COMPDYN 2011
III ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis, V. Plevris (eds.)
Corfu, Greece, 25–28 May 2011

1D SEISMIC RESPONSE OF SOIL: CONTINUOUSLY INHOMOGENEOUS VS EQUIVALENT HOMOGENEOUS SOIL

George E. Mylonakis[1], Emmanouil Rovithis[2], Haralambos Parashakis[3]

[1]Department of Civil Engineering, University of Patras
Rio, Greece, 26500
e-mail: mylo@upatras.gr

[2] Institute of Engineering Seismology and Earthquake Engineering (ITSAK)
Thessaloniki, Greece, 55102
rovithis@itsak.gr

[3] Department of Civil Works, Technological Educational Institute of Crete
Heraclion, Greece
parashaki_m@hotmail.com

Keywords: Seismic response, wave propagation, inhomogeneous soil, transfer function, equivalent homogeneous soil

Abstract. *Equivalent homogeneous soils are investigated as simplified approximations of continuously inhomogeneous soils. The examined system comprises of an inhomogeneous surface layer over a homogeneous one of higher stiffness. Five alternative definitions are adopted for the representative shear wave velocity (V_{hom}) in the inhomogeneous layer: (i) V_{hom1} at the base of the inhomogeneous layer (ii) V_{hom2} in the middle of the inhomogeneous layer (iii) V_{hom3} equal to the mean shear wave velocity within the inhomogeneous layer (iv) V_{hom4} providing equal travel time from base to surface between homogeneous and inhomogeneous soil and (v) V_{hom5} corresponding to an equivalent homogeneous soil having the same fundamental frequency as the inhomogeneous profile. Seismic response between inhomogeneous and equivalent homogeneous soils is compared by means of exact analytical solutions for single- and two-layer inhomogeneous soils. Fundamental frequencies and resonant peak amplitudes are examined, as affected by salient model parameters such as inhomogeneity factor, surface-to-base shear wave velocity ratio in the inhomogeneous layer, shear wave velocity contrast between the inhomogeneous and the homogeneous layer and relative layer thickness. It is observed that resonant frequencies of a smoothly-to-moderately inhomogeneous soil may be adequately captured by an equivalent homogeneous soil of either equal shear wave propagation velocity at the mid depth of the inhomogeneous layer, or of equal mean shear wave velocity within the whole layer. On the contrary, resonant amplitudes of a moderately-to-strongly inhomogeneous soil may be significantly underestimated or overestimated when an equivalent homogeneous soil is adopted, especially at higher resonances. The response of inhomogeneous soils with vanishing shear wave velocity near soil surface is explored.*

1 INTRODUCTION

Based on a detailed in-situ investigation of dynamic properties of soft deposits, Towhata [1] demonstrated analytically that shear wave propagation velocity may vary continuously with depth even for complex stratifications involving different soil materials. Utilizing a one-dimensional model of inhomogeneous soils with zero or finite stiffness at the surface, the above author showed analytically the possibility of higher amounts of seismic energy reaching the ground surface with respect to soils with discontinuous variation in shear modulus. Continuously inhomogeneous soils have been studied for different types of soil inhomogeneity or of seismic waves in multiple directions providing closed-form solutions for natural frequencies, modal shapes and amplification functions. Following the early work of Ambrasseys [2] and Seed and Idriss [3], Dobry et al.[4] studied the dynamic response of inhomogeneous soils with shear wave propagation velocity of the form $V_s=c\,z^{\,n}$, z being depth and n a positive inhomogeneity coefficient, corresponding to zero shear modulus at ground surface. A special case of the above equation, corresponding to $n = 2/3$, was adopted by Travasarou and Gazetas [5] as part of an investigation of seismic response of soft marine clay sediments verifying analytically that exceedingly large amplification of seismic motion may occur on the free surface. The effect of rate and type of heterogeneity on the seismic response of heterogeneous soils with shear wave velocity increasing from a non-zero value at the free surface has been examined by Ambraseys [2], Toki & Cherri [6], Schreyer [7] and Gazetas [8]. More recently, Parashakis [9] and Semblat and Pecker [10] extended the aforementioned models to obtain analytical solutions of the wave equation for a heterogeneous soil with shear wave velocity increasing with depth according to a generalized power law.

On the other hand, according to most modern seismic codes [11-13], site classification is based on the average shear wave propagation velocity within the top 30 metres (i.e. $V_{s,30}$) of the soil profile. The above regulations essentially refer to a homogeneous or inhomogeneous profile without strong gradients in shear wave propagation velocity with depth. However, in case of a moderately-to-strongly inhomogeneous soil, the choice of a pertinent, “representative” shear wave velocity is not straightforward especially when thick and soft soil deposits are encountered. In this case, conventional analyses based on discretizing soil in a multi-layer system with constant properties within each layer, may underestimate soil amplification with respect to the actual response of a continuously inhomogeneous medium, depending primarily on frequency content of input motion [1].

In light of the above considerations, “equivalent” homogenous soils are investigated as simplified approximations of continuously inhomogeneous soils. The investigation focuses on layered inhomogeneous soils as an extension of a previous research effort by the authors referring to single-layer systems [14]. The examined system comprises of a surficial inhomogeneous zone followed by a homogeneous layer on rigid base. A generalized parabolic function is adopted to describe the shear wave propagation velocity in the inhomogeneous layer, allowing modeling of inhomogeneous soils having vanishing values of shear modulus at ground surface. The problem is treated analytically by implementing closed-form solutions derived both for single- and two-layer inhomogeneous soil deposits in terms of base-to-surface transfer functions [9, 15, 16]. Seismic response between inhomogeneous and equivalent homogeneous soils is compared by means of fundamental frequencies and resonant amplitudes ratios, as affected by governing model parameters such as layer thickness, surface-to-base shear wave velocity ratio in the inhomogeneous layer, impedance contrast between surface and base layer and rate of inhomogeneity. The dependence of near-surface shear strains for inhomogeneous soils with very small surface-to-base shear wave velocity ratios is investigated by means of asymptotic analyses.

2 PROBLEM DEFINITION

A continuously inhomogeneous viscoelastic soil zone of thickness H over a rigid base (Fig.1a) is considered as a basis of the layered inhomogeneous soil examined. Soil mass density, ρ, and hysteretic damping ratio, ξ, are considered constant with depth. Shear wave propagation velocity is assumed to increase with depth according to the generalized power law function:

$$V_s(z) = V_H\left[b + (1-b)\frac{z}{H}\right]^n \tag{1}$$

where b is defined as a function of the shear wave velocity at the surface (V_o) and base (V_H) of the inhomogeneous soil layer [i.e. $b=(V_o/V_H)^{1/n}$], n is a dimensionless inhomogeneity factor varying in the common range of 0 to 1 ([9], [17], [18]) and z stands for the vertical coordinate (depth) measured from ground surface. For small values of the inhomogeneity factor n, Eq.1 simplifies to a uniform distribution while values of n close to unity correspond to linear increase in shear wave velocity with depth.

