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A SINGLE MODE ENERGY-BASED PUSHOVER PROCEDURE

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Abstract. *In this paper a new energy-based pushover procedure is presented in order to achieve an approximate estimation of structural performance under strong earthquakes. The steps of the proposed methodology are quite similar to those of the well-known displacement modification method. However, the determination of the characteristics of the equivalent single degree of freedom (E-SDOF) system is based on a different concept. Its main idea is to determine the E-SDOF system by equating the external work of the lateral loads acting on the multi degree of freedom (MDOF) system under consideration to the strain energy of the E-SDOF system. After a brief outline of the theoretical background, the sequence of the steps needed for the implementation of the proposed methodology along with the necessary equations are systematically presented. Finally, the accuracy of the proposed method is evaluated by an extensive parametric study which shows that, in general, it provides better results compared to those produced by other similar procedures.*

1 INTRODUCTION

In the last decades many research efforts have focused on developing simplified procedures for the approximate estimation of the inelastic performance of buildings under seismic excitation, in order to avoid the significant computational cost and the various inherent disadvantages of an accurate inelastic dynamic analysis. As a result of these efforts the idea of pushover analysis has been born. In the last decade a variety of more or less similar inelastic static pushover procedures have been developed, some of which have been already adopted by several seismic codes ([1], [2], [3], etc.).

Static Pushover Analysis, or Nonlinear Static Procedure (NSP) as it is denoted in seismic codes, seems to be a useful tool for engineering practice. Nevertheless, as it has already been stressed by many researchers (e.g., [4]), this procedure involves many shortcomings and can provide reasonable results only for low and medium rise planar systems. This is due to the fact that the determination of the structure's response is based on the assumption that the dynamic behavior depends only on a single elastic vibration mode. In addition, this elastic mode is supposed to remain constant despite the successive formation of plastic hinges during the seismic excitation. Also, the choice of roof displacement - as characteristic response quantity for the construction of the capacity curve - instead of any other displacement is arbitrary and it is doubtful whether the base shear-roof displacement curve is the most meaningful index of the nonlinear response of a structure, especially for irregular in height and asymmetric in plan systems. To overcome these shortcomings various modified pushover procedures have been proposed in the recent past (e.g., [5], [6], [7], [8]). Some of them [6], [7], [8] are based on the energy equivalence between the multi degree of freedom (MDOF) and the equivalent single degree of freedom (E-SDOF) systems (energy-based procedures). According to energy-based procedures, the strain energy of the structure or, equivalently, the work done by the external loads is considered to be the most representative index of its nonlinear response.

Hernadez-Montes et al. [6] suggested an energy-based formulation of pushover analysis which was motivated by the reversals in the higher mode capacity curves that were observed when applying Modal Pushover Analysis [5]. This method uses an energy-based displacement derived from the work done by the lateral loads to establish the capacity curve, instead of using the roof displacement. In each step of the pushover procedure, the work done by lateral loads associated with each mode is computed using an incremental formulation. The corresponding increment in the energy-based displacement is calculated by dividing the increment of work at each step by the base shear at that step. The incremental displacements are accumulated to obtain the energy-based displacement of the E-SDOF system. Thus, a modified capacity curve is plotted for each mode, which is used in lieu of the conventional pushover curve. These modified curves resemble traditional first mode pushover curves and do not exhibit the anomalies observed in higher mode curves.

Parducci et al. [7] proposed the determination of an equivalent energy-based displacement of the E-SDOF system. This displacement does not correspond to any actual point of the structural model, but it is a virtual value equalizing the work done by the lateral loads to the strain energy of the E-SDOF system. Then, the strain energy versus equivalent displacement diagram is plotted and - in combination with a pseudo-energy response spectrum - the performance point is determined. This point is used to estimate the response of the structure.

Earlier, Oliveto et al. [8] determined a displacement parameter based on power equivalence (which in finite terms translates into energy equivalence) between MDOF and E-SDOF systems. The properties of the E-SDOF system are then calculated as function of this energy-based displacement. Recently, this procedure was extended to include Modal Pushover Analysis [9].

The objective of the present paper is the formulation and preliminary evaluation of a new energy-based Nonlinear Static Procedure (NSP) for the approximate estimation of the seismic response of structures. The proposed procedure uses the strain energy which is considered as a more reliable index of the structural response than the base shear. This is due to the fact that the strain energy depends on the values of all forces acting to the structure as well as on the values of the displacements of all the system's degrees of freedom. The steps of the proposed methodology are quite similar to those of the well-known displacement modification method [1], [3]. However, the determination of the characteristics of the E-SDOF system is based on a different concept. Specifically, the definition of the E-SDOF system is based on the equalization of the external work of the lateral loads acting on the MDOF system under consideration to the strain energy of the E-SDOF system. In contrast to other energy-based procedures, the energy equivalence is used to derive a modified resisting force of the E-SDOF system, instead of an energy-based displacement. Thus, a modified capacity curve is plotted. As a first step, the procedure is formulated in a manner that takes into account only the predominant vibration mode and in its current form it can be rigorously applied to low and medium rise planar systems.

Firstly, the theoretical background and the assumptions of the proposed methodology are presented and briefly discussed. Secondly, both the sequence of the steps needed for the implementation of the proposed methodology along with the necessary equations are systematically presented. Finally, the accuracy of the proposed methodology is evaluated by an extensive parametric study. The paper closes with comments on results and conclusions. The whole investigation proved that the here proposed methodology gives, in general, better results as compared to other similar procedures.

