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ON THE PERFORMANCE OF A TECHNIQUE FOR MORE EFFICIENT TIME INTEGRATION WHEN APPLIED TO BRIDGE STRUCTURES SEISMIC ANALYSIS

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Abstract. *The true behavior of structural systems is dynamic, that in many cases can not be simplified to static. The most versatile tool for structural dynamic analyses, is time integration, and, hence, under special attention in the seismic analyses of structural systems, becoming more complicated everyday. Nevertheless, the responses of time integration are inexact and generally being obtained after considerable computational cost. Considering these, besides the digitized nature of ground strong motion records, a technique for considerably reducing the computational cost with small loss of accuracy is recently proposed. With attention to the notion of convergence and its role in numerical analyses, the technique replaces seismic records, with records, digitized at larger steps. The good performance of the technique is displayed in implementation in simple and complicated structural systems analyses. In continuation of the studies, the objective, in this paper, is to examine whether we can successfully implement the technique in time integration of bridges structural systems. The technique is briefly reviewed, and after introdcung a bridge structural system, designed according to the Iranian practice, the finite element model of the system is time integrated, once ordinarily, and then again, after implementing the technique. The numerical results evidence the good performance of the technique, and the essentiality of further inverstigations.*

1 INTRODUCTION

The true behavior of structural systems is dynamic, that, when subjected to severe seismic excitations, cannot be simplified to static. In order to study the dynamic behaviors, the broadly accepted approach is to discretize the structural and mathematical models in space and arriving at the semi-discretied model below:

$$
\begin{gathered}
\mathbf{M}\,\ddot{\mathbf{u}} + \mathbf{f}_{\text{int}} = \mathbf{f}(t) \\
\text{Initial Conditions :} \quad \left| \begin{array}{l} \mathbf{u}(t=0) = \mathbf{u}_0 \\ \dot{\mathbf{u}}(t=0) = \dot{\mathbf{u}}_0 \\ \mathbf{f}_{\text{int}}(t=0) = \mathbf{f}_{\text{int}_0} \end{array} \right. \\
\text{Additional Constraints :} \quad \mathbf{Q} \\
0 \le t < t_{end}
\end{gathered}
\tag{1}
$$

analyze equations (1) with an appropriate method [1-4]. In equations (1), t and t_{end} imply the time and the duration of the dynamic behavior, $\mathbf{M}$ is the mass matrix, $\mathbf{f}_{\text{int}}$ and $\mathbf{f}(t)$ stand for the vectors of internal force and excitation, $\mathbf{u}$, $\dot{\mathbf{u}}$, and $\ddot{\mathbf{u}}$ denote the unknown vectors of displacement, velocity, and acceleration, $\mathbf{u}_0$, $\dot{\mathbf{u}}_0$, and $\mathbf{f}_{\text{int}_0}$, representing the vectors of displacements, velocities, and internal forces at $t=0$, together, define the initial status of the model (regarding the essentiality of considering $\mathbf{f}_{\text{int}_0}$ in equations (1), also, see [4]), and $\mathbf{Q}$ indicates some restricting conditions, e.g. additional constraints in problems involved in impact or elastic-plastic behavior [5,6]. The most versatile method to analyze equations (1) is direct time integration [7,8]. However, especially, for nonlinear multi-degree-of-freedom problems, the formulations of time integration methods are inexact and hence the obtained responses are approximations [9,10]. This inexactness, when considered together with the step-by-step nature of time integration methods, summarized in marching throughout the integration interval, computing the responses at distinct time stations from the responses at previous stations, consecutively, (see Figure 1), implies the importance of integration step sizes, in time integration

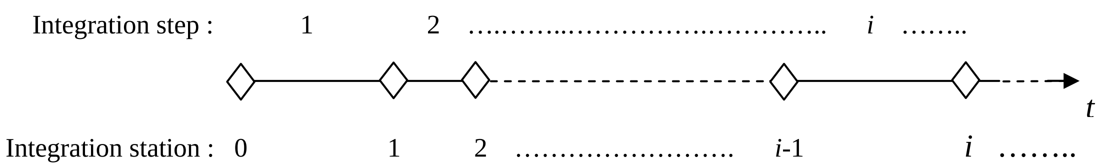

Figure 1: Typical arrangement of time steps and time stations in time integration analyses.

