental analysis is studied by means of a seismic performance assessment of three reinforced concrete frame structures and a set of 30 ground motion records. The structural performance is assessed for two limit states, which were defined on the basis of the damage observed in the beams and columns. The peak ground acceleration and the maximum top displacement were adopted, respectively, as the intensity measure and the engineering demand parameter.

3.1 Description of the structures and the structural models

The eight-storey RC frame [7], six-storey building [8] and the eight storey building [9, 10] were selected from literature. Elevation and the plan views of the structures as well as typical reinforcement of columns and beams are shown in Figure 1. All the structures under consideration are symmetrical reinforced concrete frames, but there appear differences in the number of stories, height of the stories, position of columns, dimensions and reinforcement of columns and beams cross-sections, thickness of slabs, strength of concrete and reinforcement. In addition, these structures were designed according to different design requirements.

The first structure (S1), which was designed for gravity loads only, is eight-storey RC frame with constant storey height of 2.8 m. Typical cross-sections as shown in Figure 1a are equal for all beams and columns, except for inner two columns in lower two stories and outer two columns in the top storey, where for the longitudinal reinforcement 4Φ20 bars are used instead of 4Φ16 bars. Mean strength of the concrete is 25 MPa and the mean yield strength of the steel is 400 MPa.

The second structure (S2) is six-storey RC frame and is shown in Figure 1b. The bottom storey is 4 m high, while the height of other storeys is 3 m. The dimensions of cross-sections and steel reinforcement in the beams and columns do not change from element to element. The structure was probably design according to Eurocode 8 [11] provisions for high seismic hazard area, since the amount of longitudinal and shear reinforcement in the cross-sections of the columns and beams is high. The concrete strength class is C20/25, while the steel strength class is S500.

The third structure (S3), i.e. an eight-storey building (Figure 1c), was designed according to the European standard Eurocode 8. The building's height of the first or second storeys is 5 m, whereas the other storeys are 3.1 m high. The building has three bays in each horizontal direction and therefore the largest structure considered in the presented study. All the cross-sections of the columns and beams of the structure have dimensions of 60/60 cm and 40/60 cm, respectively. For the columns, the steel reinforcement is the same for all sections, except for the cross-sections at the base, where the density of the stirrups is greater (Φ8/5 cm and Φ10/5 cm). The steel reinforcement for the beams is the same for all cross-sections, except for the beams in first two storeys, where at the top of the beams there are 6 instead of 4Φ20 bars. The concrete strength class of the building is C30/37, and the steel strength class is B500.