The single-layer system is extended to account for the presence of an underlying homogeneous layer of thickness (h_b) and shear wave propagation velocity (V_b) forming a generalized two-layer inhomogeneous soil with bounded shear wave velocity at large depths (Fig.1b). In this manner, a wide set of soil types can be modeled encompassing different soil properties between the inhomogeneous and the homogeneous layer.

Seismic response of layered inhomogeneous soils is compared to different equivalent homogenous cases. The latter are defined through a representative shear wave velocity V_{hom} in the inhomogeneous layer using the following definitions [14]:

• V_{hom1}, equal to the shear wave propagation velocity V_H at the base of the inhomogeneous layer:

$$V_{\mathrm{hom1}} = V_H \tag{2}$$

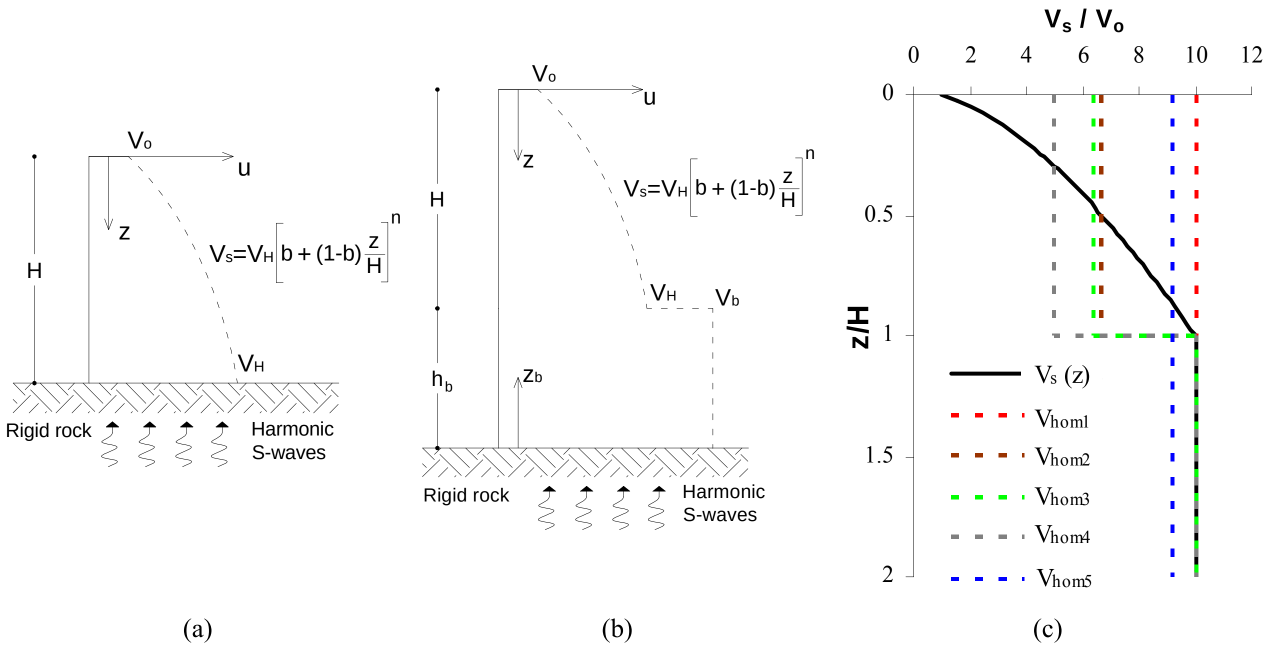

Figure 1. (a) Single inhomogeneous layer over rigid rock (b) Inhomogeneous surface layer over a homogeneous layer (c) Comparison of a two-layer inhomogeneous soil to the shear wave velocities of five equivalent homogeneous profiles (V_o/V_H = 0.1, V_b/V_H=1, h_b/H=1, n=0.6)

representing an always-stiffer soil with respect to the actual one

• V_{hom2}, equal to the shear wave propagation velocity at the mid depth of the inhomogeneous layer [8]:

$$V_{\text{hom}2} = V_s(H/2) \tag{3}$$

pertaining to an elementary yet potentially useful solution.

• V_{hom3}, equal to the mean shear wave propagation velocity within the inhomogeneous layer:

$$V_{\text{hom}3} = \frac{1}{H}\int_O^H V_s(z)dz \tag{4}$$

where $V_s(z)$ is given by Eq.1.

• V_{hom4}, providing equal base to surface travel times between homogeneous and inhomogeneous soil [19]

$$V_{\text{hom}4} = H\left[\int_O^H \frac{1}{V_s(z)}dz\right]^{-1} \tag{5}$$

• V_{hom5}, corresponding to an equivalent homogeneous soil having the same fundamental frequency as the inhomogeneous profile

$$V_{\text{hom}5} = f_{1In\text{hom}}4H \tag{6}$$

Alternatively, V_{hom5} may be viewed as the shear wave propagation velocity in the inhomogeneous soil corresponding to the "equivalent" depth (z_{eq}) proposed by Dobry et al. [20]. The two-layer equivalent homogeneous soils are compared to the inhomogeneous case in Fig. 1c based on the above V_{hom} profiles and the generalized parabola $V_s(z)$ in Eq.1. In this graph, the model parameters n, V_o/V_H, V_b/V_H and h_b/H were selected at 0.6, 0.1, 1 and 1 respectively. The deviation observed among the shear wave velocity profiles of the equivalent homogeneous soils is due to the small surface-to-base shear wave velocity ratio (V_o/V_H). Naturally, larger V_o/V_H ratios correspond to a smoother variation of $V_s(z)$ leading to comparable V_{hom} profiles.

The input motion is imposed at the base of the system in the form of a harmonic horizontal displacement, $u=u_o exp(i\omega t)$, ω being the cyclic excitation frequency, generating vertically propagating S waves.