analyses (specially, regarding the accuracy and computational cost of the analysis). Considering Δt, as a positive definite parameter, scaling (linearly controlling) the size of integration steps, throughout t $0 \le t < t_{end}$ [11], one of the broadly accepted comments, for the selection of Δt, is as noted below [11-13]:

$$
\Delta t = \text{Min}\left(h_s, \frac{T}{10}, {}_f\Delta t \right) \tag{2}
$$

In equation (2), T is the smallest dominant period in the response, in general, approximated with the smallest natural period of the system at $t=0$, likely effectual in the response, h_s is the largest value of the integration step, providing numerically stable responses ($h_s=\infty$ for unconditionally stable methods), and ${}_f\Delta t$ is the digitization step size for excitations available as digitized records (for other excitations ${}_f\Delta t=\infty$). When, ${}_f\Delta t$ dominates equation (2), i.e.

$$\Delta t={}_f\Delta t<\mathrm{Min}\left(h_s,\frac{T}{10}\right) \tag{3}$$

Δt is to be set smaller than required for accuracy (see the inequality in equation (3), merely because of the size of digitization steps, ${}_f\Delta t$. This leads to additional computational cost. With the aim of decreasing this computational cost, a technique is recently proposed [11].

The technique changes ${}_f\Delta t$ to the larger value ${}_f\Delta t'$,

$${}_f\Delta t'=n\ {}_f\Delta t\cong\mathrm{Min}\left(h_s,\frac{T}{10}\right)\quad,\quad n=1,2,3\ldots \tag{4}$$

($n=1$ implies the limiting case, when the inequality in equation (3) is being replaced with equality), and replaces the excitation with a new excitation, digitized at steps equal to ${}_f\Delta t'$, such that in time integration analysis of the system subjected to the new excitation,

$$\Delta t={}_f\Delta t'\cong\mathrm{Min}\left(h_s,\frac{T}{10}\right) \tag{5}$$

and meanwhile the replacement of the excitation preserves responses convergence and the rate of convergence. (Convergence is the most important essentiality for all approximate analyses [14,15].) The technique is already successfully implemented in the analysis of some simple structural systems [11], i.e. a tall building [16], a fuel storage tank [17], and a silo [18], and also subjected to further theoretical investigation [19]. In continuation of the carried out investigations, this paper presents a study on the performance of the technique, when applied to the analysis of a traditionally designed bridge against a ground strong motion.

A brief review of the recent technique is presented in Section 2, after which, a bridge structural system is introduced and examined for the performance of the technique in Section 3, and finally, with a set of conclusions and guidelines for future research, the paper is ended in Section 4.

2 THE RECENT TECHNIQUE IN BRIEF

In order to replace the digitized excitation with a new excitation digitized at larger steps, such that the responses rate of convergence (generally two) [9,10,20] is preserved, a theory is set [11,21,22] and formulated in view of four assumptions. In brief, provided the assumptions below (implied in Figure 2):

1- The excitation steps, ${}_f\Delta t_i$ $i=1,2,\ldots$, are equally sized,

$$\forall\ i,j\quad {}_f\Delta t_i={}_f\Delta t_j={}_f\Delta t>0 \tag{6}$$

2- The integration steps, Δt_i $i=1,2,\ldots$, are equally sized,

$$\forall\ i,j\quad \Delta t_i=\Delta t_j=\Delta t>0 \tag{7}$$

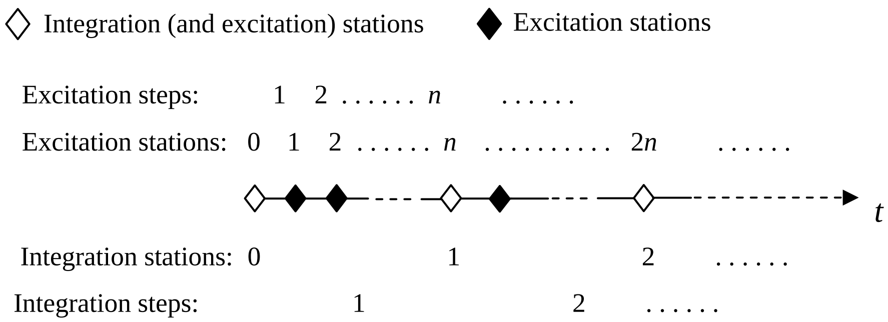

Figure 2: Typical distribution of excitation and integration stations in the recent technique [11].