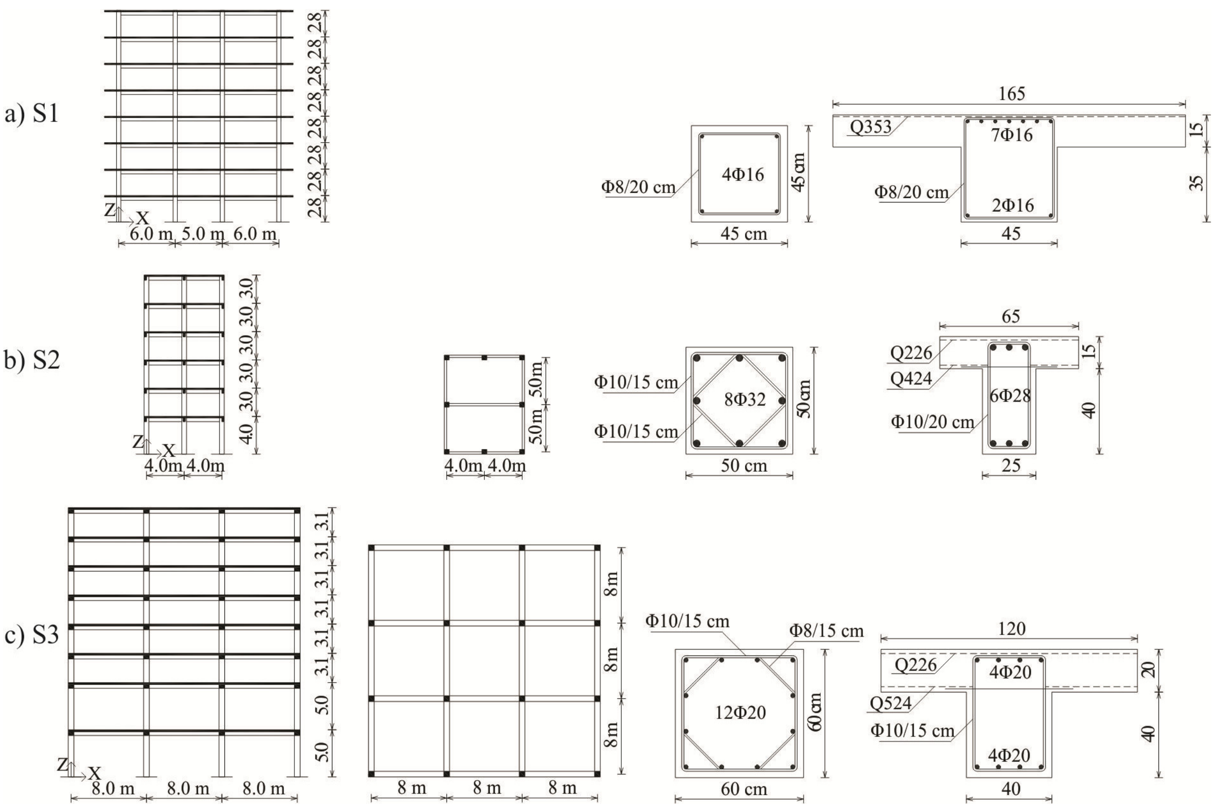

Figure 1: The elevation, plan view and the reinforcement in typical cross-sections of the columns and beams for a) eight storey frame, b) six-storey and c) eight-storey RC building.

Structural models of the buildings were prepared in the PBEE toolbox [4]. Structures S2 and S3 were modelled as 3D frame structures although they are symmetric. The height of the storeys was determined by the distance between the centrelines of the beams. The masses were concentrated at the storey levels, at the centre of gravity and the effective width of the beams was modelled as described in Eurocode 2 [12], assuming zero moment points at the midpoint of the beams. The effective width of all the beams in structure S1 was considered equal to 1.65 m, while the effective width of the beams of the structure S2 differed and amounted to 0.65 m, 0.75 m and 1.85 m. In the case of the structure S3, the effective width of the beams in the exterior and interior frames amounted 1.2 m and 2 m, respectively.

Beam and column flexural behaviour was modelled by one-component lumped plasticity elements, consisting of an elastic beam and two inelastic rotational hinges. The moment-rotation relationship before strength deterioration was modelled by a bi-linear relationship, whereas the post-capping stiffness was assumed to be linear, with a descending branch. The yield and maximum moment in the columns were calculated taking into account the axial forces due to the vertical loading. The ultimate rotation Θ_u in the columns at the near collapse (NC) limit state corresponded to 80% of the maximum moment measured in the post-capping range of the moment-rotation relationship. It was estimated by means of the Conditional Average Estimate (CAE) method [13]. For the beams, the EC8-3 [14] formulas were used to compute the ultimate rotations in the plastic hinges. In case of structure S1 ultimate rotations were multiplied by a factor of 0.85, because there was no seismic detailing considered in the design of the frame. The parameter β_u, which controls the unloading stiffness in the plastic hinges, was assumed to have a value of 0.8. Note, that computation of the moment-rotation envelopes in plastic hinges is embedded in the PBEE toolbox. Nonlinear dynamic analyses were performed by assuming 5 % damping proportional to mass.

3.2 Ground motion records

A set of 30 ground motion records [15] was used for seismic loading. The peak ground acceleration of the ground motions varies between 0.05 - 0.52 g. The records have been selected within events having a magnitude of between 6.5 and 6.9. All the ground motion records have been recorded on firm soil, with a distance range from the epicentre of 12 - 55 km. The acceleration spectra for each of the 30 ground motion records, and the corresponding 16^{th}, 50^{th} and 84^{th} fractiles, are presented in Figure 2.

