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FOOTBRIDGE LATERAL VIBRATIONS INDUCED BY SYNCHRONISED PEDESTRIANS: AN OVERVIEW ON MODELLING STRATEGIES

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Abstract. *This paper aims to provide a review and critical analysis of the state of the art concerning crowd-structure interaction phenomena on footbridges. The problem of lateral vibrations induced by synchronised pedestrians, namely the Synchronous Lateral Excitation, is specifically addressed. Due to the multi-physic and multi-scale nature of the complex phenomenon, several research fields can contribute to its study, from structural engineering to biomechanics, from transportation engineering to physics and applied mathematics. Among the different components of the overall coupled dynamical system - the structure, the crowd and their interactions - the latter ones are separately analysed from both a phenomenological and modelling point of view. A special attention is devoted to those models, which explicitly account for the interaction between mechanical and living systems.*

1 INTRODUCTION

The interaction between the structure and the crowd walking on it, and among the pedestrians within the crowd, gives rise to a multi-scale multi-physic complex dynamic system. The latter is characterised by collective phenomena that are not only due to the features of the single system components but also to their interactions. Specifically, the crowd behaviour, in particular the pedestrian force exerted on the structure, affects the structural dynamic properties and response, and the latter modifies the behaviour of the pedestrians walking on the moving structural surface.

This contribution provides a review and critical analysis of the state of the art on the modelling of the interactive dynamics involving crowds and structures, specifically focused on the so-called lively footbridges. Indeed, a specific kind of crowd-structure interaction phenomenon, the so-called Synchronous Lateral Excitation (SLE), is likely to occur on lively footbridges in the lateral direction, due to the pedestrian sensitivity to lateral vibrations which affect their balance during gait. The SLE has attracted the increasing attention of structural engineers and researchers in the last few years. While all over the 20th century the research was mainly directed towards the effects of pedestrian vertical excitation, the closure of the London Millennium Footbridge in 2000 [1, 2] focused the attention towards the problem of lateral vibration due to synchronised pedestrians. The frequency of occurrence of SLE has recently grown due to the construction of a great number of lively footbridges. In fact, bacause of the aesthetic requests for greater slenderness and lightness, newly built footbridges are often characterised by reduced mass, stiffness and damping, so that they are extremely prone to vibration.

The increasing interest for footbridge dynamics in the Civil Engineering field is testified by the growing number of papers published in international journals: a non-exaustive survey of the number of peer-reviewed papers cited in this review versus time is provided in Figure 1a. The organisation since 2002 of a specific international conference named 'Footbridge' mainly devoted to this issue, the recent publication of guidelines for the design of footbridges under human action [3, 4] and the start-up of dedicated European research projects [5, 6] confirm that the crowd-structure interaction is one of the topical research subjects in structural dynamics.

According to the writers' opinion, in spite of the great scientific effort in this field and the significant advances in the comprehension of the phenomenon, the study could benefit of the contribution of other research fields to achieve general and conclusive results. First, the understanding and modelling of the multi-scale multi-physic phenomena involved by crowd-structure interaction makes it necessary to collect contributions from several research fields in a multidisciplinary frame, besides that of structural engineering: for instance, the pedestrian walking behaviour has been extensively studied in the field of biomechanics, while crowd modelling belongs to transportation, physics and applied mathematics research fields. This review aims to offer a contribution to the convergence of these knowledges: the distribution of the references cited in this work among the above mentioned research fields is shown is Figure 1b, where the papers are classified according to the journal or authors main scientific affiliation. Second, engineering and, more in general, technological sciences traditionally experience some difficulties in considering the peculiar behaviour of living systems in the interaction between inner mechanical systems and outer systems, even though this aspect is relevant in some specific cases such as the modelling of crowd and structures. It is well understood, in the case of crowd, that human and animal behaviour follow specific strategies, in some cases we can call it intelligent behaviours, that modify laws of classical mechanics [7, 8, 9, 10, 11, 12]. This is a specific characteristic of all living systems even in the case of low scales such as insects [13]

Figure 1: Published paper in the Civil Engineering field (a) and scientific affiliation of the cited references (b)

or cells [14, 15]. It is worth stressing that the coupling of living and mechanical systems is characterised by peculiar issues that have to be carefully taken into account in a comprehensive modelling approach. Summarising two specific aspects:

(i) The dynamics of living systems follows rules generated by self-organised ability, while mechanical systems follow rules of continuum mechanics according to conservation laws that provide background paradigms that are constant in time.

(ii) The overall system shows hybrid multi-scale characteristics considering that the crowd appears as a discrete system, that is, a system with finite degrees of freedom, while the modelling of structures is developed by continuum models, namely by a system with an infinite number of degrees of freedom.

Both above specific aspects are taken into account in the review proposed in this contribution.

The paper is inspired to the authors' previous review paper [16] and develops in three more Sections. Section 2 is devoted to the state of the art concerning the phenomenological analysis of the behaviour of the crowd dynamics and of the crowd-structure interaction (CSI). Section 3 is addressed to the mathematical models proposed so far to describe the different components of the overall crowd-structure system and the crowd-structure interaction models. Finally, the conclusions and some research perspectives are discussed in Section 4.

2 PHENOMENOLOGICAL ANALYSIS

This section is devoted to a review of the studies that are useful for the understanding and phenomenological description of the crowd-structure interaction phenomenon, named SLE. In the following the studies relative to the crowd behaviour, and those relative to the crowd-structure interaction are briefly analysed. A review of the studies concerning the behaviour of a single pedestrian can be found in [16] and [17].