3 ANALYTICAL SOLUTION

Starting from the following ordinary differential equation:

$$\frac{d}{dz}\left[G(z)\frac{du}{dz}\right] + \rho\omega^2 u = 0 \tag{7}$$

which describes one-dimensional shear waves under harmonic oscillations in a soil layer with constant mass density ρ and variable shear modulus $G(z)$, it can be shown [14-16] that the displacement field of an inhomogeneous layer such as that described in Eq.1 is given by:

$$u(z) = \frac{C_1\left(b+q\dfrac{z}{z_r}\right)^{\mu}}{N_{\nu+1}\left(\lambda b^{\ell/2}\right)}\left\{J_\nu\left[\lambda\left(b+q\frac{z}{z_r}\right)^{\ell/2}\right]N_{\nu+1}\left[\lambda b^{\ell/2}\right] - J_{\nu+1}\left[\lambda b^{\ell/2}\right]N_\nu\left[\lambda\left(b+q\frac{z}{z_r}\right)^{\ell/2}\right]\right\} \tag{8}$$

where $q=1-b$, C_1 is an integration constant determined from the boundary conditions, $J_v(\,)$ and $N_v(\,)$ denote the Bessel functions of the first and second kind and order v, respectively, and $\ell = 2(1-n)$ is a dimensionless parameter representing the step of the associated power series solutions [21]. Parameters μ and v are obtained from the asymptotic convergence of the solution close to zero, as $\mu=(1-2n)/2$ and $v=(2n-1)/2(1-n)$, while the asymptotic behavior of the solution at infinity requires $\lambda = 2k_r z_r / \ell q$, k_r and z_r being a reference wave number ($=\omega/V_r$) and a reference depth.

By definition, the base-to-surface transfer function is expressed as [22, 23]:

$$F(\omega) \equiv \frac{u(0)}{u(H)} \tag{9}$$

where $u(0)$ and $u(H)$ stand for the horizontal soil displacement at the free surface and the base of the single-layer system, computed from Eq.8 by setting $z=0$ and $z=H$, respectively. After some algebra, Eq.9 yields:

$$F(\omega) = \frac{2b^{\mu-\ell/2}}{\pi\lambda}\left[J_{v+1}\left(\lambda b^{\ell/2}\right)N_v\left(\lambda\right) - J_v\left(\lambda\right)N_{v+1}\left(\lambda b^{\ell/2}\right)\right]^{-1} \tag{10}$$

Referring to the two-layer inhomogeneous soil, the base-to-surface transfer function is defined in the same spirit as in Eq. 9:

$$F(\omega) \equiv \frac{u(0)}{u_b(0)} \tag{11}$$

where $u_b(0)$ denotes the soil displacement at base level ($z_b=0$). The response of the underlying homogeneous layer is given by the function [22, 23]:

$$u_b(z_b) = A_1 \sin(k_b z_b) + A_2 \cos(k_b z_b) \tag{12}$$

where A_1 and A_2 represent the amplitudes of the waves travelling upward and downward in the layer, respectively, and k_b ($=\omega / V_b$) is the corresponding wave number. Upon enforcing the continuity of shear stresses and displacements at the base of the system and the interface of the surface and the base layer:

$$\begin{aligned} u_b(0) &= u_o \\ u_b(h_b) &= u(H) \\ \tau_b(h_b) &= -\tau(H) \end{aligned} \tag{13}$$

by means of Eqs. 8 and 12 yield the following solution for the base-to-surface transfer function in Eq.11:

$$\begin{aligned} F(\omega) = \left[\frac{2(1-n)(1-b)\rho_b}{\pi b^{1/2} H}\right] & \left\{k_H \rho_b \left[J_v\left(\theta_H\right)N_{v+1}\left(\theta_O\right) - J_{v+1}\left(\theta_O\right)N_v\left(\theta_H\right)\right]\cos(k_b h_b) - \right. \\ & \left. - k_b \rho_H \left[J_{v+1}\left(\theta_H\right)N_{v+1}\left(\theta_O\right) - J_{v+1}\left(\theta_O\right)N_{v+1}\left(\theta_H\right)\right]\sin(k_b h_b)\right\}^{-1} \end{aligned} \tag{14}$$

where $\theta_O = \lambda b^{\ell/2}, \theta_H = \lambda\left(b + qH / z_r\right)^{\ell/2}$; ρ_H and ρ_b stand for soil mass density of the inhomogeneous and the homogeneous layer, respectively, and k_H ($=\omega/V_H$), k_b ($=\omega/V_b$) the corresponding wave numbers. Material damping can be accounted for in the above solutions by replacing the real wave numbers with the complex counterparts $k_H^*\left(=\omega / V_H^*\right)$ and $k_b^*\left(=\omega / V_b^*\right)$, respectively. Further details on the analytical derivations of Eqs.10 and 14 can be found in references [9] and [14].

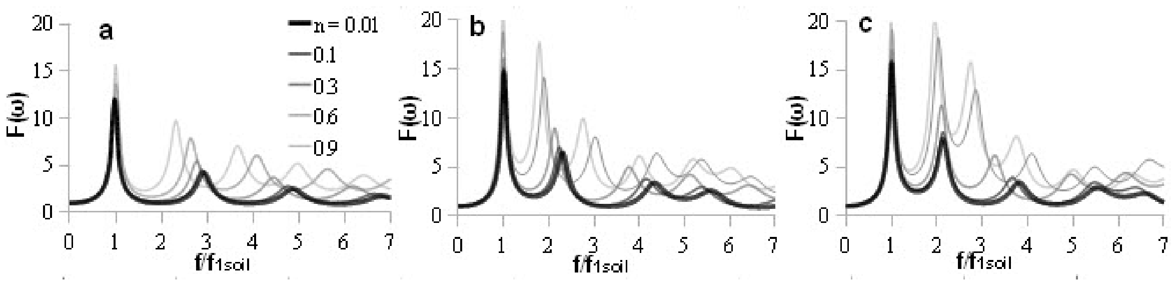

Figure 2. Base-to-surface transfer functions of two-layer inhomogeneous soil for (a) V_b/V_H=1 (b) V_b/V_H=2 (c) V_b/V_H=3. In all graphs V_o/V_H=0.1, h_b/H=2, ρ_b/ρ_H=1, ξ=0.05

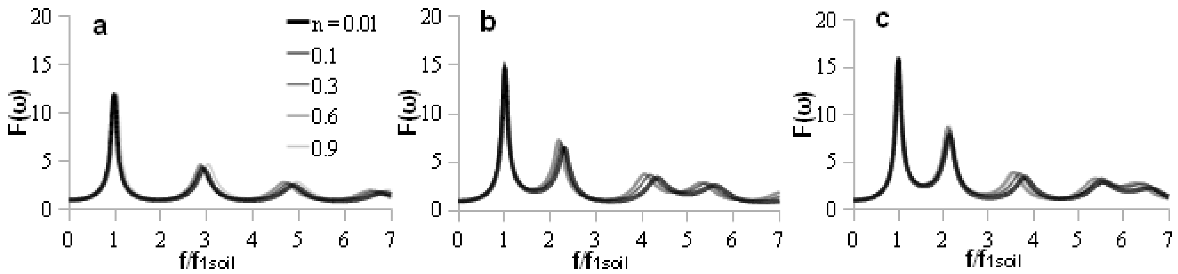

Figure 3. Base-to-surface transfer functions of two-layer inhomogeneous soil for (a) V_b/V_H=1 (b) V_b/V_H=2 (c) V_b/V_H=3. In all graphs V_o/V_H=0.75, h_b/H=2, ρ_b/ρ_H=1, ξ=0.05