2 THEORETICAL BACKGROUND

It is well known that the linear elastic response of a MDOF system can be decomposed to responses of SDOF systems, one for each elastic vibration mode (modal analysis). Although this concept lacks a theoretical basis in the inelastic range of behavior, it has been widely used by many researchers (e.g., [5]) in order to develop approximate, simplified nonlinear static procedures. It is obvious that this approach includes some fundamental assumptions. A major assumption is that the response of a MDOF system can be expressed as superposition of the responses of appropriate SDOF systems just like in the linear range. Of course, such an assumption violates the very logic of nonlinearity, as the superposition principle is not valid to nonlinear systems. However, it must be thought as a fundamental postulate, which constitutes the basis on which many simplified pushover procedures are built. Thus, each SDOF system corresponds to a vibration "mode" i with "modal" vector $\boldsymbol{\varphi}_i$ (the quotation marks indicate that the application of the superposition principle is not strictly valid). The displacements $\mathbf{u}_i$ and the inelastic resisting forces $\mathbf{F}_{si}$ are supposed to be proportional to $\boldsymbol{\varphi}_i$ and $\mathbf{M}\boldsymbol{\varphi}_i$, respectively (where $\mathbf{M}$ is the mass matrix of the system). Furthermore, "modal" vectors $\boldsymbol{\varphi}_i$ are supposed to be constant, despite the successive development of plastic hinges.

Taking into account the aforementioned assumptions and applying well-known principles of structural dynamics the following conclusion is derived [10]: the nonlinear response of a MDOF system with N degrees of freedom subjected to an horizontal earthquake ground motion $\ddot{u}_g(t)$ can be expressed as superposition of the responses of N E-SDOF systems, each one corresponding to a vibration "mode" having mass equal to the effective modal mass M_i^*, displacement D_i which depends on the roof displacement u_{Ni} and inelastic resisting force equal to the "modal" base shear parallel to the direction of excitation V_i. Furthermore, the external work of "modal" forces $\mathbf{F}_{si}$ on the differential displacements $d\mathbf{u}_i = \nu_i\, \boldsymbol{\varphi}_i\, dD_i$ (where ν_i is the modal participation factor of mode i) is equal to the work of the resisting force (or the strain energy) of the corresponding E-SDOF system for the differential displacement dD_i.

Some basic equations correlating the properties of the "modal" E-SDOF systems to the properties of the MDOF system are derived and summarized in Table 1. However, these equations are derived on the basis of the aforementioned assumptions and cannot be valid all together at the same time when a pushover analysis is conducted. Thus, Modal Pushover Analysis [5] leaves out the 3rd equation and uses the two others to establish the "modal" E-SDOF systems. The conventional procedures adopted by codes follow a similar approach with some additional assumptions. More specifically, they take into account only the predominant vibration mode and permit modifications to the corresponding mode shape vector. The existing energy-based single or multimodal procedures keep the last two equations and determine the E-SDOF systems' displacements on the basis of the energy equivalence between E-SDOF and MDOF system. On the contrary, the proposed method keeps the 1st and the 3rd equation and uses the energy equivalence to determine a modified resisting force of the E-SDOF systems.

MDOF system		E-SDOF systems
"modal" displacements $\mathbf{u}_i^T = \boldsymbol{\varphi}_i^T \nu_i D_i$ (roof displacement u_{Ni})	$\Rightarrow$	displacement $D_i = u_{Ni} / \nu_i \varphi_{Ni}$ (1st)
"modal" base shear V_i	$\Rightarrow$	resisting force $V_{SDOFi} = V_i$ (2nd)
work of "modal" forces on the differential "modal" displacements $d\mathbf{u}_i^T = \boldsymbol{\varphi}_i^T \nu_i dD_i$ $E(d\mathbf{u}_i)$	$\Rightarrow$	work of resisting force on the differential displacement dD_i $E(dD_i) = E(d\mathbf{u}_i)$ (3rd)

Table 1: Definition of the E-SDOF systems.

3 THE PROPOSED METHODOLOGY

As a first step, the proposed methodology is formulated in a manner that takes into account only the predominant vibration mode in the excitation direction. Thus, in its current form, it is suitable for structural systems with small contributions of higher modes, such as low and medium rise planar frames. The steps needed for its implementation are as follows [10]:

- Step 1: Create the structural model.
- Step 2: Apply to the model a set of lateral incremental forces proportional to the vector $\mathbf{M}\boldsymbol{\varphi}_1$ of the fundamental elastic vibration mode 1 and determine the (strain energy)-(roof displacement) curve E_1-u_{N1}. E_1 is equal to the work of the external forces and moments.
- Step 3: Divide the abscissas of the E_1-u_{N1} diagram by the quantity $\nu_1\varphi_{N1} = u_{N1}/D_1$ and determine the (strain energy)-(displacement) diagram E_1-D_1 of the E-SDOF system (Figure 1).
- Step 4: Calculate the work $\Delta E_{1,\lambda}$ (Figure 1) of the external forces in each of λ discrete intervals between the successive formation of plastic hinges. $dE_{1,\lambda}$, as part of $\Delta E_{1,\lambda}$ (Equation (1)), is considered to derive from Equation (2).