3- The excitation steps are embedded by the integration steps (the first time station, i.e. t_0, is a station for both excitation and integration),

$$\exists \quad n \in Z^+ \qquad \frac{\Delta t}{{}_f\Delta t} = n < \infty \tag{8}$$

4- The $\mathbf{f}(t)$ in equations (1) is a digitized representation of an actual excitation, $\mathbf{g}(t)$, smooth [23] with respect to time, i.e.,

$$\begin{aligned} & \mathbf{f}(t) = \mathbf{g}(t)\delta(t-\alpha_i) \\ & \mathbf{g}(t): \text{ smooth with respect to time} \\ & \alpha_i = i\,{}_f\Delta t \;\;,\;\; i = 0,1,2,\ldots \\ & \delta(t-\alpha_i) = \begin{cases} 1 & t = \alpha_i \\ 0 & t \neq \alpha_i \end{cases} \end{aligned} \tag{9}$$

(and hence, the temporal derivatives of $\mathbf{f}(t)$, though rarely known, exist).

we can replace the excitation in equations (1), $\mathbf{f}$, with the new excitation, $\tilde{\mathbf{f}}$, digitized at steps equal to $n\,{}_f\Delta t$, according to:

$$\begin{aligned} & t_i = 0: && \tilde{\mathbf{f}}_i = \mathbf{f}(t_i), \\ & 0 < t_i < t_{end}: && \tilde{\mathbf{f}}_i = \frac{1}{2}\mathbf{f}(t_i) + \frac{1}{4n'}\sum_{k=1}^{n'}\left[\mathbf{f}(t_{i+k/n}) + \mathbf{f}(t_{i-k/n})\right], \\ & t_i = t_{end}: && \tilde{\mathbf{f}}_i = \mathbf{f}(t_i), \end{aligned} \tag{10}$$

where,

$$\begin{aligned} & t = \Delta t \;: && n' = n-1 \\ & \Delta t < t < t_{end} - \Delta t \;: && n' = \begin{cases} \dfrac{n}{2} & n = 2j \quad j \in Z^+ \\ \dfrac{n-1}{2} & n = 2j+1 \quad j \in Z^+ \end{cases} \\ & t = \Delta t \;: && n' = n-1 \end{aligned} \tag{11}$$

and Δt and n ($n \in Z^+$) are the largest values satisfying

$$\Delta t = n_f \Delta t \leq \mathrm{Min}\left(h_s , \frac{T}{10} \right) \leq t_{end} \tag{12}$$

with generally small loss of accuracy in time integration. Since, the new excitation, $\tilde{\mathbf{f}}$, is digitized at steps equal to $n_f \Delta t$, when considered instead of the original excitation, can lead to a reduction in computational cost (including the time spent and memory essential for the analysis), A_C, about and not more than

$$A_C = 100\left(\frac{n-1}{n}\right)\% \tag{13}$$

3 NUMERICAL STUDY

Consider the structural system introduced in Figure 3. The structure is a three span precast concrete girder bridge with equal spans of 30 meters. The superstructure is 12 meters wide and carries two traffic lanes. The superstructure consists of a 25 centimeters thick concrete slab and five reinforced concrete girders. The girders are connected by transverse diaphragm and at each end simply supported on 400×400 millimeters steel reinforced elastomeric bearings. Each bearing consists of two exterior layers with 6 millimeters thickness, four interior layers with 12 millimeters thickness, and five 2 millimeters steel reinforcement. The stiffness properties of the bearings are as noted below:

$$\begin{array}{l} \text{Vertical Stiffness}: \quad 3227000 \quad KN/m \\ \text{Shear Stiffness}: \quad 5220 \quad KN/m \\ \text{Rotational Stiffness}: \quad 1420 \quad KNm/Rad \\ \text{Torsional Stiffness}: \quad 118 \quad KNm/Rad \end{array} \tag{14}$$