4 HARMONIC RESPONSE OF THE TWO-LAYER INHOMOGENEOUS SOIL

Analytical base-to-surface transfer functions obtained for the two-layer inhomogeneous soil by means of Eq.14 are plotted in Fig.2 referring to the combined effect of inhomogeneity factor n and shear wave velocity ratio (V_b/V_H) at the interface of the inhomogeneous and the homogeneous layer, with the abscissa normalized by the fundamental frequency of the soil, f_{1soil}. Shear wave velocity ratio V_o/V_H at the surface and the base of the inhomogeneous layer was set at 0.1 corresponding to strong gradients in shear wave velocity with depth. For this range of soil inhomogeneity, increasing the inhomogeneity factor n amplifies response and shifts higher mode resonances to lower frequencies [15, 16], especially for large shear wave velocity contrast (i.e large V_b/V_H ratios) between the surface and the base layer (Fig.2c). Of particular interest is the strong amplification observed at higher modes, indicating reduced soil damping effects, contrary to the response of a piece-wise homogeneous two-layer soil, where the role of higher soil modes progressively diminishes.

On the contrary, for higher surface-to-base wave velocity ratio (V_o/V_H) corresponding to a mild variation of shear wave propagation velocity within the surface inhomogeneous layer, the harmonic response of the two-layer inhomogeneous soil resembles that of the homogeneous case (n=0.01). This is clearly demonstrated in Fig.3 where Eq.14 is computed for a V_o/V_H ratio of 0.75 providing comparable base-to-surface transfer functions. Note that the only difference between the results shown in Fig.2 and Fig.3 is the value of V_o/V_H ratio. The above behavior was found to exist regardless of thickness (h_b) of the underlying homogeneous layer. The effect of the latter is explored in Fig. 4 for a two-layer inhomogeneous soil described by three layer thickness values (h_b/H) and a V_o/V_H ratio of 0.1. It is observed that as the inhomogeneity factor n approaches 1, deeper soil deposits tend to respond at lower frequencies with larger peak amplitudes (Figs. 4b-4c). However, the effect of both relative layer thickness and inhomogeneity factor is minimized with increasing V_o/V_H ratio, as shown in Figure 5 where base-to-surface transfer functions are computed for a higher V_o/V_H ratio (0.75).

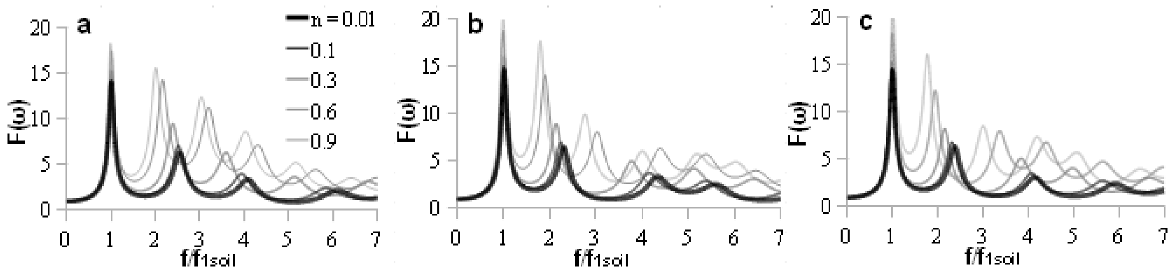

Figure 4. Base-to-surface transfer functions of two-layer inhomogeneous soil for (a) $h_b/H=1$ (b) $h_b/H=2$ (c) $h_b/H=3$. In all graphs $V_o/V_H=0.1$, $V_b/V_H=2$, $\rho_b/\rho_H=1$, $\xi=0.05$

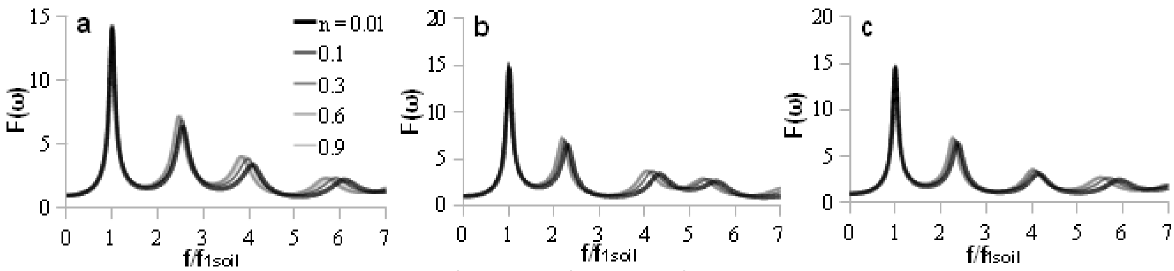

Figure 5. Base-to-surface transfer functions of two-layer inhomogeneous soil for (a) $h_b/H=1$ (b) $h_b/H=2$ (c) $h_b/H=3$. In all graphs $V_o/V_H=0.75$, $V_b/V_H=2$, $\rho_b/\rho_H=1$, $\xi=0.05$

Conclusively, the harmonic response of the generalized two-layer inhomogeneous system under investigation is primarily controlled by V_o/V_H ratio corresponding to a critical measure of soil inhomogeneity. Similar observations have been reported by Rovithis et al [14] referring to the single-layer system (Fig.1a) based on Eq.10.

5 COMPARISON WITH "EQUIVALENT" HOMOGENEOUS SOIL

The response of the inhomogeneous two-layer system was compared to the equivalent homogeneous soils in terms of natural frequencies and resonant peak amplitudes. Recall in this regard that for a homogeneous viscoelastic two-layer soil, base-to-surface transfer function is given by the expression [22]:

$$F(\omega)=\left[\cos\left(q_{\hom}H\right)\cos\left(q_b h_b\right)-I_R\sin\left(q_{\hom}H\right)\sin\left(q_b h_b\right)\right]^{-1} \tag{15}$$

where $q_{hom}(=\omega/V_{hom})$ and $q_b(=\omega/V_b)$ stand for the wave numbers of the surface and the base layer, respectively, and I_R $(=\rho_b V_b/\rho_H V_{hom})$ is the impedance contrast between the two layers.

Resonant frequencies and peak amplitudes ratios between equivalent homogeneous and inhomogeneous soils were obtained by means of Eqs.14 and 15 for each representative shear wave propagation velocity V_{hom} according to Eqs 2-6. For the purpose of this parametric investigation both (V_b/V_H) and (h_b/H) ratios were consecutively set at 1, 2 and 3.