2.1 Crowd behaviour

Crowds are complex systems, that is, large ensemble of individuals interacting in a non-linear manner. Some of the peculiar features of the pedestrian behaviour are outlined in the following [18]:

1. pedestrians in a crowd represent an example of *collective behaviour*, that is, an individual's action is dominated by the influence of his neighbours, so that the individual behaves differently from the way he/she would behave on his/her own;

2. pedestrians are *active* agents, i.e., under normal conditions without panic onset, they share the same objective of walking with the maximum velocity towards a target (e.g., doors, exit, displays), bypassing possible obstacles and avoiding the most crowded zones (see the concept of *personal space* developed in human sciences and psychology, e.g. in [19]). These strategies enable them to determine actively their walking direction and velocity, without being passively subject to the laws of inertia;

3. pedestrians are *intelligent* agents, i.e., their mind evaluates, selects, and/or makes synthesis of what it perceives according to various psychological criteria (e.g., the level of anxiety [20] or the capacity to perform ensamble evaluations [21]).

4. under normal external and subjective conditions (e.g., the area where pedestrians walk is illuminated allowing visual perception), pedestrians do not perceive the real world locally in space, due to their ability to see up to a given extent around them (the so-called *sensory region* [22]). Moreover, pedestrians react after a time interval has elapsed from the perception time (see e.g., [23, 21]). Therefore, pedestrians in a given position at a given time react to the conditions perceived in front of them at a delayed time, i.e., in a *non-local* way in both space and time ;

5. pedestrians are *anisotropic* agents, i.e., they are not equally affected by stimuli coming from all directions in space. Specifically, they distinguish between ahead and behind, in normal situations being essentially sensitive to what happens in a symmetric visual field focused on their direction of movement;

6. walking pedestrians adapt the depth and width of the sensory region to their travel purpose and to their walking speed (e.g., a pedestrian walking for leisure purposes is expected to scan a wider field than a commuter attaining a train, and the faster a pedestrian walks the deeper the space required to evaluate and react is).

Because of the crowd intrinsic multiscale features, crowd related phenomena can be schematically observed with reference to three different scales [24]:

- *macroscopic scale*, which describes the state of the ensamble of individuals with averaged quantities.

- *mesoscopic scale*, where the state of the system is identified by the probability distribution functions of the microscopic state of the individuals.

- *microscopic scale*, where the contribution of each single individual to the behaviour of the system is described.

Nevertheless, it is worth stressing that this classification obeys to conceptual, experimental and modelling requirements, while unexpected emerging phenomena, expecially in out-of-equilibrium conditions, can result from a mix of individual and collective behaviour involving several scales. In the following, the crowd behaviour is analysed from a macroscopic point of view, since experimental measurements found in literature usually refer to averaged variables, namely the crowd density, velocity and flow, which describe the macroscopic state of the crowd. In this macroscopic framework, the single pedestrian behaviour can be recovered in a limit sense, i.e. when density tends to zero.

In the following, a stationary and homogeneous pedestrian traffic flow described by macroscopic quantities is considered. The so-called *fundamental relation* is valid:

$$q = \rho v, \tag{1}$$

where q is the flow, intended as the number of pedestrians passing a cross-section of an area in a unit of time [ped/ms]; ρ is the crowd density [ped/m^2]; v is the average walking velocity [m/s]. The three variables are macroscopic characteristics of the flow: the graphical representations of their relations are called *fundamental diagrams*. It reflects one of the main feature of crowd behaviour, that is the walking velocity is affected by the crowd density, namely the higher the crowd density, the lower the walking velocity.

Looking at the flow-density diagram in Fig. 2, some relevant quantities can be identified [25]:

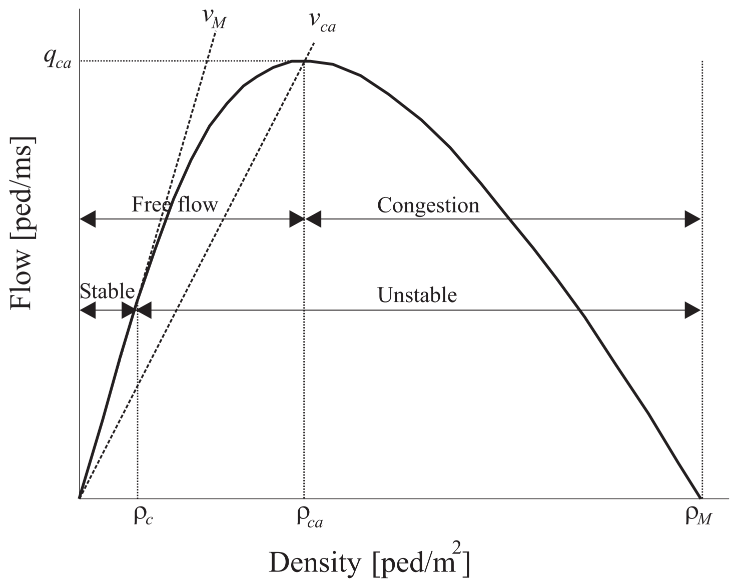


Figure 2: Flow-density fundamental diagram

- free speed v_M: the slope of the function $q(\rho)$ at the origin that corresponds to the velocity if $q = 0$ ped/ms and $\rho = 0$ ped/m^2, that is to the walking velocity of the single unimpeded pedestrian (a review of the available statistics about the single pedestrian velocity can be recovered in [16]);

- critical density ρ_c: the lower bound for unconstrained free walking. For $\rho < \rho_c$, pedestrians walk with constant free speed $v = v_M$ (*stable region*); for $\rho > \rho_c$, the walking speed decreases with increasing density (*unstable region*);

- capacity speed v_{ca} and density ρ_{ca}: the speed and density when $q = q_{ca}$, that is, the maximum flow. The region of density below ρ_{ca} is called *free flow region*, while the *congestion region* corresponds to a higher density than the capacity density;

- jam density ρ_M: the maximum admissible density corresponding to null speed and flow.

The values of the aforementioned variables are not expected to be universal, since walking behaviour is influenced by a great number of microscopic factors, such as age, culture, gender, travel purpose, type of walking facility and single or multiple walking direction [25, 26]. Many studies have been directed to the determination of a law that links the walking velocity to the crowd density. Most of these studies [26, 25] belong to the tranportation research field, with the aim of controlling the layout and dimensions of pedestrian walking facilities.