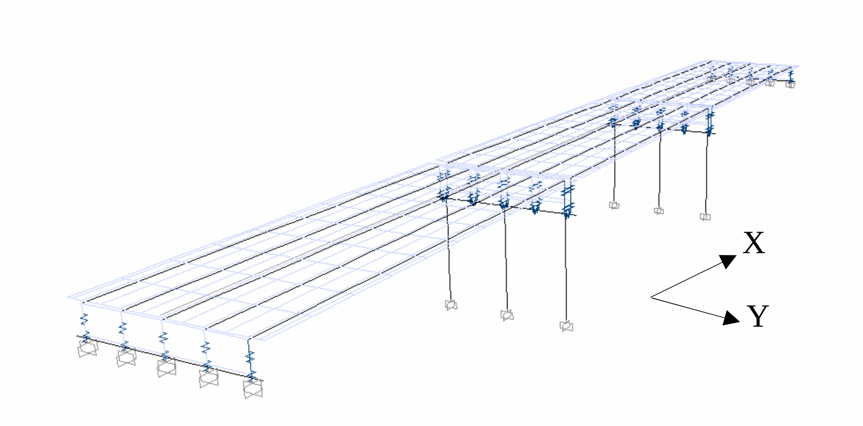

Figure 3: The bridge structural model considered in the numerical study in this paper.

The substructure consists of two closed end seat type abutments and two interior bents. Each bent consists of three concrete columns, each with 7.75 meters height and 1.2 meters diameter. The abutments are assumed to be rigid. The finite elements model, in Figure 3, is consisted of frame elements for the girders and shell elements for the superstructure slab. The shell elements are vertically offset to locate them at their actual position, and the columns are modeled using frame elements, and are assumed fixed at pile cap interface. The elastomeric bearings are modeled by spring elements with appropriate stiffness properties for all six degrees of freedoms (see equations (14)). The model is subjected to the excitation in Figure 4, once ap

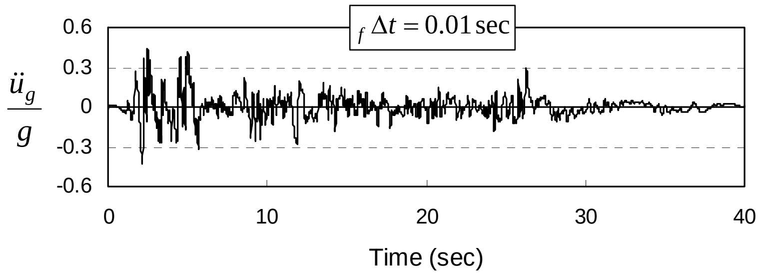

Figure 4: The digitized ground strong motion applied to the structural model in Figure 3 once in the *x* and then in the *y* direction.

plied in the *x* direction and then again in the *y* direction (see Figure 1, where, $\ddot{u}_g$ and g respectively stand for the ground acceleration and the constant of gravity, i.e. $9.81\ m/\text{sec}^2$). The excitation step size, $_f\Delta t$, equals $0.01\,\text{sec}$. The geometry and structural design of the bridge model is according to the practice, conventional in Iran [24-26].

In view of the approximate response partially reported in Figure 5, a good selection for T is as noted below:

$$T \cong 0.25 \tag{15}$$

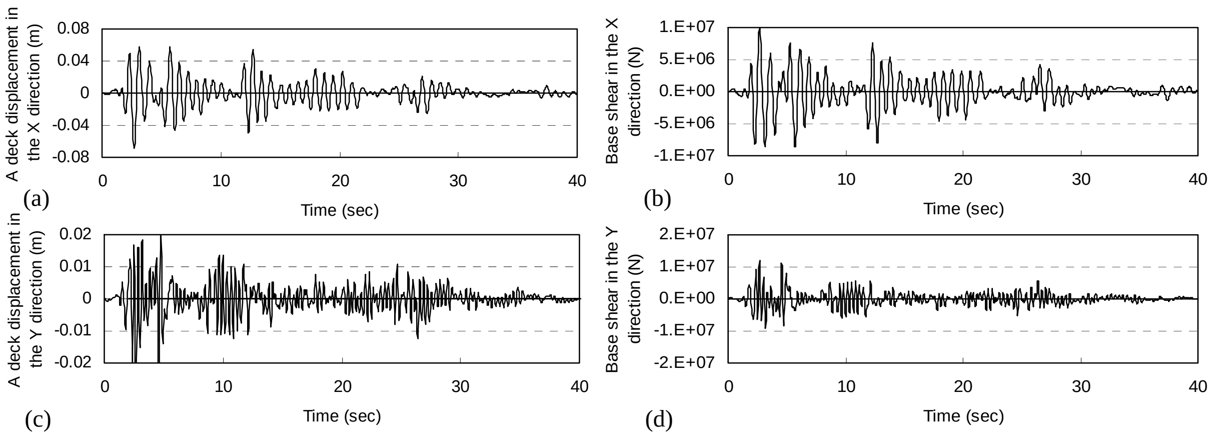

Figure 5: The approximate responses when applying the excitation to the structural model in Figure 3: (a) *x* direction, a deck displacement, (b) *x* direction, base shear, (c) *y* direction, a deck displacement, (d) *y* direction, base shear.