Ratio of the fundamental frequency of the equivalent homogeneous profile to the first natural frequency of the inhomogeneous soil (f_{1hom}/f_{1Inhom}) is plotted in Fig.6 against the inhomogeneity factor n for the examined V_o/V_H ratios. In all graphs, V_{hom} is defined by Eq.2 (i.e $V_{hom}=V_H$) and V_b/V_H is equal to 3. Each plot corresponds to a different h_b/H ratio; 1, 2 and 3 respectively. Naturally, the use of V_H as the equivalent shear wave propagation velocity of the inhomogeneous layer results in a stiffer soil, leading to frequency ratios above unity especially for moderately-to-strongly inhomogeneous soils (small V_o/V_H ratios and large inhomogeneity factors n). However, larger h_b/H values referring to deeper soil deposits lead to lower f_{1hom}/f_{1Inhom} ratios (Fig. 6b-6c), indicating a prevailing contribution of the underlain homogeneous layer to the overall response.

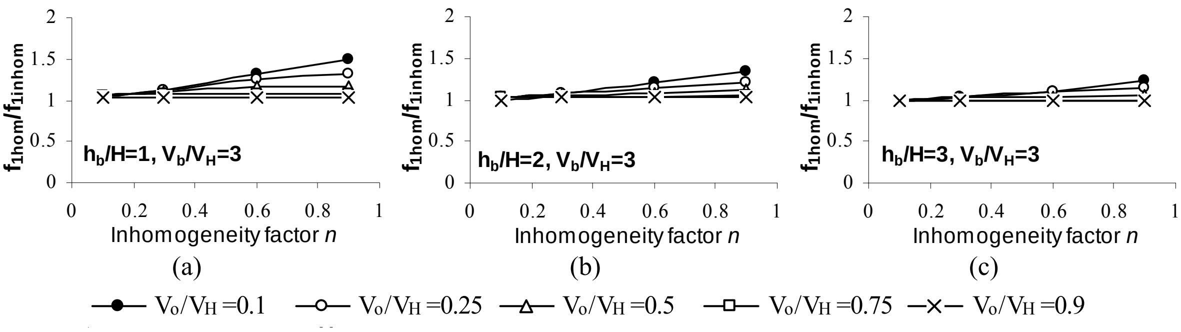

Figure 6. Ratio of fundamental frequency of the equivalent homogeneous soil (f_{1hom}) to first natural frequency of the inhomogeneous soil (f_{1Inhom}) as function of inhomogeneity factor n: (a) $h_b/H=1$ (b) $h_b/H=2$ (c) $h_b/H=3$. In all plots, $\boldsymbol{V_{hom}=V_{hom1}}$, $V_b/V_H=3$, $\rho_b/\rho_H=1$, $\xi = 0.05$

Resonant frequencies of a smoothly-to-moderately inhomogeneous soil (i.e $V_o/V_H > 0.25$) are well-predicted by an equivalent homogeneous soil with a surface layer of either equal shear wave propagation velocity at the mid depth of the inhomogeneous layer (Eq.3), or of equal mean wave propagation velocity within the whole layer (Eq.4). Figures 7a-7c show f_{1hom}/f_{1Inhom} ratios computed by means of Eq.3 for various h_b/H and V_b/V_H ratios. Similar results obtained from Eq.4 are plotted in Figs. 7d-7f referring to the second natural frequency of the deposit (i.e. f_{2hom}/f_{2Inhom}). In all cases, frequency ratios are close to unity indicating a good approximation of the resonant frequencies of the inhomogeneous soil. The latter should be correlated with the continuous nature of the generalized parabola adopted to describe the wave propagation velocity in the inhomogeneous layer. Insignificant deviations from the exact solution are observed for low V_o/V_H ratios ($V_o/V_H=0.1$) leading to slightly overestimated frequencies with increasing inhomogeneity factor n, especially at high resonances (Figs. 7d-7f).

On the contrary, when the equivalent homogeneous soil is defined through Eq.5 ($V_{hom}=V_{hom4}$) the actual fundamental frequency of the inhomogeneous deposit is underestimated. Fig.8 shows the corresponding $f_{1hom}/f_{11inhom}$ ratios for three values of shear wave velocity contrast at the interface of the inhomogeneous and the homogeneous layer. It is observed

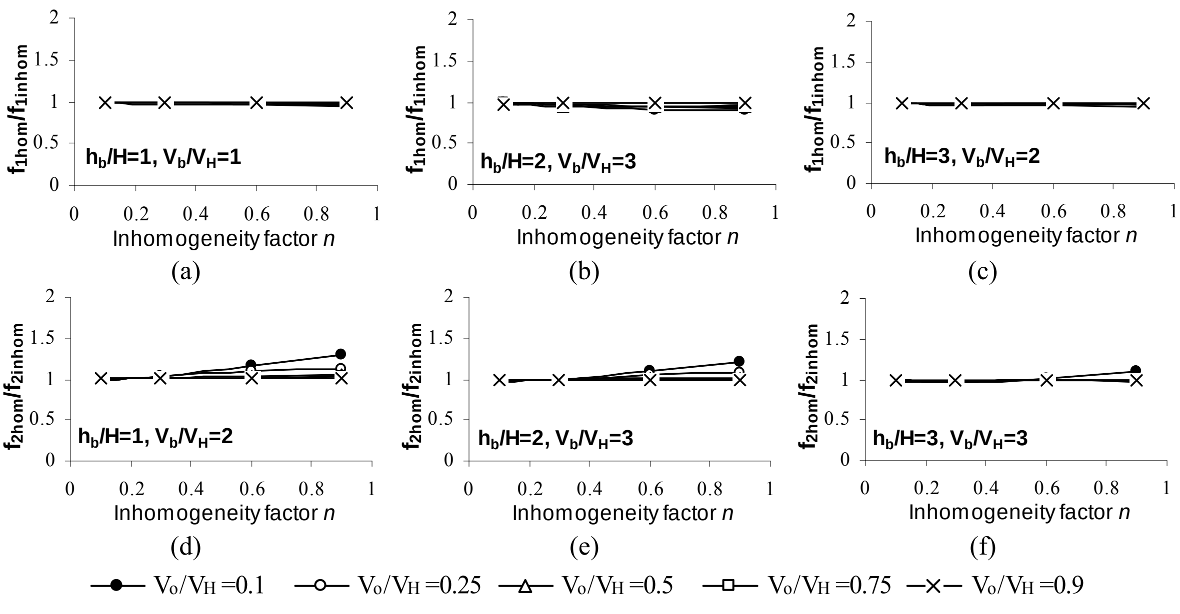

Figure 7. (a-c) f_{1hom}/f_{1Inhom} ratios as function of inhomogeneity factor n: $\boldsymbol{V_{hom}=V_{hom2}}$ (d-f) f_{2hom}/f_{2Inhom} ratios as function of inhomogeneity factor n: $\boldsymbol{V_{hom}=V_{hom3}}$. In all plots $\rho_b/\rho_H=1$, $\xi = 0.05$