$$dE_{1,\lambda} = \Delta E_{1,\lambda} - V_{1,\lambda-1} (D_{1,\lambda} - D_{1,\lambda-1}) = \Delta E_{1,\lambda} - V_{1,\lambda-1}\, dD_{1,\lambda} \tag{1}$$

$$dE_{1,\lambda} = \frac{1}{2} k_{1,\lambda}\, dD_{1,\lambda}{}^2 \Rightarrow k_{1,\lambda} = 2\, dE_{1,\lambda} / dD_{1,\lambda}{}^2 \quad (2)$$

where $k_{1,\lambda}$ is the stiffness of the E-SDOF system corresponding to mode 1 in the interval λ. The resisting force $V_{1,\lambda}$ is given by Equation (3):

$$V_{1,\lambda} = V_{1,\lambda-1} + k_{1,\lambda}\, dD_{1,\lambda} \quad (3)$$

For $\lambda = 1$ (i.e., when the first plastic hinge is created) the force $V_{1,1}$ is equal to the base shear parallel to the direction of excitation. By utilizing Equations (1), (2) and (3) for each interval, determine the (resisting force)-(displacement) diagram V_1-D_1 of mode 1 (Figure 2).

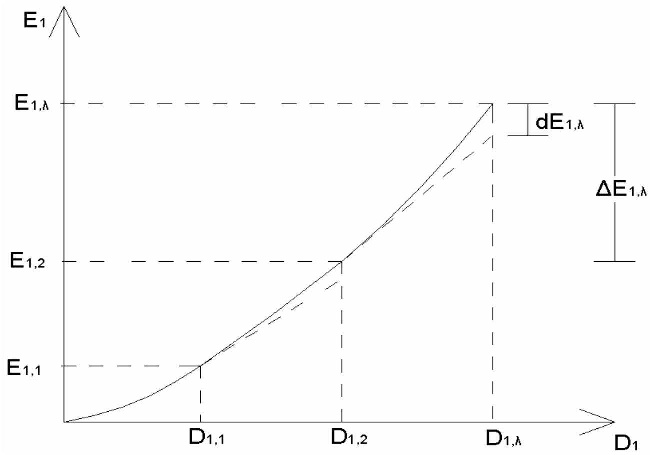


Figure 1: (Strain energy)-(displacement) diagram E_1-D_1 of the E-SDOF system.

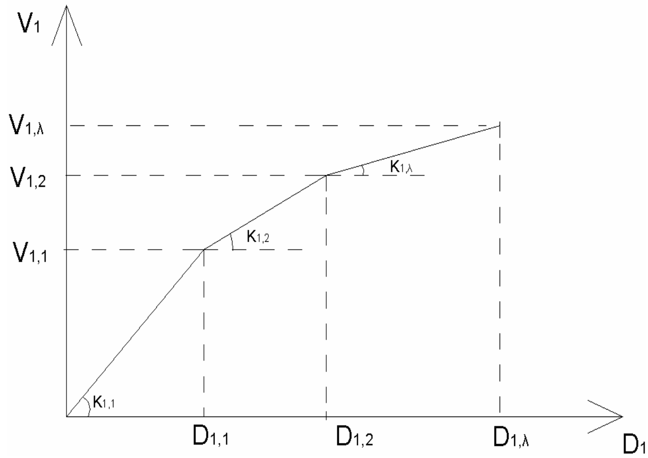


Figure 2: (Resisting force)-(displacement) diagram V_1-D_1 of the E-SDOF system.

- Step 5: Idealize V_1-D_1 to a bilinear curve using one of the well known graphic procedures (e.g., [1], Section 3.3.3.2.5) and calculate the period T and the yield strength reduction factor R of the E-SDOF system corresponding to mode 1.
- Step 6: Calculate the target displacement and other response quantities of interest (drifts, plastic rotations, etc.) of mode 1, using one of the well known procedures of displacement modification (e.g., [1], Section 3.3.3.3.2 / [11], Section 10.4). If the procedure is applied for research purposes using recorded earthquake ground motions, it is recommended to estimate the inelastic displacement of the E-SDOF system by means of nonli-

near dynamic analysis, instead of using the relevant coefficients (e.g., C_1 in ASCE 41-06 and FEMA 440). This is due to the fact that the coefficient values given by codes are based on statistical processing of data with excessive deviations and, therefore, large inaccuracies might result [12].

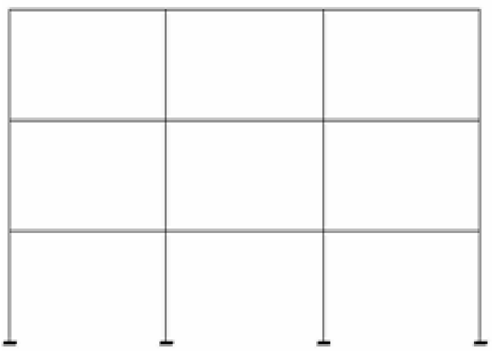
- Step 7: Repeat steps 2 to 6 applying the incremental forces in the opposite direction. It is obvious that this step is necessary to apply only for asymmetric structures.