and hence, with attention to equations (4) and (12),

$$n = 2 \tag{16}$$

is an appropriate selection for the n in equations (11) and (12). Nevertheless, for a broader study, the cases:

$$n = 2, 4, 8, 20 \tag{17}$$

are also considered in time integration analyses with the Newmark average acceleration method [27]. The accuracies are compared in Figures 6 and 7. Considering the time spent and CPU memory essential for the computation, the computational costs are reported in Table 1, and, for the sake of completion, the original and new excitation records are reported in Figure 8. Figures 6 and 7 and Table 1 clearly display the good performance of the recent technique and imply that the technique has a good chance to reduce the computational costs in analyses of bridges seismic behaviors by time integration.

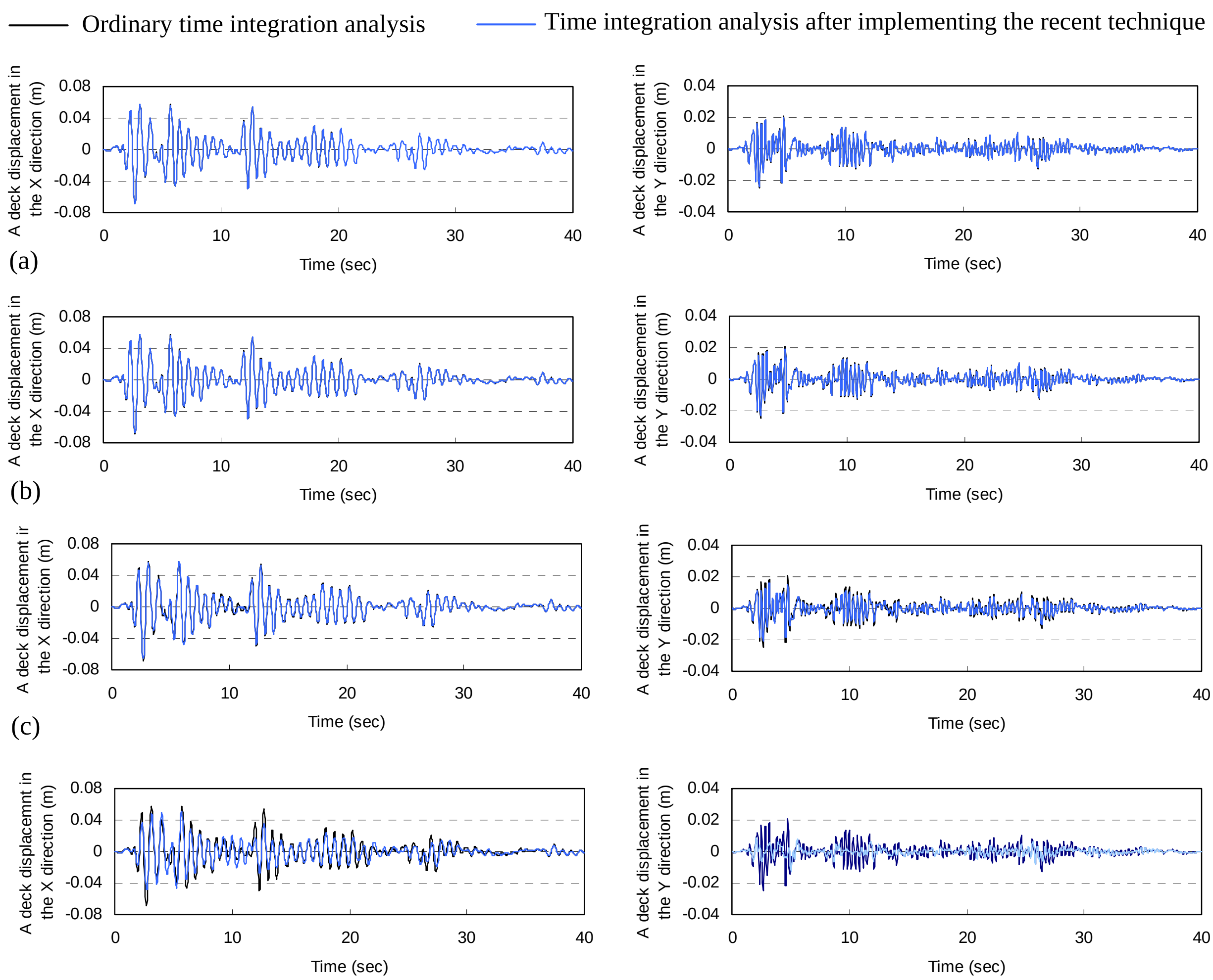

Figure 6: The performance of the recent technique when implemented in direct time integration analysis of the structural system introduced in Figures 3 and 4 and equations (14), by the average acceleration Newmark method [27], for a deck displacement: (a) $n = 2$, (b) $n = 4$, (c) $n = 8$, (d) $n = 20$.

The bridge structural system and excitation, considered in the numerical study, presented above, are common in Iran. Nevertheless, since, in view of the explanations in Section 2 and specifically equations (10)-(13), the performance of the technique might be different for different values of *n*, and *n* depends on both the structural system and the excitation, further study regarding the sensitivities is essential. Meanwhile, from the point of view of the analysis method, specifically, when considering behaviors against severe earthquakes, it is essential to study the performance of the technique, in presence of nonlinearities, considering a variety of integration methods, and different iterative nonlinearity solution methods.