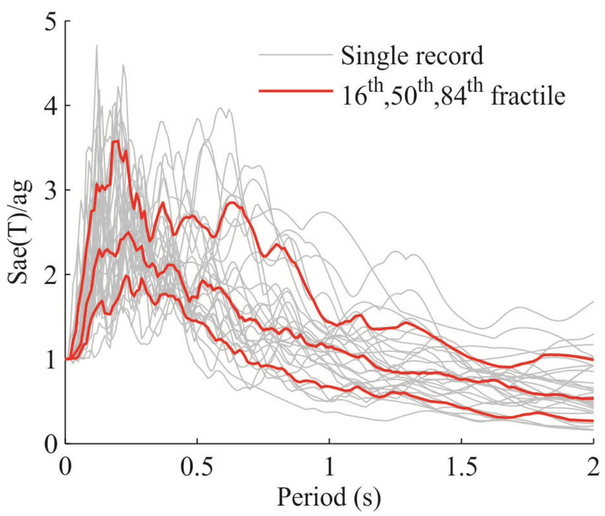

Figure 2: Acceleration spectra for all 30 ground motion records and corresponding 16^{th}, 50^{th} and 84^{th} fractile.

3.3 Seismic performance assessment using progressive IDA

The seismic response parameters, the top displacement and the corresponding peak ground acceleration, were estimated for the significant damage (SD) and near collapse (NC) limit states (LS). It is considered that the SD limit state is violated at the structural level when the rotation in the plastic hinge of the first column or in all the beams in one of the storeys exceeds the rotation which corresponds to the maximum moment in the columns or beams, respectively. Similarly, it is considered that the NC limit state is violated when the rotation in the plastic hinge of the first column or all the beams in one of the storeys exceeds the ultimate rotation in the columns or beams, respectively. The NC limit state is also violated if the average residual top displacement in last five seconds of the analysis exceeds the height of a building by more than 1 %, or if global dynamic instability is reached before other conditions for near collapse limit state are violated.

In order to assess seismic performance of structure by using progressive IDA, a precedence list of ground motion records need to be determined based on the IDA, which is performed for a simple model, e.g. equivalent single-degree-of-freedom (SDOF) model. The equivalent SDOF models were therefore defined from results of pushover analyses. The pushover analyses were performed by assuming a modal distribution of the horizontal forces for all structures. The analysis is limited to horizontal direction X (Figure 1) since the structures are symmetric, pushover analyses were performed in positive X direction only. The pushover curves, together with the points which indicate the defined limit states, are presented in Figure 3. Pushover curves were idealized with the tri-linear force-displacement relationships as presented in Figure 3. The strength of the idealized force-displacement relationship was set equal to the maximum strength determined by the pushover analysis. In Table 1 mass of the equivalent SDOF model m^*, the transformation factor Γ, period of the equivalent SDOF model T^*, which is equal to fundamental period of MDOF structure in the analysed direction, and the ratio between the maximum base shear and weight of the structure F_b/W are presented for the three structures.