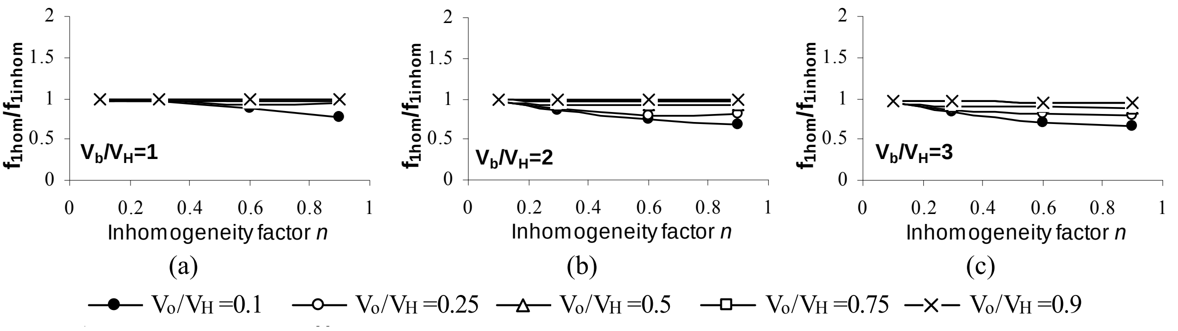

Figure 8. f_{1hom}/f_{1Inhom} ratios as function of inhomogeneity factor n: (a) V_b/V_H=1 (b) V_b/V_H=2 (c) V_b/V_H=3. In all plots, $\boldsymbol{V_{hom}}$=$\boldsymbol{V_{hom4}}$, h_b/H=1, ρ_b/ρ_H=1, ξ = 0.05

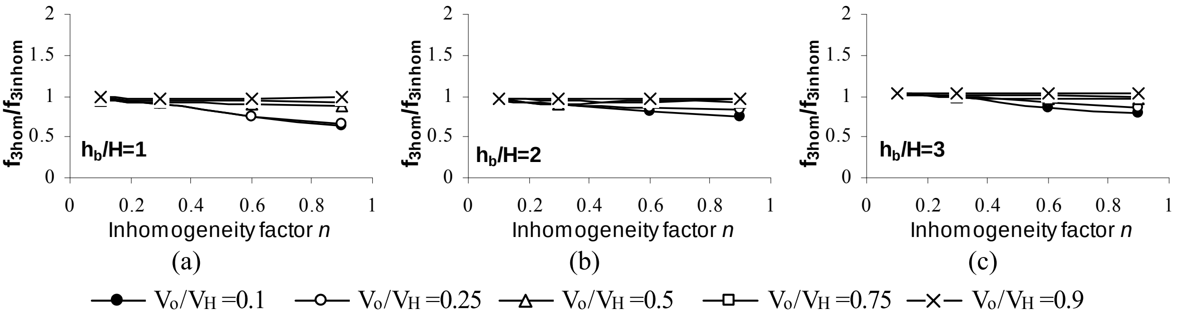

Figure 9. f_{3hom}/f_{3nhom} ratios as function of inhomogeneity factor n: (a) V_b/V_H=1 (b) V_b/V_H=2 (c) V_b/V_H=3. In all plots, $\boldsymbol{V_{hom}}$=$\boldsymbol{V_{hom5}}$, h_b/H=2, ρ_b/ρ_H=1, ξ = 0.05

that indeed the fundamental frequency ratio is lower than 1 especially for large V_b/V_H ratios (Fig.8b-8c). Similar trends can be seen in Fig. 9 where f_{3hom}/f_{3Inhom} ratios referring to the third natural frequency of a two-layer homogeneous soil having the same fundamental frequency as the inhomogeneous one are plotted by means of Eq.6 (V_{hom}=V_{hom5}).

Further comparisons between continuously inhomogeneous and equivalent homogeneous soils were performed, relating peak resonant amplitudes of base-to-surface transfer functions. Resonant amplitude ratios (A_{hom}/A_{inhom}) defined in the same spirit as the resonant frequency ratios are plotted in Fig. 10 corresponding to the first natural frequency (i.e. A_{1hom}/A_{1Inhom}) of a two-layer system with (V_b/V_H) and (h_b/H) ratio of 3 and 2, respectively. Each plot in Fig.10 corresponds to a different equivalent homogeneous soil based on Eqs 2-6. Linear hysteretic damping was taken at 0.05 for both inhomogeneous and equivalent homogeneous cases. The same results are shown in Fig.11 for the second natural frequency of the deposit (A_{2hom}/A_{2Inhom}). It is observed that the replacement of a continuously inhomogeneous soil with an equivalent homogeneous may lead to substantial overestimated or underestimated resonant amplitudes depending on the value of V_{hom}. Note, for example, that for a strongly inhomogeneous soil (i.e. V_o/V_H = 0.1, n = 0.9), A_{1hom}/A_{1inhom} ratio based on Eq.5 can be about 0.6 (Fig.10d), which suggests an underestimation of the actual resonant amplitude while Eq.6 yields a value of 1.7 (Fig.10e) overestimating strongly the amplitude of the fundamental resonance. The above deviation becomes larger at higher resonances. For example, A_{2hom}/A_{2Inhom} ratios may vary in the range 0.4 (Fig.11a) to 4.5 (Fig.11e) depending on the approach followed to define V_{hom}. Thereby, the replacement of a continuously inhomogeneous soil layer with an equivalent homogeneous one in terms of peak resonant amplitudes may be valid only for a sufficiently smooth variation of shear wave velocity with depth. The latter was observed independently of V_b/V_H ratio, h_b/H ratio and V_{hom}.