As far as the jam densities are concerned, they have been estimated to vary between 4 and 5.4 ped/m^2 [27]. The average human body has a width of 45.6 cm and a depth of 28.2 cm [27]. These dimensions refer to a motionless pedestrian. The maximum pedestrian density could be derived from the minimum average body surface (0.13 m^2): this leads to a density of 7.69 ped/m^2, that is difficult to obtain in practice, since people can hardly move at densities over 5 ped/m^2. When pedestrians are walking they require more lateral and forward space than a motionless person. The required lateral additional space has been estimated to be about 62% of the average width of pedestrians [26]. The required forward space (distance among pedestrians d) instead depends on the walking velocity: a linear relation has been proposed by Seyfried et al. [28], as a fitting to experimental data, which is valid in the $0.1 < v < 1$ m/s domain:

$$d = 0.36 + 1.06v. \tag{2}$$

The distance among pedestrians can be expressed as the sum of the step length l_p and the so-called *sensory zone* d_s, defined by Fruin [22] as ”the area required by the pedestrians for perception, evaluation and reaction”. The pioneering Fruin’s definition of this *buffer zone* quantifies one of the distinctive properties of pedestrians as active particles, i.e. their bounded visual field over which environmental information are obtained in order to draw specific, real-time strategies.

Because of the great number of factors affecting pedestrian flows, rather different experimental data and fundamental diagrams can be found in literature. Looking at the experimental data classified with respect to the kind of pedestrian traffic [29, 28, 30] (Fig. 3), it is clear that the Reimer’s measurements, reported in [29], are different from the other ones, since they refer to fast pedestrian transit in train stations and the author himself refers to them as exceptional cases: it is odd that a recent design guideline [3] only reports Reimer’s diagram. One of the first studies, which explicitly accounts for the travel purpose as a parameter that affects the fundamental diagram, was developed by Oeding [29]. He proposed an interesting diagram (Fig. 4), recently recovered in [3] and [34] , which graphs the capacity of pedestrian walkways as a function of density and traffic type. In particular he distinguished four types of pedestrian traffic (shopping, event, rush hour and factory traffic), corresponding to increasing walking velocity and capacity. The diagram also allows a classification of walking regimes to be outlined, that is: free ($\rho < 0.3$ ped/m^2); acceptable ($0.3 < \rho < 0.6$ ped/m^2); dense ($0.6 < \rho < 1$ ped/m^2); very dense ($1 < \rho < 1.5$ ped/m^2); crowded ($\rho > 1.5$ ped/m^2).

An exaustive survey of the speed-density relations proposed so far can be found in [25] and [27]. They are graphically represented in Fig. 5a, which plots the linear relations [25, 27], and in Fig. 5b, where the non-linear laws [7, 26] are reported. It should be reminded that all graphics refer to steady uniform conditions described by macroscopic quantities, therefore fluctuations are lost. Many of the studies report a linear relationship between velocity and density (Fig. 5a), according to the following form:

$$v = v_M - k\rho; \; k > 0. \tag{3}$$

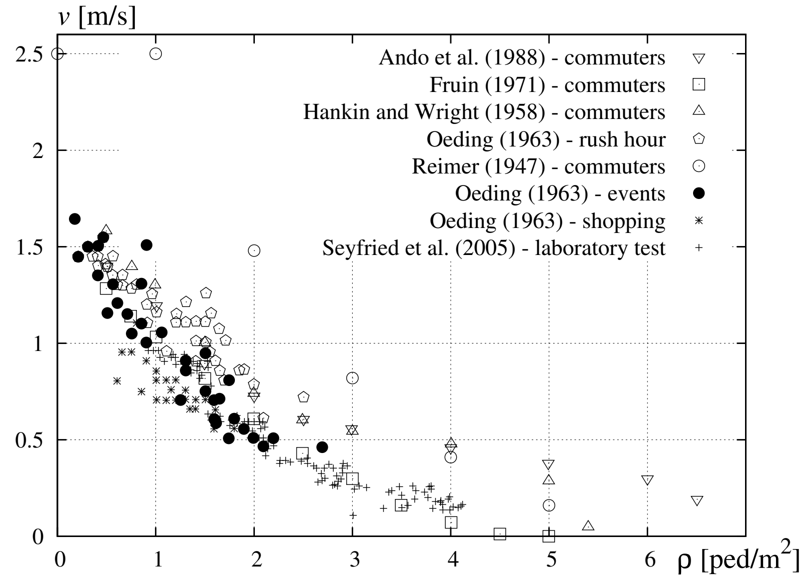


Figure 3: Speed-density relation: experimental data [31, 22, 32, 29, 33, 28]