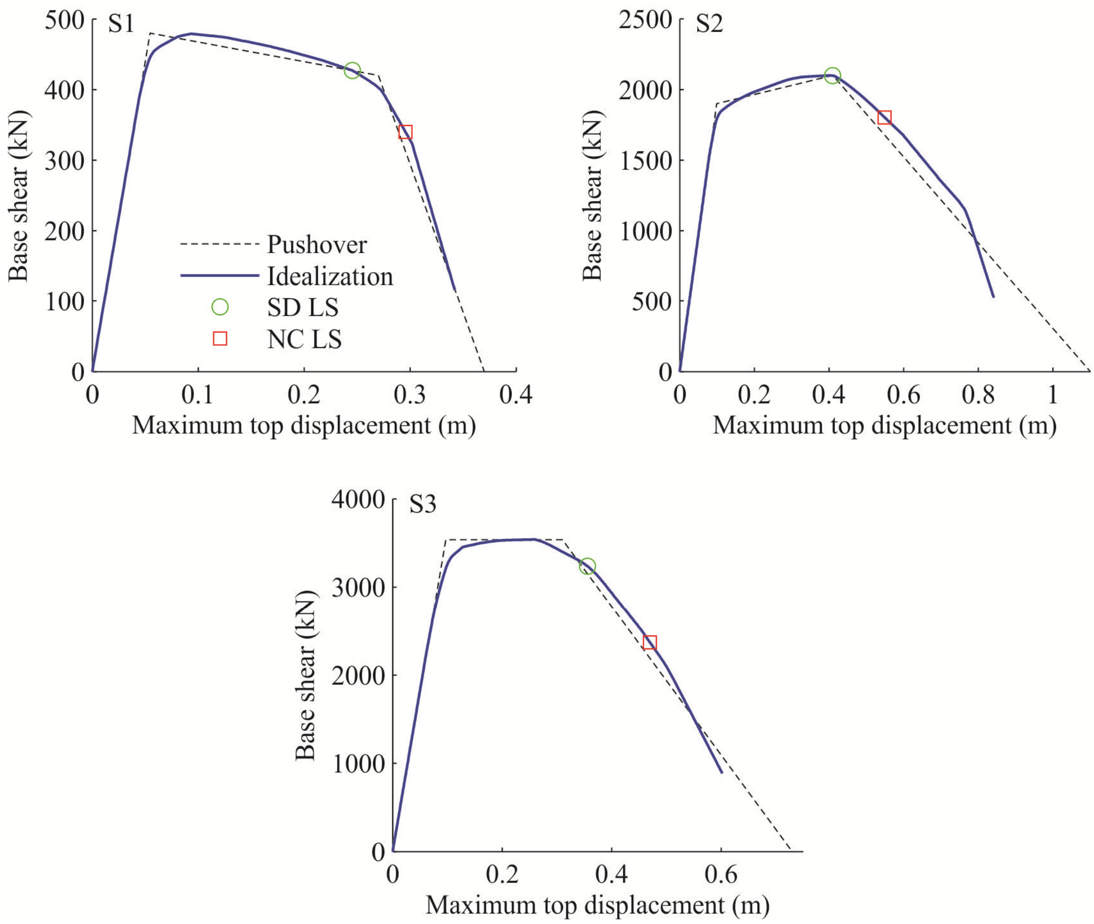

Figure 3: Pushover curves, showing the displacements which correspond to the SD limit state and NC limit state, and the idealized force-displacement relationships for structures S1, S2 and S3.

Table 1: Mass of the equivalent SDOF model m*, the transformation factor Γ, period of equivalent SDOF model T* and the ratio between the maximum base shear and weight of the structure F_b/W for structures S1, S2 and S3.

Structure	m* (t)	Γ	T* (s)	F_b/W
S1	606	1.27	1.65	0.05
S2	327	1.28	0.82	0.44
S3	2860	1.21	1.76	0.09

The damage at the plastic hinges of the beams and columns at NC limit state, which is obtained based on the pushover analysis, is presented in Figure 4. Note that, green, yellow and red colours represent, respectively, yielding of reinforcement, the state of exceeding the maximum moment, and the state of exceeding the ultimate rotation, whereas the grey part of the structure does not suffer any damage. The majority of the damage is concentrated in first and second storeys, while the upper stories remain almost undamaged.

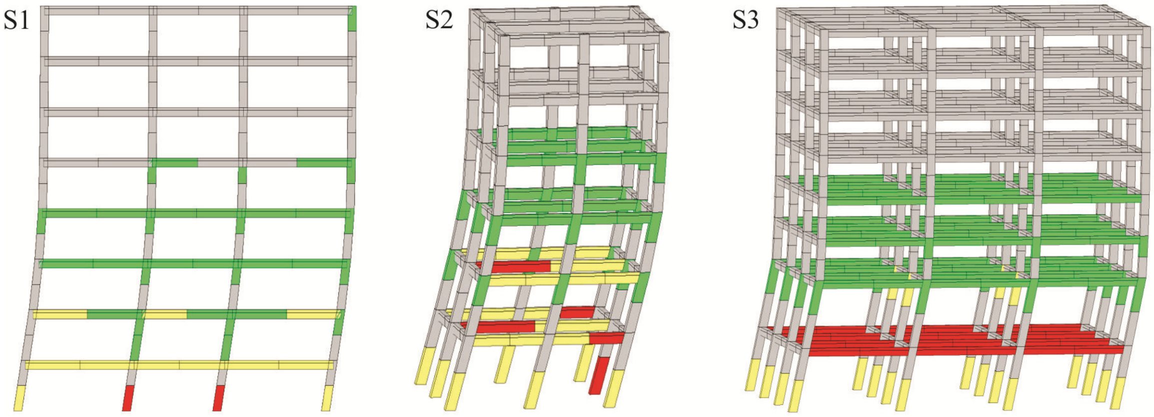

Figure 4: The distribution of damage for pushover analysis at NC limit state for structures S1, S2 and S3.