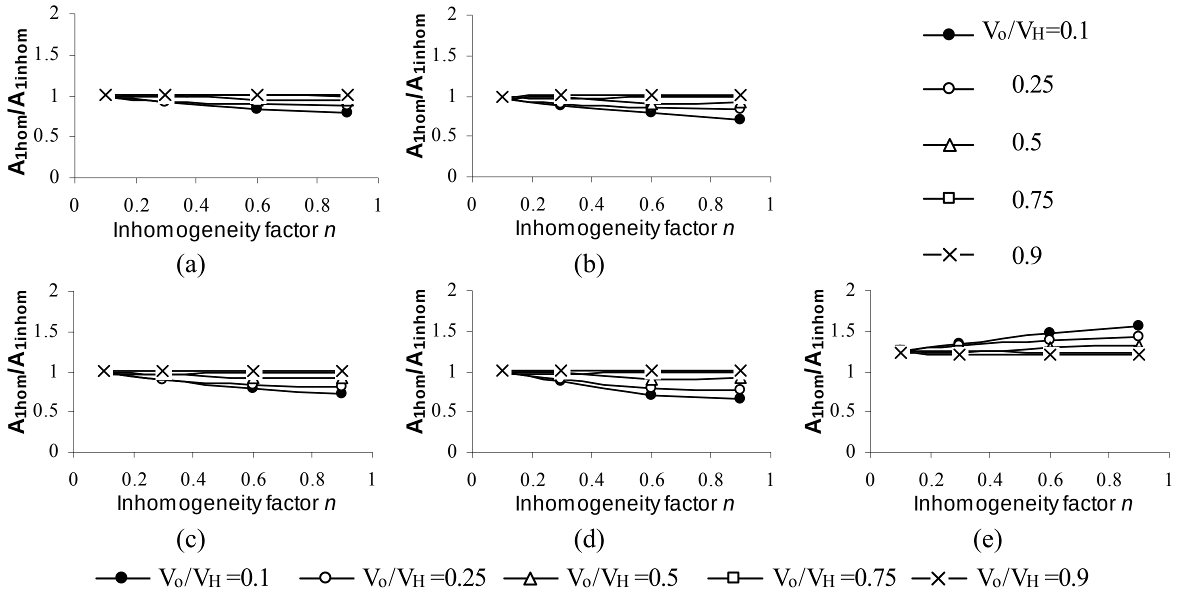

Figure 10. Resonant amplitude ratios (A_{1hom}/A_{11nhom}) corresponding to the first natural frequency of the system as function of inhomogeneity factor *n*: (a) $V_{hom}=V_{hom1}$ (b) $V_{hom}=V_{hom2}$ (c) $V_{hom}=V_{hom3}$ (d) $V_{hom}=V_{hom4}$ (e) $V_{hom}=V_{hom5}$. In all plots, $V_b/V_H=3$, $h_b/H=2$, $\rho_b/\rho_H=1$, $\xi = 0.05$

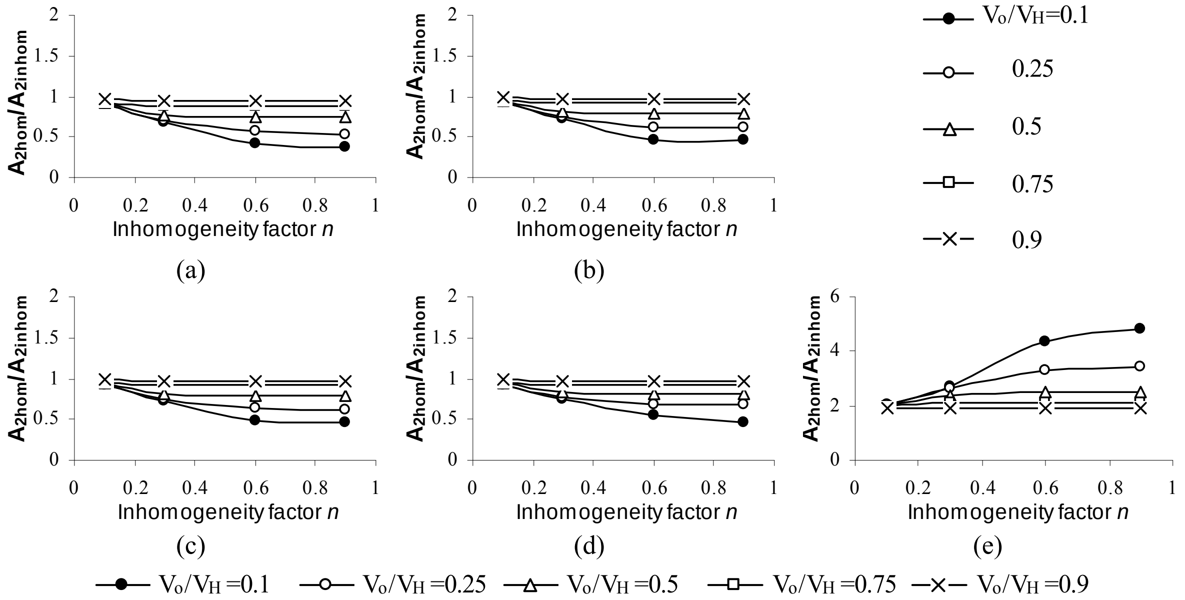

Figure 11. Resonant amplitude ratios (A_{2hom}/A_{21nhom}) corresponding to the second natural frequency of the system as function of inhomogeneity factor *n*: (a) $V_{hom}=V_{hom1}$ (b) $V_{hom}=V_{hom2}$ (c) $V_{hom}=V_{hom3}$ (d) $V_{hom}=V_{hom4}$ (e) $V_{hom}=V_{hom5}$. In all plots, $V_b/V_H=3$, $h_b/H=2$, $\rho_b/\rho_H=1$, $\xi = 0.05$

6 INHOMOGENEOUS SOIL WITH VANISHING STIFFNESS AT SOIL SURFACE

For an inhomogeneous soil layer having zero stiffness at the free surface (b=0), it can be shown [14-16] that shear strain γ(z) in the soil is given by:

$$\gamma(z) = -\frac{\omega^2 H}{V_H^2}\left(\frac{z}{H}\right)^{-2n} C_1 z^{1/2} J_\nu\left(\lambda b^{\ell/2}\right) \tag{16}$$

where $\nu=1/2(1-n)$, $\ell = 2(1-n)$ and $\mu=1/2$.

For values of z close to zero the term $z^{1/2}J_\nu\left(\lambda b^{\ell/2}\right)$ becomes asymptotically equal to [24]:

$$z^{1/2}J_\nu\left(\lambda b^{\ell/2}\right) \sim z^{1/2}\frac{1}{\Gamma(1+\nu)}\left(\frac{\lambda z^{\ell/2}}{2}\right)^\nu \tag{17}$$

where Γ( ) is the Gamma function. Accordingly the solution in Eq.16 takes the form:

$$\gamma(z) = -\frac{\omega^2 H}{V_H^2}\left(\frac{z}{H}\right)^{-2n} C_1 z^{1/2}\frac{1}{\Gamma(1+\nu)}\left(\frac{\lambda z^{\ell/2}}{2}\right)^\nu \tag{18}$$

which for small z's yields the expression (recall that $\nu\ell = 2(1-n)/2(1-n) = 1$ in this solution):

$$z^{-2n+1/2+\nu\ell/2} = z^{-2n+1/2+1/2} = z^{1-2n} \tag{19}$$

indicating that for n>1/2 the exponent (1-2n) becomes negative and, thereby, the magnitude of shear strain becomes infinite, regardless of frequency and excitation amplitude and despite the fact that the corresponding shear stress is zero. On the contrary, for n<1/2 the exponent is positive and shear strain is zero at the ground surface. Therefore, strain amplitude at the surface can be either zero or infinite, depending on the value of the inhomogeneity factor, but never finite. These findings are in agreement with those obtained in [5] for n=2/3.