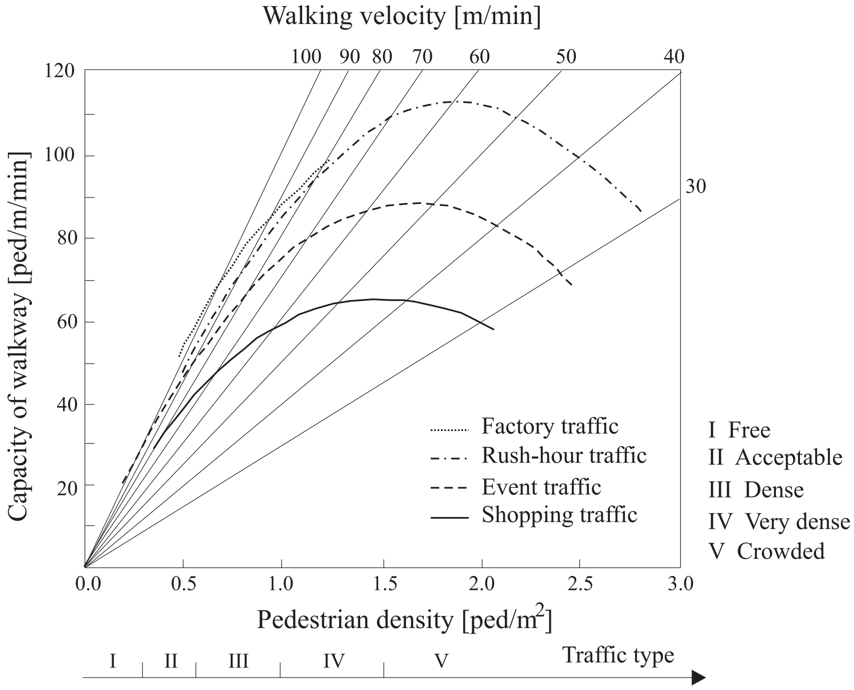
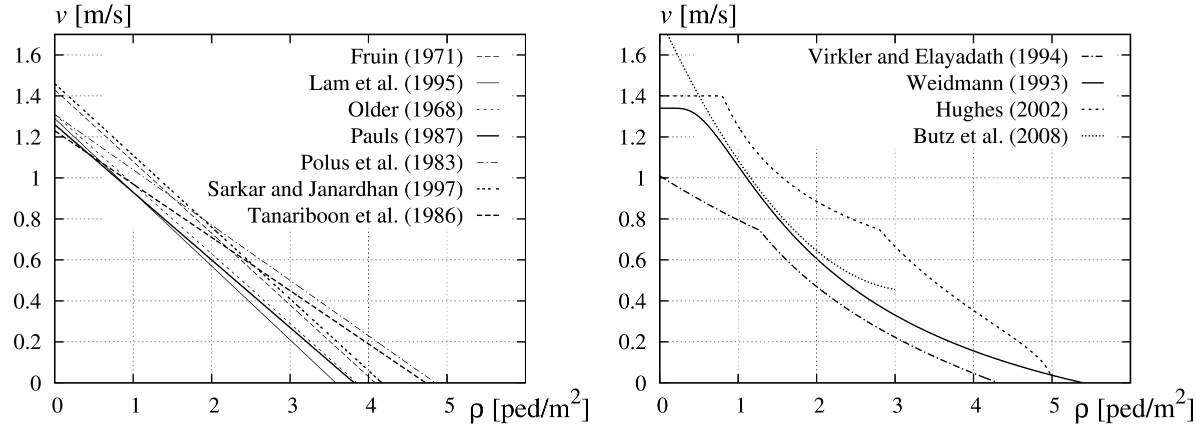


Figure 4: Relationship between bridge capacity, pedestrian density and their velocity, after Oeding [29]

Other authors have proposed non linear laws (e.g. [26]) or multi-regime models (e.g. [7]). In general, the non-linear multi-regime models are more accurate than the linear laws, since they better capture the almost constant speed at low densities and they have the upward concave form that better fits the observation data. In particular, the law proposed by Weidmann, called *Kladek formula*, has the advantage of being a continuous function of ρ, avoiding unrealistic discontinuities, such as in the Hughes' diagram. A parametrical form of the Kladek formula has been proposed in [42], in order to account for the influence on the walking velocity of both psychological and physiological factors, represented by the travel purpose and geographic area,



(a) (b)

Figure 5: Speed density relations in literature [7, 5, 26, 22, 35, 36, 37, 38, 39, 40, 41]: linear (a) and non linear (b)

respectively:

$$v = v_M \left\{ 1 - \exp\left[-\gamma \left(\frac{1}{\rho} - \frac{1}{\rho_M} \right) \right] \right\}, \tag{4}$$

where γ is an exponent that makes the relation sensitive to different travel purposes (leisure/shopping, commuters/events, rush hour/business), which is obtained through a fitting of the data in [22, 29]. The jam density ρ_M is expressed as:

$$\rho_M = \frac{1}{\beta_G S_m}, \tag{5}$$

where $S_m = 0.13$ m^2 is the mean surface occupied by a motionless pedestrian and the geographic area coefficient β_G is derived, considering the dimension occupied by the human body in different countries [27], as the ratio between the surface averaged per geographic area and the mean surface. It results that β_G equals 1.075 for the European and American case, while it equals 0.847 for Asian countries. The free speed v_M is expressed in the general form:

$$v_M = \bar{v}_M \, \alpha_G \, \alpha_T, \tag{6}$$

where $\bar{v}_M = 1.34$ m/s is the average free speed [27] and the coefficients α_G and α_T, which make the velocity sensitive to the geographic area and travel purpose, are determined analysing the data reported in [27] as the ratio between the proposed free speeds and $\bar{v}_M$ (Table 1).

Table 1: Coefficients of geographic area and travel purpose

	Travel purpose				**Geographic area**		
	Rush hour/ Business	Commuters/ Events	Leisure/ Shopping		Europe	USA	Asia
α_T	1.20	1.11	0.84	α_G	1.05	1.01	0.92
γ	0.273ρ_M	0.214ρ_M	0.245ρ_M				

It is worthwhile pointing out that the speed-density relations described so far refer to a one-directional flow. In a bi-directional flow, the effects due to passing pedestrians lead to a reduction of the flow capacity. Weidmann [26] estimates a capacity loss of about 4-9% in the case of equal flows in both directions (50%/50% or directional ratio = 1) and a higher capacity loss (about 14.5%) for a directional ratio of 10%/90%. This result has to be ascribed once more to the pedestrian intelligent behaviour and it follows from the self-organised phenomena which take place in the crowd: a directional ratio close to unit induces a two-lane natural configuration, where each pedestrian tends to keep his/her right due to cultural influences; low directional ratios and wide pedestrian walkways do not allow the upstream pedestrians to organise together, so that they are viewed by the main stream as moving obstacles, involving a significant capacity loss.

Finally, it is worth drawing some considerations about flow dimensionality. Many of the proposed relations refer to pedestrian movement on a plane. Seyfried et al. [28] showed that the measurement of the speed-density relation for a single-file movement of pedestrians leads to results in complete agreement with Weidmann's diagram: this means that specific two-dimensional features, such as internal friction and lateral interference, do not have a strong influence on the fundamental diagram in the considered density range.