Precedence lists of ground motion records for all structures were determined from the IDA curves for the equivalent SDOF models by utilizing the simple optimization procedure [3]. Single record IDA curves were then computed progressively, starting from the first ground motion record in the precedence list for each structure. Progressive IDA was terminated after an acceptable tolerance for 16th, 50th and 84th fractile IDA curves was achieved. It was recognized the fractile IDA curves are predicted with sufficient accuracy with first 4, 5 and 4 subsets of ground motion records form the precedence list of records, respectively, for structure S1, S2 and S3. Note that each subset of ground motion records consists of three records.

The global dynamic instability of each computed IDA curves was estimated with a precision of 0.02 g. The largest interval between the peak ground acceleration, for which the seismic response parameters were computed, was defined as being equal to 0.05 g. However, if the peak ground acceleration, which corresponds to global dynamic instability, was large, the IDA curves were computed for only 20 points. Each nonlinear dynamic analysis within the IDA was calculated by employing the Newmark integration scheme, assuming $\gamma_n = 0.5$, $\beta_n = 0.25$ and an integration time interval of 0.005 s.

The "selected" IDA curves for the ground motion records from the precedence list of records and the corresponding 16th, 50th and 84th fractile response, with indication of the significant damage (SD) and near collapse (NC) limit states, are presented in figures 5a, 5c and 5e for all three structures.

In order to demonstrate the accuracy of results for 16th, 50th and 84th fractile response obtained with progressive IDA, IDA was performed for all 30 records. The comparison of fractile response obtained by progressive IDA and IDA is presented in figures 5b, 5d and 5f for all considered structures. The differences between the fractile IDA curves is practical negligible, even in the range close to the NC limit state, where the structures are already severely damaged. The indicated points in figures 5b, 5d and 5f represent the 16^{th}, 50^{th} and 84^{th} fractiles of the peak ground acceleration, which cause the median top displacement corresponding to the SD or NC limit state for IDA and progressive IDA, and are also presented in Table 2. It can be observed that the error between the presented median estimates based on progressive IDA and IDA is always less than 10 % in all cases, while a mean error is only 3 %. Progressive IDA produces practically the same results as IDA. However, a bit larger errors are observed in the case of the prediction of global dynamic instability (figures 5b, 5d and 5f).

Table 2: The 16^{th}, 50^{th} and 84^{th} fractiles of peak ground acceleration (PGA) causing the SD and NC limit states, and the corresponding median top displacement (MTD) for IDA and progressive IDA for all three structures. The values in brackets represent an error with respect to results of the IDA.

			SD limit state		NC limit state	
Structure	Method	Fractile	MTD (m)	PGA (g)	MTD (m)	PGA (g)
S1	IDA	16	0.24	1.01	0.34	1.24
		50		0.38		0.55
		84		0.18		0.27
	Progressive IDA	16	0.23 *(2%)*	0.97 *(3%)*	0.37 *(9%)*	1.28 *(3%)*
		50		0.37 *(3%)*		0.59 *(7%)*
		84		0.18 *(3%)*		0.27 *(0%)*
S2	IDA	16	0.40	1.99	0.53	2.32
		50		1.22		1.43
		84		0.82		0.91
	Progressive IDA	16	0.40 *(1%)*	2.02 *(2%)*	0.53 *(1%)*	2.29 *(2%)*
		50		1.26 *(3%)*		1.44 *(1%)*
		84		0.82 *(0%)*		0.90 *(1%)*
S3	IDA	16	0.32	1.23	0.45	1.87
		50		0.54		0.74
		84		0.24		0.32
	Progressive IDA	16	0.33 *(4%)*	1.26 *(2%)*	0.44 *(2%)*	1.82 *(3%)*
		50		0.55 *(2%)*		0.72 *(2%)*
		84		0.26 *(6%)*		0.33 *(3%)*

The seismic performance assessment by using progressive IDA requires significantly less computational time. It is shown that, by using progressive IDA, computational time can be reduced for about 50 % compared to that needed to perform IDA. It was proved, at least for the presented examples, that the seismic response parameters are sufficiently accurate when determined by progressive IDA, which requires only 12 to 15 IDA curves instead of 30, which was the number of records used in the IDA. Nevertheless there is some additional work with the preparation of the precedence list. Note that the method provided sufficiently accurate results, although it is based on response of an equivalent SDOF model.