However, as real soils inherently possess a finite amount of stiffness at the surface, the behavior of the solution at very small V_o/V_H ratios is investigated in Fig.12, for a single inhomogeneous layer described by four inhomogeneity factors. Damping ratio, ξ, was set at 0.05 in this graph. Strong amplification is evident at higher mode resonances, as V_o/V_H and n tend to 0 and 1, respectively. This suggests that strong amplification will develop even for finite surface stiffness (under zero shear strain), which will merely get maximized at the theoretical

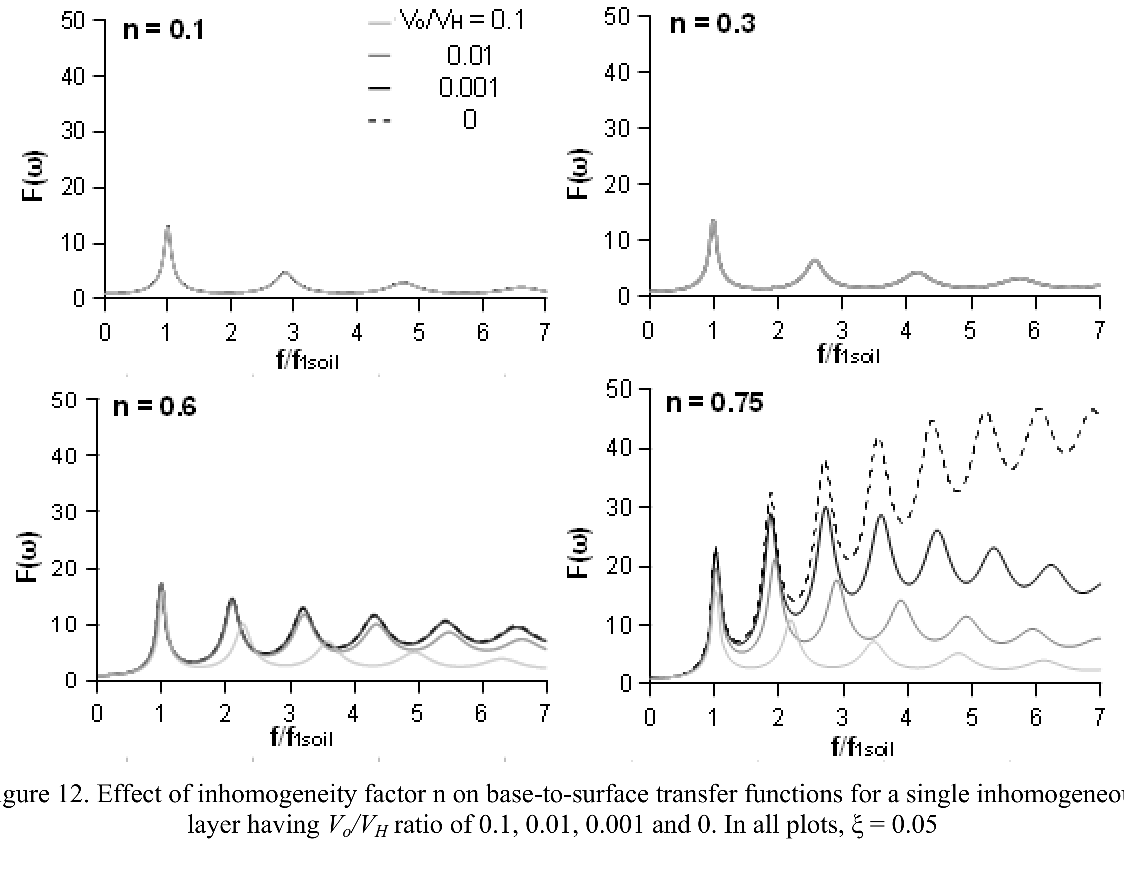

Figure 12. Effect of inhomogeneity factor n on base-to-surface transfer functions for a single inhomogeneous layer having V_o/V_H ratio of 0.1, 0.01, 0.001 and 0. In all plots, ξ = 0.05

limit V_o/V_H=0. An explanation is that the strong amplification is associated with transition phenomena (i.e., accumulation of wave energy in areas of progressively smaller elastic modulus near the surface leading to an increase in wave amplitude) as opposed to reflection phenomena associated with development of resonance in the layer.

7 CONCLUSIONS

Equivalent homogeneous soils were examined as simplified approximations of layered continuously inhomogeneous soils implementing alternative definitions for the representative shear wave propagation velocity. The investigation focused on resonant frequencies and peak resonant amplitudes, as affected by salient model parameters. The special case of an inhomogeneous soil having zero stiffness at the surface was explored as to the variation of shear strain with depth.

The harmonic response of a two-layer strongly inhomogeneous soil (V_o/V_H = 0.1) is amplified with increasing inhomogeneity factor *n* shifting higher mode resonance to lower frequencies. The above effect is more pronounced for deeper soil deposits with large shear wave velocity contrast between the surface and the base layer. Comparison of resonant frequencies and amplitudes between two-layer inhomogeneous and equivalent homogeneous soils revealed that the response of a smoothly-to-moderately inhomogeneous soil in terms of resonant frequencies may be adequately captured by an equivalent homogeneous soil with a surface layer of either equal shear wave propagation velocity at the mid depth of the inhomogeneous deposit, or of equal mean wave propagation velocity within the whole layer. For moderately-to-strongly inhomogeneous soil (V_o/V_H<0.5 and inhomogeneity factor n>0.3) the above equivalent homogeneous approximations remain a promising solution given that deep soil deposits (h_b/H>2) are encountered. On the contrary, resonant amplitudes of a moderately-to-strongly inhomogeneous soil may be significantly overestimated or underestimated when an equivalent homogeneous soil approach is adopted, especially at higher resonances. For the special case of inhomogeneous soils with vanishing shear wave velocity at the free surface (V_o/V_H = 0), near-surface shear strain may be either zero (for n < 0.5) or infinite (for n >0.5) but never finite. Strong amplification will develop even for finite surface stiffness (under zero shear strain) which will get maximized at the theoretical limit V_o/V_H = 0. From a practical viewpoint, a V_o/V_H ratio of less than 0.1 combined with an inhomogeneity factor *n* of over 0.5 will suffice to trigger this effect.

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