In conclusion, it can be stated that it is not possible to determine a universally valid speed-density relation, since the pedestrian behaviour is affected by a great amount of parameters. For this reason, a specific law should be tuned in order to characterise a particular crowd condition. Furthermore, it should be borne in mind that the fundamental diagrams are derived in steady state conditions: this means that the quantities characterising the system vary slowly with respect to space and time. Therefore, fundamental diagrams, like the ones presented in this section, are not suitable for direct use in out-of-equilibrium conditions.

Althought density and velocity are the main macroscopic state variables of the crowd, other quantities are of interest in the perspective of the modelling of the crowd dynamic load acting on a structure and of the crowd-structure interaction. Among these quantities, let us recall the walking frequency, which affects in turn the pedestrian force frequency content, and a measure of the degree of synchronisation among pedestrians (e.g., the walking phase, the standard deviation of the walking frequency or, more in general, a correlation coefficient).

As for the walking frequency, it has to be intended as a vertical one f_{pv}, that is, the number of times a foot touches the ground in a time unit, while the horizontal or lateral walking frequency f_{pl} is intended as the number of times the same foot touches the ground. At the individual level (single unimpeded pedestrian at $\rho = 0$), the former depends on the free walking velocity v_M and on the step length l_s according to the fundamental walking law

$$f_{pv} = \frac{v_M}{l_s}, \tag{7}$$

where the mentioned walking parameters are random variables whose statistical moments are obtained by ensemble averaging of experimental measurements [5, 43, 44, 45, 46, 47, 48]. In the case of constrained pedestrians in a crowd ($\rho > 0$), the free speed v_M in Eq. 7 should be replaced by $v(\rho)$ and all the walking parameters should be intended as macroscopic averaged quantities. In this macroscopic framework, different empirical $f_{pv} - v$ laws have been proposed as a fitting to experimental measurements. Linear relations have been proposed, for instance, by Butz et al. [5], on the basis of the Oeding's measurements [29]:

$$f_{pv} = 0.7886 + 0.7868v, \tag{8}$$

and by Ricciardelli et al. [46]:

$$f_{pv} = 0.024 + 0.754v, \tag{9}$$

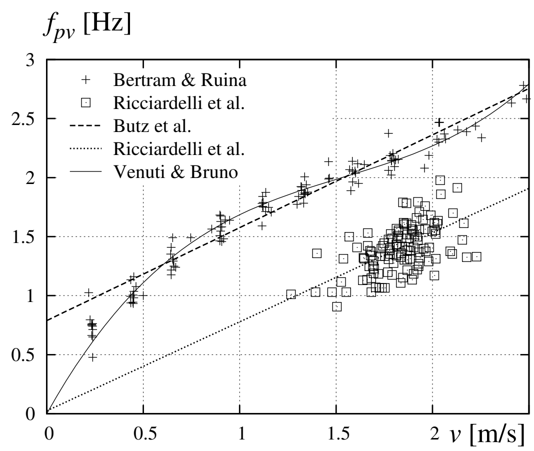


Figure 6: Examples of $f_{pv} - v$ relations

who observed that the constant value 0.024 m/s derived from the fitting can be neglected for engineering purposes. A non linear relation based on a cubic fitting to the data of Bertram and Ruina [49] has been proposed in [50]:

$$f_{pv} = 2.93v - 1.59v^2 + 0.35v^3. \tag{10}$$

The three $f_{pv} - v$ laws are compared in Fig. 6.

As for the degree of synchronisation among pedestrians in a crowd, when a pedestrian's walking is constrained because of high density values, people tend to walk with the same frequency and a null relative phase angle, that is, they synchronise to each other [51]. This behaviour is due to the attempt to avoid foot-to-foot contact in the forward direction and shoulder-to-shoulder contact in the lateral direction. Some experimental evidence of the former attempt has been obtained by Seyfried et al. [28] who observed that pedestrians tend to optimise the available forward space in case of high density, giving some overlap in the space occupied with the pedestrian in front.

So far a limited number of experimental tests has been devoted to the investigation and quantification of the synchronisation among pedestrians in dense crowds, even though it could play a crucial role in crowd-structure interaction, acting as a inner trigger of the pedestrian-structure synchronisation. In the last two years some new studies [5, 52, 53] have been carried out by means of experimental tests performed within different ranges of the crowd density. Arajo *et al.* [52] found that, in the crowd density range 0.3-0.9 ped/m^2, there is no evidence of synchronisation among pedestrians, since the standard deviation of the walking frequencies is almost constant for different densities and the phase angles are totally random. Ricciardelli and Pizzimenti [53] observed that, in the crowd density range 0.5-1.5 ped/m^2, initially different walking frequencies and phases tend to get closer for increasing crowd densities, giving rise to synchronisation nuclei within the crowd. Finally, Butz *et al.* [5] performed tests in which pedestrian streams with densities varying between 1.2 and 3 ped/m^2 walked along a 30 m long and 1.5 m wide path and their walking frequencies were measured with pressure sensors located in the right shoes. They found that the standard deviation of step frequencies for high crowd density was lower than for low density, indicating a higher correlation among pedestrians in the first case.

Besides the above mentioned experimental approach, some interesting suggestions to the comprehension of the phenomenon can be found in the fields of physics and applied mathematics. In fact, the synchronisation among pedestrians can be ascribed to *collective synchronisation* phenomena, which have been studied from the Sixties in the pioneering works of Winfree [54] and Kuramoto [55]. Collective synchronisation occurs when ”an enormous system of oscillators spontaneously locks to a common frequency, despite the inevitable differences in the natural frequencies of the individual oscillators” [56]. Several examples can be found in the natural world, from the pacemaker cells in the heart, to flashing fireflies. Winfree studied the behaviour of a large population of weakly-coupled, nearly identical limit-cycle oscillators: when the spread of natural frequencies is large with respect to the coupling strength K, the system behaves uncoherently; as the spread decreases, the system behaves uncoherently until a threshold K_c is crossed (Fig. 7). Then, a small cluster of oscillators synchronises and the coherency grows towards perfect synchronisation: the threshold K_c, therefore, corresponds to a *phase transition*.