4 EVALUATION OF THE PROPOSED METHODOLOGY

In order to evaluate the accuracy of the proposed methodology an extensive parametric study is carried out. In particular, the methodology is applied to a series of 3-, 6-, 9- and 12-storey R/C planar frames (Figures 3 to11). Each frame is characterized by a string symbol comprising one or two letter(s) and a number which indicates the number of its storeys. The meaning of the letter(s) is as follows:

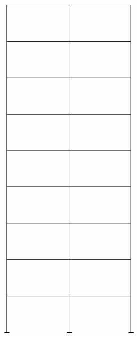
- R – Regular frames.
- M – frames with irregular distribution of Mass along the height. (Odd and even storeys have different masses).
- S – frames with irregular distribution of Stiffness along the height. (Odd storeys have greater height).
- SS – frames with Soft Storey. (1^{st} storey has greater height).

For each frame three sets of pushover analyses are performed: i) one based on the proposed methodology (PM), ii) a second based on a procedure similar to the existing energy-based methods, i.e. according to it the energy equivalence between MDOF and E-SDOF systems is achieved by modifying the displacements (EB) and iii) a third based on the conventional displacement modification procedure (CP). The only difference between the three applied pushover procedures is the determination of the V_1-D_1 diagram (step 4), while the rest of the steps and assumptions are identical. V_1-D_1 diagram affects the characteristics of the E-SDOF system (T and R) and as a consequence the estimation of the target displacement. Each set of analyses comprises 12 different accelerograms corresponding to strong earthquake motions recorded in Greece. The maximum response of the E-SDOF system is calculated by means of nonlinear dynamic analysis for each excitation. Then, the target roof displacement is either estimated by multiplication of the resulting response by the quantity $\nu_1\varphi_{N1}$ (PM, CP) or obtained by the roof displacement – energy-based displacement correspondence (EB) [6].



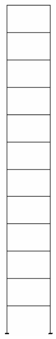
Storey height: 3m - Bay width: 5m
Restraints: columns fixed at base
Constraints: diaphragm at each level
Seismic mass: 30t per level (90t total)
Gravity loads: not considered
Column cross-sections: 40/40 cm
Column reinforcement: 8Φ16
Beam cross-sections: 25/40 cm
Beam reinforcement: 2Φ14 (over and under)
Concrete: C16/20 (f_{ck}=16 MPa)
Reinforcement bars: S400 (f_{yk}=400 MPa)

Figure 3: Frame R3.



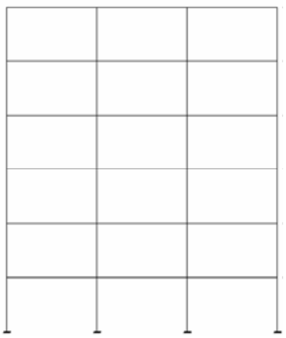
Storey height: 3m - Bay width: 5m
Restraints: columns fixed at base
Constraints: diaphragm at each level
Seismic mass: 30t per level (270t total)
Gravity loads: not considered
Column cross-sections: 60/60 cm
Column reinforcement: 8Φ20
Beam cross-sections: 25/50 cm
Beam reinforcement: 2Φ14 (over and under)
Concrete: C16/20 (f_{ck}=16 MPa)
Reinforcement bars: S400 (f_{yk}=400 MPa)

Figure 4: Frame R9.



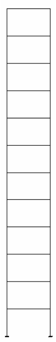
Storey height: 3m - Bay width: 5m
Restraints: columns fixed at base
Constraints: diaphragm at each level
Seismic mass: 15t per level (180t total)
Gravity loads: not considered
Column cross-sections: 60/60 cm
Column reinforcement: 8Φ20
Beam cross-sections: 25/50 cm
Beam reinforcement: 2Φ14 (over and under)
Concrete: C16/20 (f_{ck}=16 MPa)
Reinforcement bars: S400 (f_{yk}=400 MPa)

Figure 5: Frame R12.



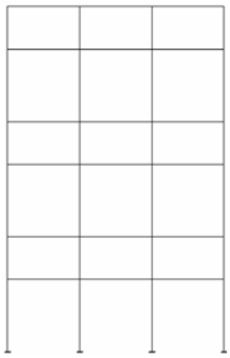
Storey height: 3m - Bay width: 5m
Restraints: columns fixed at base
Constraints: diaphragm at each level
Seismic mass: 20t (odd storeys) or 40t (even storeys) (180t total)
Gravity loads: not considered
Column cross-sections: 50/50 cm
Column reinforcement: 8Φ20
Beam cross-sections: 25/40 cm
Beam reinforcement: 2Φ12 (over and under)
Concrete: C16/20 (f_{ck}=16 MPa)
Reinforcement bars: S400 (f_{yk}=400 MPa)

Figure 6: Frame M6.



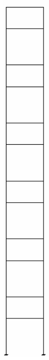
Storey height: 3m - Bay width: 5m
Restraints: columns fixed at base
Constraints: diaphragm at each level
Seismic mass: 9t (odd storeys) or 16t (even storeys) (150t total)
Gravity loads: not considered
Column cross-sections: 60/60 cm
Column reinforcement: 8Φ25
Beam cross-sections: 25/50 cm
Beam reinforcement: 2Φ14 (over and under)
Concrete: C16/20 (f_{ck}=16 MPa)
Reinforcement bars: S400 (f_{yk}=400 MPa)

Figure 7: Frame M12.