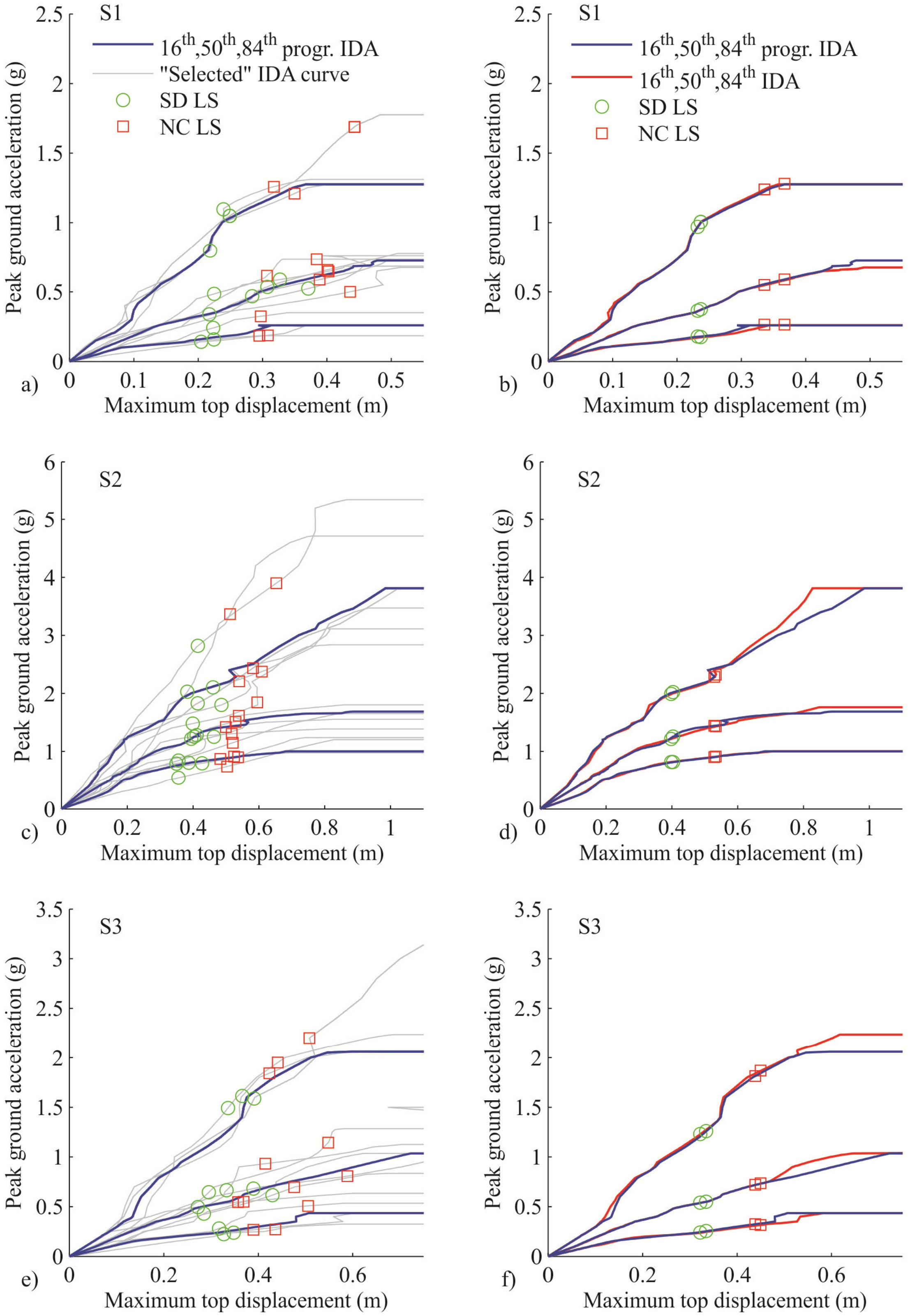

Figure 5: The "selected" IDA curves with the corresponding fractile response determined by progressive IDA and the comparison of the 16th, 50th and 84th fractile response determined by IDA and progressive IDA for structures S1, S2 and S3. Significant damage (SD) and near collapse (NC) limit states are indicated.