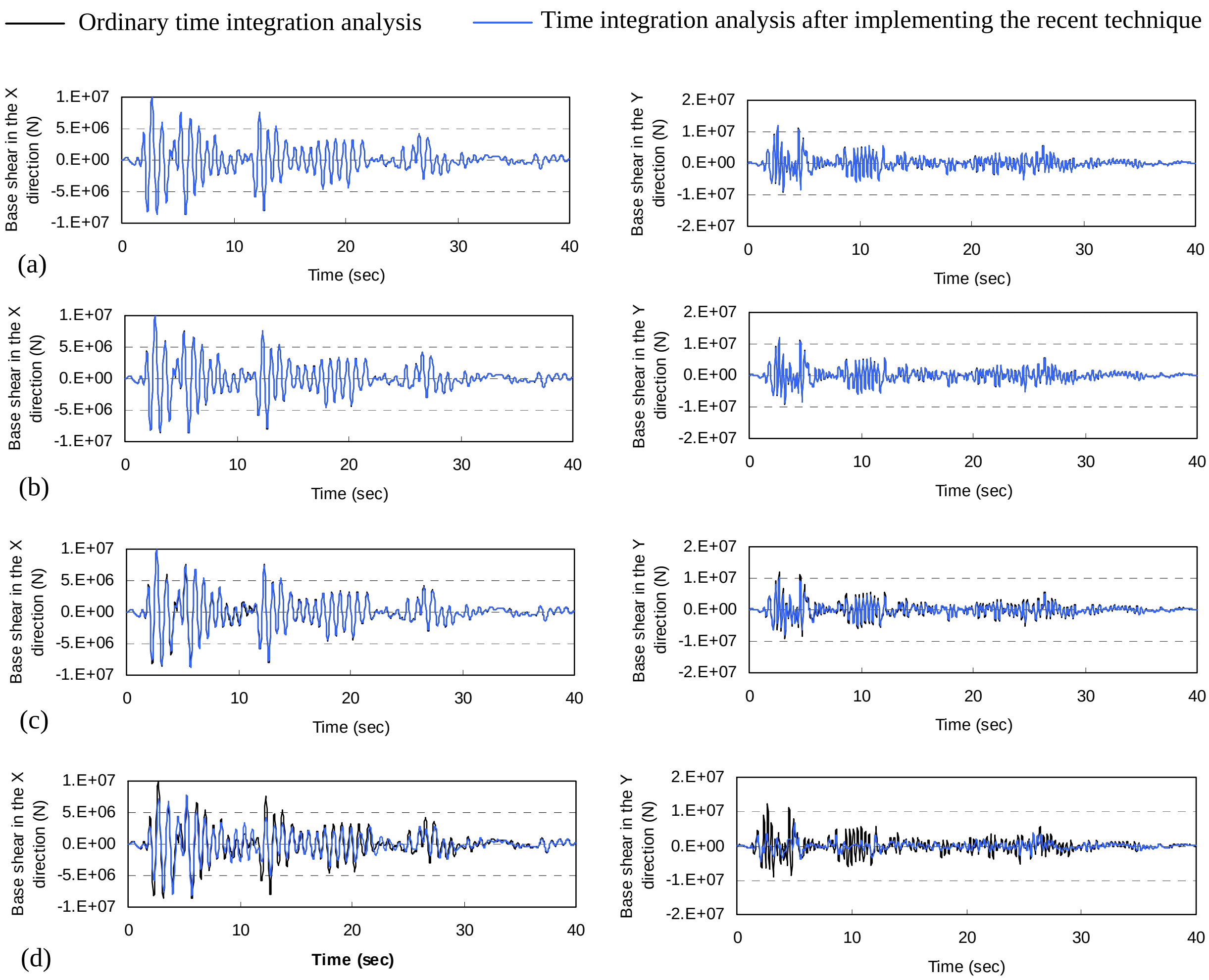

Figure 7: The performance of the recent technique when implemented in direct time integration analysis of the structural system introduced in Figures 3 and 4 and equations (14), by the average acceleration Newmark method [27], for base shear: (a) $n=2$, (b) $n=4$, (c) $n=8$, (d) $n=20$.

Ordinary	$n=2$	$n=4$	$n=8$	$n=20$
100	50	25	12.5	5

Table 1: An approximate study on the computational costs, $100-A_C$, when implementing the recent technique [11], compared to the ordinary analysis, with computational cost 100.

4 CONCLUSIONS

The performance of a new technique proposed for reducing the computational cost of seismic analysis by time integration is examined considering a bridge