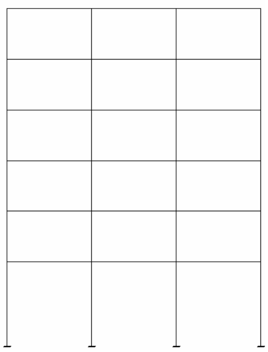
Storey height: 5m (odd storeys) or 3m (even storeys) - Bay width: 5m
Restraints: columns fixed at base
Constraints: diaphragm at each level
Seismic mass: 25t per level (150t total)
Gravity loads: not considered
Column cross-sections: 50/50 cm
Column reinforcement: 8Φ20
Beam cross-sections: 25/40 cm
Beam reinforcement: 2Φ12 (over and under)
Concrete: C16/20 (f_{ck}=16 MPa)
Reinforcement bars: S400 (f_{yk}=400 MPa)

Figure 8: Frame S6.



Storey height: 5m (odd storeys) or 3m (even storeys) - Bay width: 5m
Restraints: columns fixed at base
Constraints: diaphragm at each level
Seismic mass: 10t per level (120t total)
Gravity loads: not considered
Column cross-sections: 60/60 cm
Column reinforcement: 8Φ25
Beam cross-sections: 25/50 cm
Beam reinforcement: 2Φ14 (over and under)
Concrete: C16/20 (f_{ck}=16 MPa)
Reinforcement bars: S400 (f_{yk}=400 MPa)

Figure 9: Frame S12.



Storey height: 5m (1^{st} storey) or 3m (rest storeys) - Bay width: 5m
Restraints: columns fixed at base
Constraints: diaphragm at each level
Seismic mass: 30t per level (180t total)
Gravity loads: not considered
Column cross-sections: 50/50 cm
Column reinforcement: 8Φ20
Beam cross-sections: 25/40 cm
Beam reinforcement: 2Φ12 (over and under)
Concrete: C16/20 (f_{ck}=16 MPa)
Reinforcement bars: S400 (f_{yk}=400 MPa)

Figure 10: Frame SS6.

Storey height: 5m (1^{st} storey) or 3m (rest storeys) - Bay width: 5m
Restraints: columns fixed at base
Constraints: diaphragm at each level
Seismic mass: 13t per level (156t total)
Gravity loads: not considered
Column cross-sections: 60/60 cm
Column reinforcement: 8Φ25
Beam cross-sections: 25/50 cm
Beam reinforcement: 2Φ14 (over and under)
Concrete: C16/20 (f_{ck}=16 MPa)
Reinforcement bars: S400 (f_{yk}=400 MPa)

Figure 11: Frame SS12.

The storey displacements and drifts of the frames under consideration are compared to those obtained by nonlinear response history analysis (NL-RHA), which is considered as the reference solution. In Figures 12 to 20 the mean errors for the 12 excitations (in relevance to the NL-RHA results) of storey displacements and drifts are shown. Notice that the positive sign (+) means that the response parameters obtained by NSPs are greater than those obtained by NL-RHA. Conversely, the negative sign (-) means that the response parameters are underestimated by NSPs. From Figures 12 to 20 becomes clear that the proposed concept for the determination of the E-SDOF system leads to more accurate estimation of the target roof displacement (only in the case of frame R12 EB gives a little lower mean error). Mean errors range from -1% to 17% for PM, from 1% to 45% for EB and from 5% to 52% for CP. Concerning the rest response quantities, the mean errors resulting from the PM are sufficiently smaller in most cases (80% and 73% of cases in relevance to CP and EB, respectively). All the three applied procedures fail to provide a reasonable estimation for drifts at the upper storeys of taller frames. Such failures have been observed in many similar investigations due to the higher mode effects (e.g., [12]).

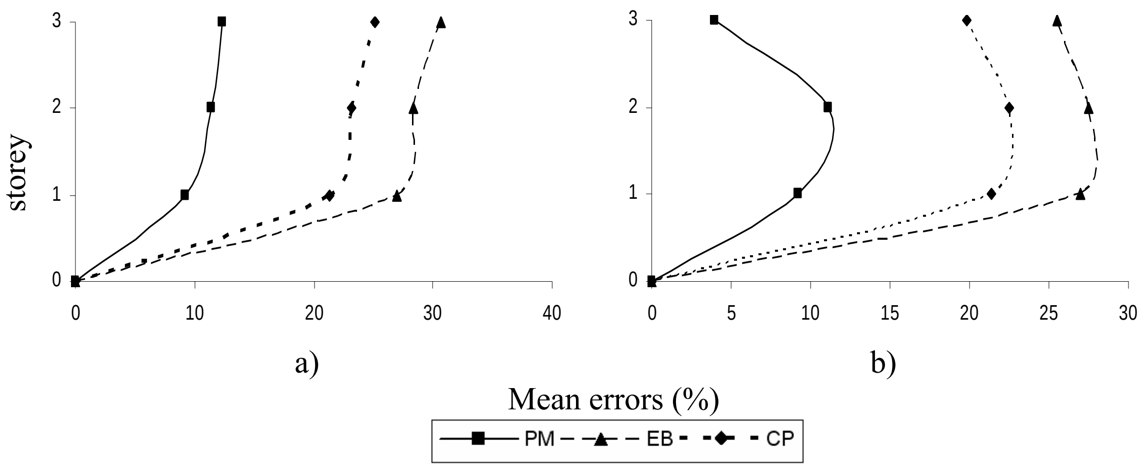


Figure 12: Mean errors (%) of storey displacements (a) and drifts (b) - Frame R3.