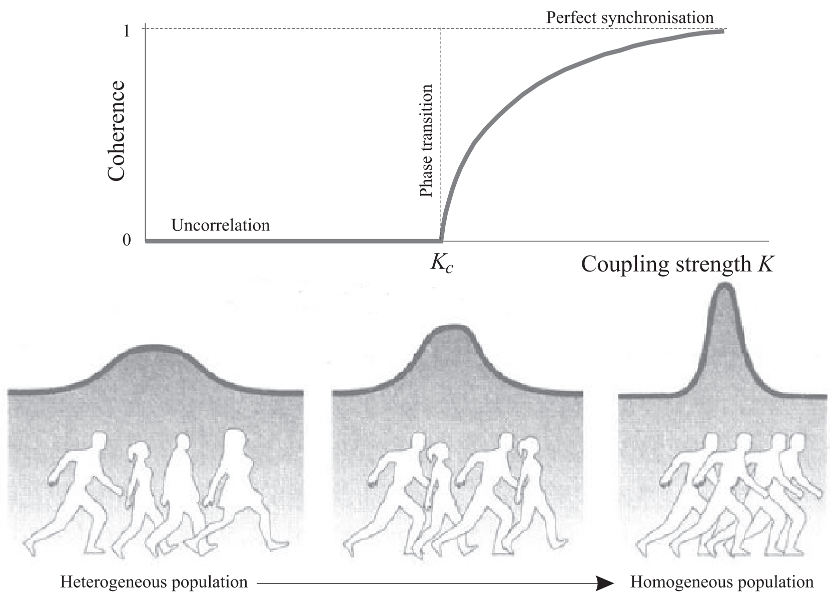


Figure 7: Schematic representation of the Winfree’s model after Pizzimenti [57]

Pizzimenti [57] observed that the phase transition described by Winfree could interpret what has been observed on the London Millennium Bridge in its opening day: first, small groups of pedestrians started to synchronise; then, when the number of pedestrians exceeded a critical value, most of them were captured in the synchronisation phenomenon.

2.2 Pedestrian behaviour on a vibrating platform: crowd-structure interaction

When a pedestrian crosses a lively footbridge, he walks on a vibrating surface, therefore, if the vibrations become perceptible, human-structure interaction can occur. As stated in the Introduction, pedestrians are more sensible to lateral vibrations of the walking surface, therefore they are more likely to synchronise with the deck lateral motion. Indeed, because of the attempt to maintain body balance, the pedestrians unconsciously adapt their lateral frequency to the lateral natural frequency of the moving surface. A scheme of the pedestrian-structure synchronisation, also known as *lock-in* in structural engineering, is represented in Figure 8 [5]: if the lateral movement of the torso has the same frequency and is in phase with the deck lateral velocity, the work $W = \int_T F\dot{z}dt$ is always positive, that is, the pedestrian provides positive energy input, causing the vibrations to enlarge. As a consequence, pedestrians walk with their legs more widespread, the lateral motion of the upper part of the torso increases and the resulting lateral force grows in turn in a self-excitation mechanism, which characterises the SLE. This phenomenon is amplified if the pedestrian walks in a dense crowd, since the synchronisation among pedestrians (§2.1) can trigger or increase the effects of the pedestrian-structure synchronisation [58, 59]. So far, the phenomenon has never led to structural failure since it has a self-limited nature, that is, when the vibrations exceed a limit value pedestrians detune, stop walking or touch the handrails, causing the vibrations to decay. It is worth pointing out that the above mentioned phenomena responsible for the self limited nature of SLE are essentially due to the intrinsic features of the pedestrians that behave as active and intelligent agents (see the beginning of §2.1). Nevertheless, the resulting reduced comfort for the users has often led to a temporary closure of the footbridge in order to provide proper countermeasures, with consequent economic and social repercussions.

It is worth pointing out that human-structure interaction has also been observed by some authors in the case of vertical vibrations [60, 61]. In particular, humans' inability to synchronise their pace with vertically moving surfaces causes the vibration to diminish, as if pedestrians provide additional damping to the system. This effect, which is well-known in the case of stationary people, is not completely understood in the case of moving people and needs further investigations.

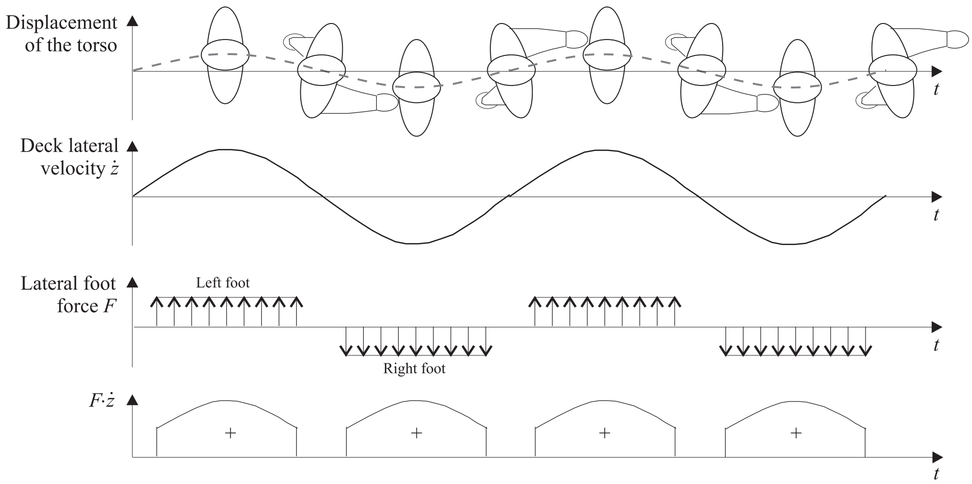


Figure 8: Schematic representation of pedestrian-structure synchronisation after Butz et al. [5]

The SLE is not related to a specific structural type, but it "could occur on any bridge with a lateral frequency below about 1.3 Hz loaded with a sufficient number of pedestrians" [1] (see [61] for a review of bridges exhibiting SLE). In the last few years, many studies have been devoted to the understanding of the SLE, mainly through an empirical approach. The experimental studies can be roughly divided into the following categories: i) laboratory tests on moving treadmills or long vibrating platforms; ii) field tests on real footbridges; iii) observation of videos recorded during real world crowd events; iv) analysis of similar phenomena occurring in nature and studied in different scientific fields.