structural system common in Iranian practice. The numerical results display that, for the bridge structural system studied here, considerable reduction of computational cost is attained and the loss of accuracy is even less than expected. Further study in this regard, considering different bridge structural systems, different ground strong motions, different integration methods, and different nonlinearity solution methods is suggested for further research.

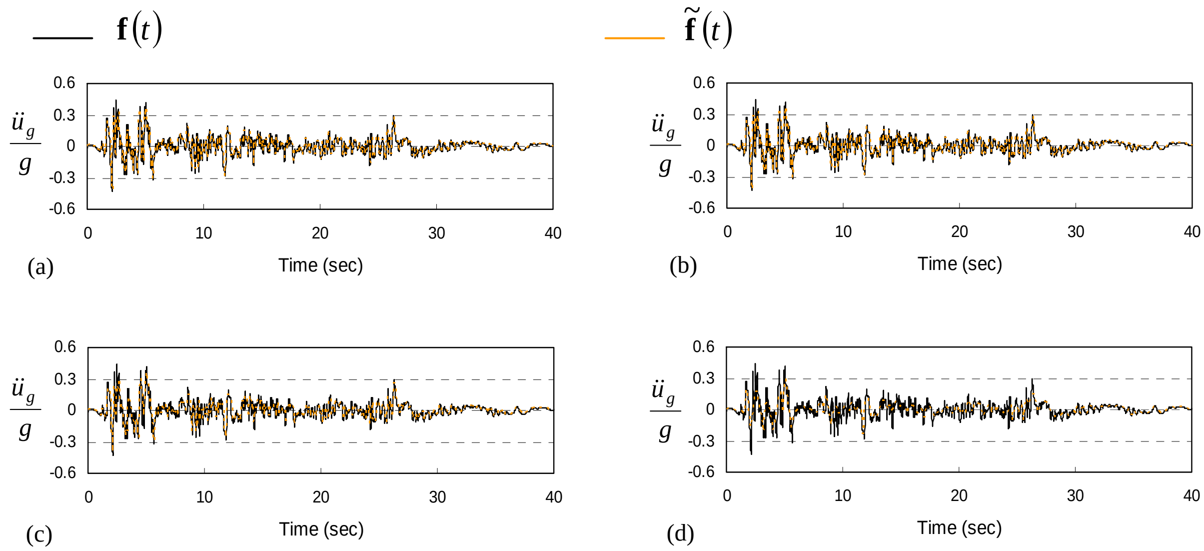

Figure 8: The excitations, $\tilde{\mathbf{f}}(t)$, when implementing the recent technique in the numerical study considering: (a) $n=2$, (b) $n=4$, (c) $n=8$, (d) $n=20$.

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