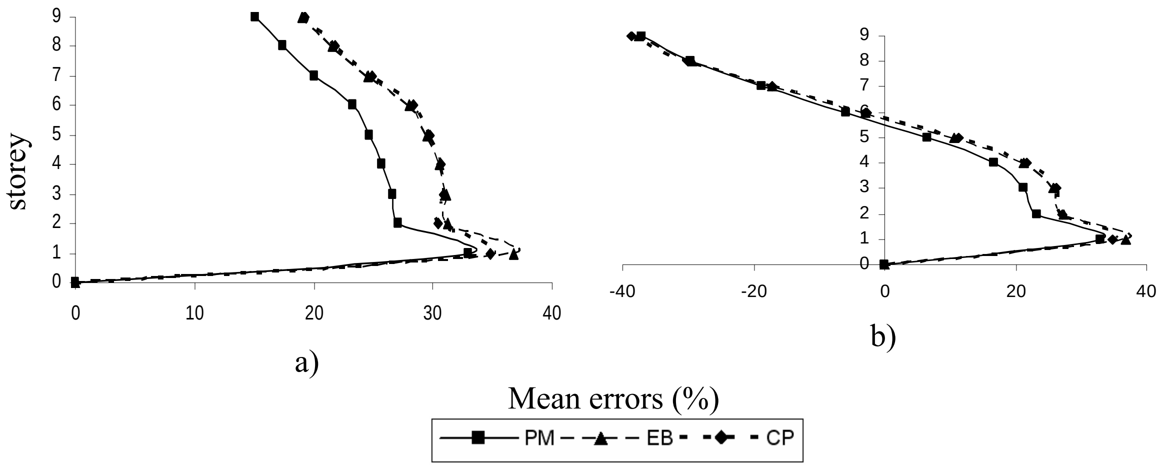


Figure 13: Mean errors (%) of storey displacements (a) and drifts (b) - Frame R9.

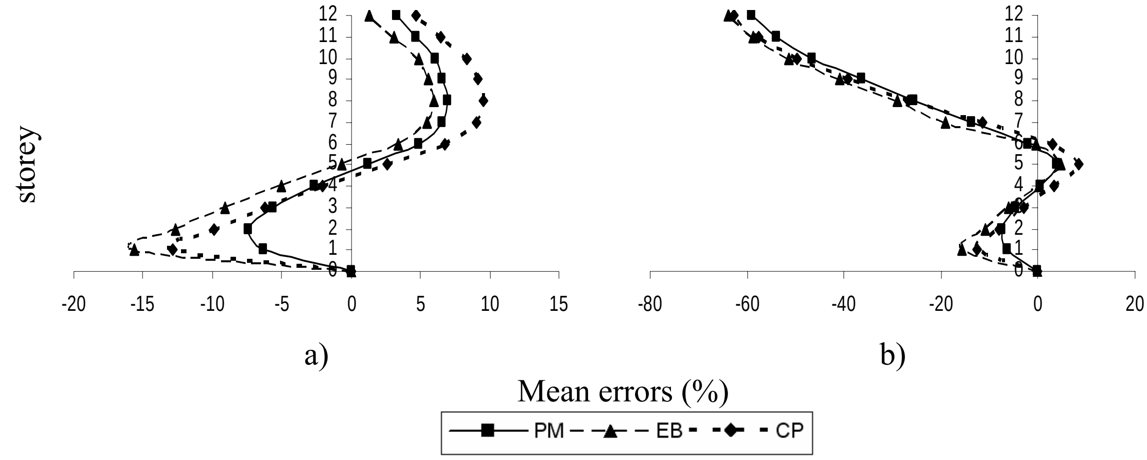


Figure 14: Mean errors (%) of storey displacements (a) and drifts (b) - Frame R12.

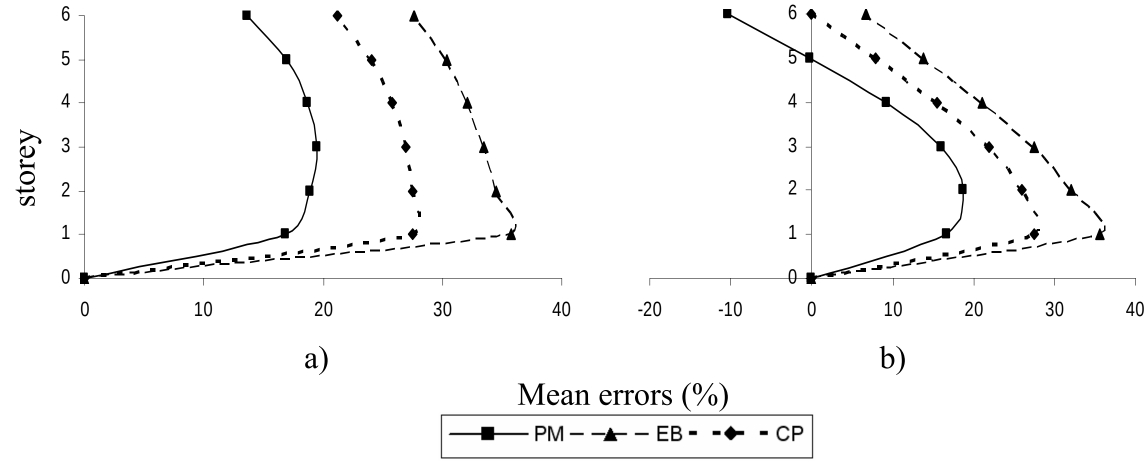


Figure 15: Mean errors (%) of storey displacements (a) and drifts (b) - Frame M6.