The first laboratory experiments in the structural engineering field were carried out at London Imperial College and at the University of Southampton to explain what had happened on the London Millennium Bridge [1]. The tests at the University of Southampton involved a person walking "on the spot" on a small shaking table. The tests at Imperial College involved persons walking along a 7.2 m long platform which could be driven laterally at different frequencies and amplitudes. Similar tests on a moving treadmill were also performed by Pizzimenti [57]: the specially built treadmill (Fig. 9a) was laterally driven at different amplitude-frequency configurations. On one hand, treadmill devices permit a steady-state walking behaviour to be easily reached; on the other, they only allow the walking behavior of a single pedestrian to be explored. Moreover, the pedestrian is forced to walk at a given velocity and his/her natural behaviour is affected by the small treadmill surface. Even though these tests did not permit the behaviour of people walking in a crowd to be investigated, they allowed important results to be obtained about the behaviour of a single pedestrian, that is, the amplitude of the exerted lateral force for increasing amplitude of the platform oscillation and the probability that he/she would synchronise his/her pace to the frequency of the moving platform.

In order to better simulate the conditions that can occur on a footbridge, on which groups of pedestrians walk continuously, Setra [4] built a 7 m long and 2 m wide slab on 4 flexible blades moving laterally, in order to measure the horizontal load exerted by pedestrians and to estimate the threshold of motion perception corresponding to the triggering of the lock-in. The device was provided with access and exit ramps in order to maintain walking continuity. These tests showed that there is an acceleration threshold (around 0.1-0.15 m/s^2) above which some synchronisation arises and causes uncomfortable vibrations. A longer platform has been built during the Synpex European project [5]. The 12 m long and 3 m wide test rig was designed to vibrate both in the vertical and horizontal direction (Fig. 9b). One of the main limits of the vibrating platforms built so far is that their reduced length does not permit the synchronisation phenomena to fully develop, so that constitutive relations between the pedestrian state variables and platform motion cannot be drawn in equilibrium conditions. On the other hand, they allow experiments to be performed in a controlled environment, so that the effect of different factors can be isolated.

The phenomenon has also been studied by means of field tests. The tests carried out on

(a) (b)

Figure 9: Example of treadmil and moving platform after Pizzimenti [57] and Butz et al. [5]

the Millennium Bridge during the closure period [1] (Fig. 10) evidenced an almost linear dependence of the pedestrian force on the deck lateral velocity. In addition, the deck acceleration abruptly increased when the number of pedestrians exceeded a critical value. Several test campaigns were also carried out on the Solferino footbridge [4], leading to the following conclusions: the lock-in occurred for the first lateral mode and appeared to start and develop more easily when the step lateral frequency was lower than the deck lateral frequency; lock-in occured beyond a particular threshold, that can be determined in terms of critical number of pedestrians (as proposed in [1]) or critical value of acceleration, which seems more relevant;

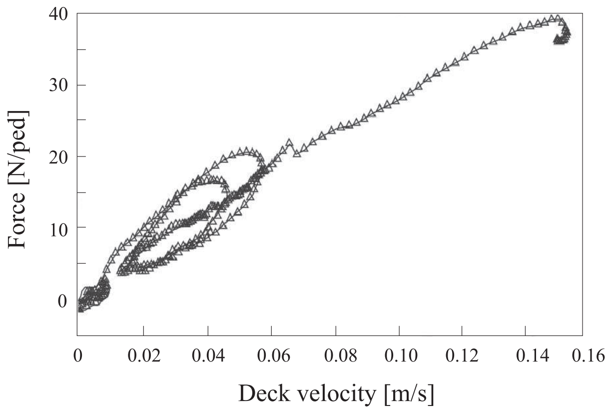


Figure 10: Lateral force per person per vibration cycle vs deck velocity, after Dallard et al.[1]

below 0.1 m/s^2, the pedestrian behaviour may be considered not related to the structure motion. Field measurements were also conducted by Nakamura [62] on the M-bridge in Japan. Accelerometers in the lateral direction were attached to the base of the handrails at five positions and accelerometers were also attached to the waist belt of a person, who walked on the bridge among other pedestrians. The measurements confirmed that the pedestrian walked at the same frequency as the girder and showed that the pedestrian's phase is between 120° and 160° ahead of the girder. A comparison between the time histories of the girder and pedestrian lateral motion also showed that the pedestrian was sometimes no longer tuned to the structure (Fig. 11): this means that he sometimes lost his balance because of large girder vibrations. By analising the behaviour of three footbridges (Millennium Bridge, T-bridge and M-bridge), Nakamura proposed a serviceability limit for lateral vibration, that is, a displacement of 45 mm (a velocity of about 0.25 m/s and an acceleration of about 1.35 m/s^2). Nakamura also observed that synchronisation is unlikely to occur at a deck natural frequency under 0.6 Hz. The same conclusion can also be inferred from the tests performed by Pizzimenti on the moving treadmill [57]. Finally, it is worth citing the experimental campaigns conducted during the Synpex project, which involved field tests on several footbridges, such as the Pedro and Ines footbridge in Coimbra (Portugal) [63] or the stress ribbon bridge at the campus of the University of Porto [64], with the aim of characterising the pedestrian perception of footbridge vibrations, validating load models and identifying the most relevant footbridge dynamic properties. Even though field tests give further information with respect to laboratory tests, it has to be pointed out that in both kind of tests the pedestrians do not walk in a completely natural way, since they are conditioned by several constraints (for instance they are asked to tune their pace to the sound of a metronome): this fact should be considered in the interpretation of the test results.