4 CONCLUSIONS

Computational efficiency of progressive IDA was assessed by means of seismic performance assessment of three symmetrical reinforced concrete frame structures. The considered structures differed in geometry, material, design requirements and consequently in period and ratio between the maximum base shear and weight of the structure significantly. However, it was shown that it is possible to predict the 16^{th}, 50^{th} and 84^{th} fractiles of seismic response parameters for the significant damage and near collapse limit state by using the progressive IDA with minor errors in comparison to IDA for all three structures.

The differences in 16^{th}, 50^{th} and 84^{th} fractiles of peak ground acceleration causing the significant damage and near collapse limit states, and in the corresponding median top displacement for progressive IDA and IDA are smaller than 10 %, for most cases only about 3 %. Progressive IDA produces practically the same results as IDA if compared in the range up to the near collapse limit state, although the results of progressive IDA are based on only 12 to 15 ground motion records. Therefore progressive IDA requires only about 50 % of computational time with respect to the computational time needed for the IDA. However, there is some additional work with determination of precedence list of ground motion.

ACKNOWLEDGMENTS

The results presented in this paper are based on work supported by the Slovenian Research Agency. This support is gratefully acknowledged.

REFERENCES

[1] D. Vamvatsikos, C.A. Cornell, Incremental Dynamic Analysis. *Earthquake Engineering and Structural Dynamics*, 31(3), 491-514, 2002.

[2] A. Azarbakht, M. Dolšek, Prediction of the median IDA curve by employing a limited number of ground motion records. *Earthquake Engineering and Structural Dynamics*, 36, 2401-2421, 2007.

[3] A. Azarbakht, M. Dolsek, Progressive Incremental Dynamic Analysis for First-Mode Dominated Structures. *Journal of Structural Engineering*, available on-line, doi:10.1061/(ASCE)ST.1943-541X.0000282, 2010.

[4] M. Dolsek, Development of computing environment for the seismic performance assessment of reinforced concrete frames by using simplified nonlinear models. *Bulletin of Earthquake Engineering* 8, 1309-1329, doi:10.1007/s10518-010-9184-8, 2010.

[5] F. McKenna, G.L. Fenves, Open system for earthquake engineering simulation. Pacific Earthquake Engineering Research Center, Berkeley, California, *http://opensees.berkeley.edu*, 2007.

[6] MathWorks, MATLAB the Language of Technical Computing, *http://www.mathworks.com/*, 2007.

[7] M. Dolsek, Estimation of Seismic Response Parameters Through Extended Incremental Dynamic Analysis. M. Papadrakakis, M. Michalis, N.D. Lagaros eds. *Computational Methods in Earthquake Engineering*. Springer, 2011.

[8] C.Ch. Mitropoulou, N.D. Lagaros, M. Papadrakakis, Advances in Life Cycle Cost Analysis of Structures. M. Papadrakakis, M. Michalis, N.D. Lagaros eds. *Computational Methods in Earthquake Engineering*. Springer, 2011.

[9] M. Causevic, S. Mitrovic, Comparison between non-linear dynamic and static seismic analysis of structures according to European and US provisions. *Bulletin of Earthquake Engineering*, doi:10.1007/s10518-010-9199-1, 2010.

[10] M. Kosič, Design of eight-storey reinforced concrete frame building according to European standard Eurocode 8. *Personal communication*, 15 December 2010.

[11] CEN, Eurocode 8: Design of structures for earthquake resistance, Part 1: General rules, seismic action and rules for buildings. *EN 1998-1*. European Committee for Standardisation, Brussels, December 2004.

[12] CEN, Eurocode 2: Design of concrete structures, Part 1-1: General rules and rules for buildings. *EN 1992-1-1*. European Committee for Standardisation, Brussels, December 2004.

[13] I. Peruš, K. Poljanšek, P. Fajfar, Flexural deformation capacity of rectangular RC columns determined by the CAE method. *Earthquake Engineering and Structural Dynamics* 35, 1453-1470, 2006.

[14] CEN, Eurocode 8: Design of structures for earthquake resistance, Part 3: Strengthening and repair of buildings. *EN 1998-3*. European Committee for Standardisation, Brussels, March 2005.

[15] D. Vamvatsikos, C.A. Cornell, Direct estimation of the seismic demand and capacity of oscillators with multi-linear static pushovers through IDA. *Earthquake Engineering and Structural Dynamics* 35, 1097-1117, 2006.