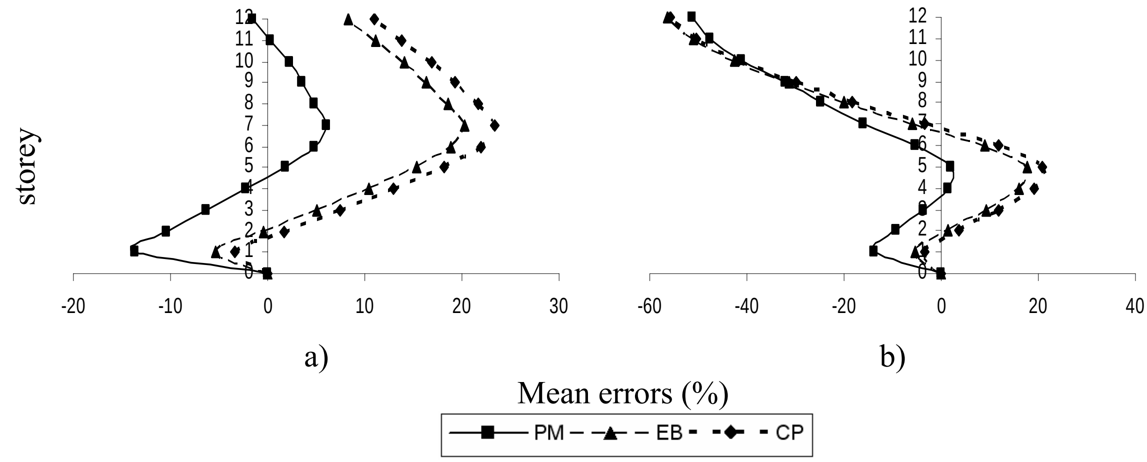


Figure 16: Mean errors (%) of storey displacements (a) and drifts (b) - Frame M12.

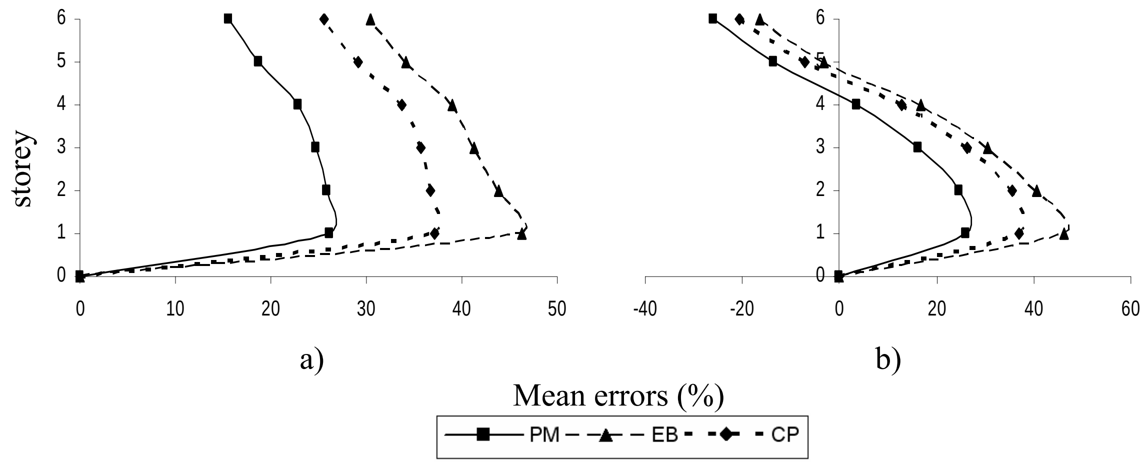


Figure 17: Mean errors (%) of storey displacements (a) and drifts (b) - Frame S6.

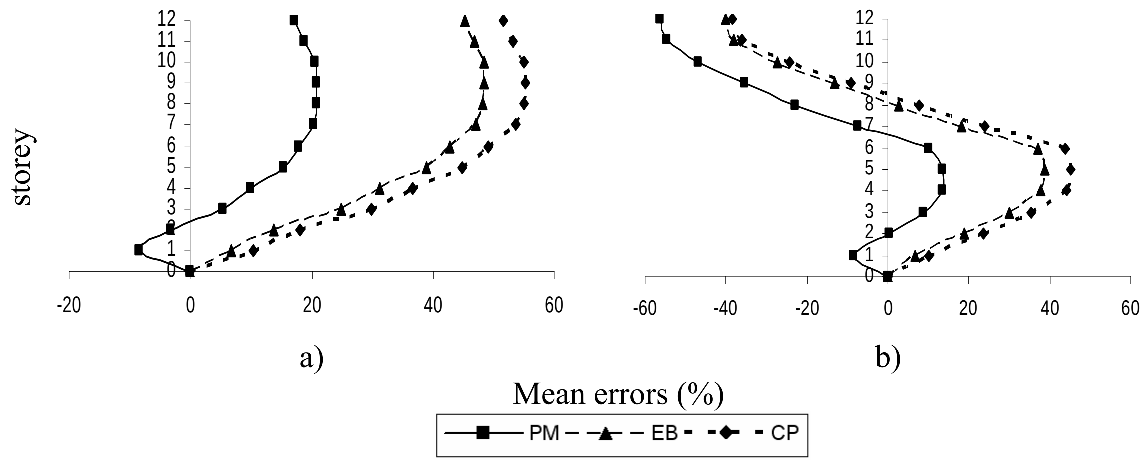


Figure 18: Mean errors (%) of storey displacements (a) and drifts (b) - Frame S12.

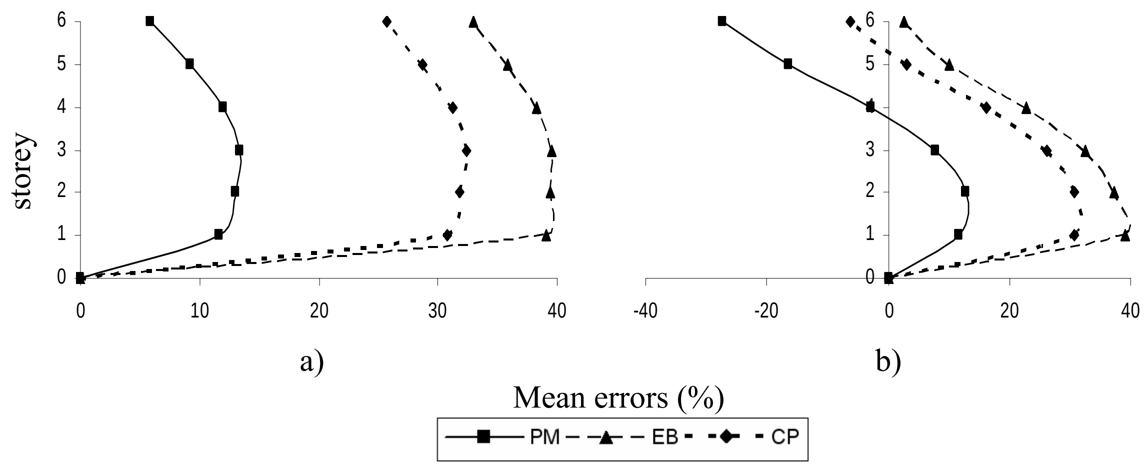


Figure 19: Mean errors (%) of storey displacements (a) and drifts (b) - Frame SS6.

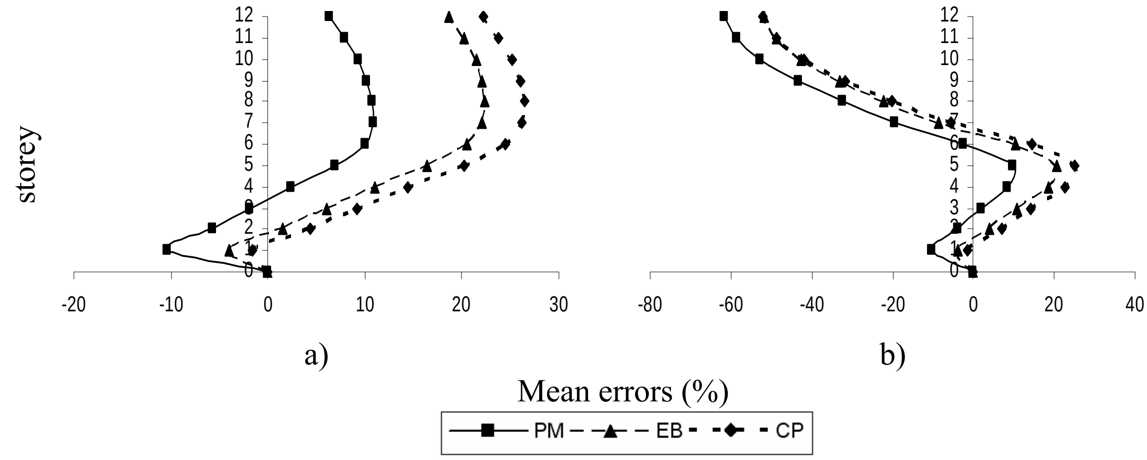


Figure 20: Mean errors (%) of storey displacements (a) and drifts (b) - Frame SS12.

5 CONCLUSIONS

A new energy-based Nonlinear Static Procedure (NSP) is formulated and evaluated in this paper. According to this procedure

- The properties of the E-SDOF system are determined by equating the external work of the lateral loads acting on the MDOF system under consideration to the strain energy of the E-SDOF system, and
- In contrast to other energy-based procedures, this energy equivalence is used to derive a modified resisting force of the E-SDOF system, instead of an energy-based displacement.

According to the results of an extensive parametric study the following conclusions can be drawn:

- The proposed method leads to a more accurate estimation of the target roof displacement.
- In most cases the values of other significant response parameters (e.g., displacements and drifts) are more accurate too.
- None of the three compared pushover procedures can provide reasonable estimations of drifts at upper storeys of tall buildings due to higher modes effects.

For the present, the proposed methodology can be rigorously applied to low and medium rise planar frame structures with rather small contributions of higher mode effects. However, it can be easily extended in a manner that allows its application to high rise planar frames with significant contributions of higher modes as well as to multi-storey asymmetric 3D-buildings.

Finally, it is worth noticing that the implementation of the proposed procedure in existing analysis software can be accomplished without particular difficulty.

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