Other suggestions come from the observation of the videos recorded during crowd events or experimental campaigns. Fujino et al. [65] were the first to observe the SLE on the T-bridge in Japan, which connects a boat race stadium to a bus terminal. They recorded human passage during a congested period by means of three cameras installed on the stadium roof, synchronised to each other and connected to a computer. The motion of a selected number of pedestrians' heads were digitised from the video by means of a microcomputer [66]. The head motion time histories showed a surprising similarity, i.e. they were synchronised although the amplitudes were different. Fujino et al. estimated a percentage of synchronised pedestrians of about 20%.

Finally, as in the case of the synchronisation among pedestrians, some useful hints can be found in the wide literature about synchronisation in natural sciences [67]. The pedestrian-

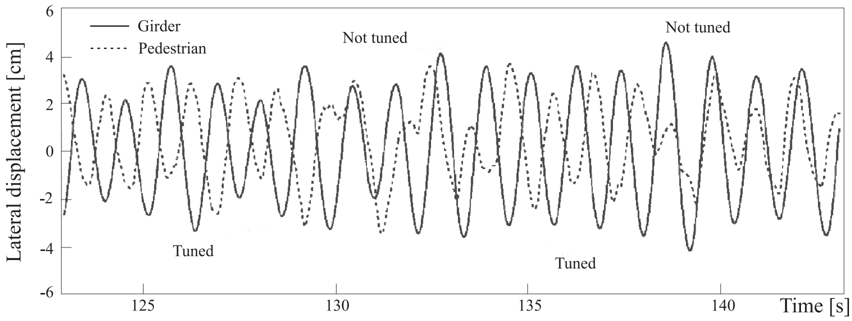


Figure 11: M-bridge: lateral displacements of girder and pedestrian at the L/4 position, after Nakamura [62]

structure synchronisation is, in fact, an example of entrainment of a self-sustained periodic oscillator by an external force, where the pedestrian is the oscillator, while the external force is represented by the structure inertial force, which is proportional to the structure acceleration. Lets ω_p be the circular frequency of the autonomous oscillator, Ω_p the frequency of the driven oscillator and ω_s the frequency of the external force. For a fixed value of the force amplitude $\bar{F}$, the frequency of the driven oscillator depends on the *frequency detuning*, i.e. on the difference $\Omega_p - \omega_s$: for small absolute value of the detuning, the external force entrains the oscillator (i.e. $\Omega_p = \omega_s$) even for low value of the force amplitude; if the detuning is above a critical threshold, the synchronisation occurs only by increasing the value of the force amplitude $\bar{F}$. Fig. 12a plots the frequency difference $\Omega_p - \omega_s$ versus ω_s for a fixed value of $\bar{F}$: the identity of frequencies that holds within a finite range of the detuning is called *frequency locking*. The trend of the frequency detuning $\Omega_p - \omega_s$ versus ω_s and $\bar{F}$ is sketched in Fig. (12b): the domain where $\Omega_p = \omega_s$ is called *synchronisation region* or *Arnold tongue* and is highlighted in grey.

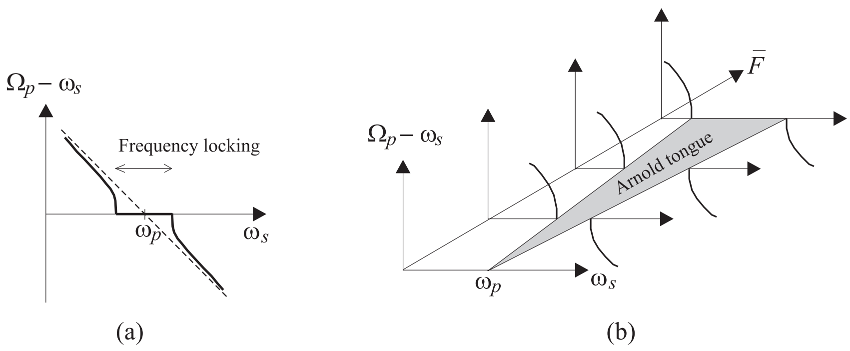


Figure 12: Frequency locking (a) and Arnold tongue (b) [67]

This framework applies to another phenomenon which has some similarities with the SLE and for this reason has inspired some authors [68, 69, 70]: it is a fluid-structure interaction (FSI) phenomenon commonly known as lock-in in Wind Engineering. In this case, the cross-flow oscillations of a bluff structure are due to and interact with the shedding of vortices in its wake. Even though the vortex-induced and crowd-induced oscillations differ in their causes and in the kind of physics they belong to (i.e. purely classical fluid and solid mechanics for FSI, hybrid classic and physics of life for CSI), they show analogous features about the structural response. In a given range of the incoming wind velocity (i.e. the synchronisation region), the vortex-shedding frequency is in fact constant and equal to the frequency at which the structure oscillates, rather than being a linear function of the wind velocity, as stated by the well-known

Strouhal law [71]. In other terms, the structural motion affects the wind flow so that synchronisation occurs and the resonance condition takes place. Furthermore, both phenomena are self-limited, in the sense that structural oscillations do not proceed to divergent amplitudes but enter a limit cycle even though the structural damping is null. These similarities brought some authors [69, 68] to propose the definition of a *pedestrian Scruton number*, in analogy to the Scruton number used in wind engineering. This number, which depends on the structural damping and on the ratio between the structure and crowd mass, can be used as an indicator of the likeliness of occurrence of SLE on a particular footbridge. According to the writers, the introduction of this sinthetic number in CSI is somewhat questionable, because the specific features of the crowd behaviour are not included in.

3 MODELLING STRATEGIES

The earlier and most common approach to deal with the problem of crowd-structure interaction considers and models the pedestrians as a simple action applied to the structure. The problem, therefore, reduces to the calculation of the structural response under the action of a suitable load model. According to this approach, several load models have been proposed (reviewed e.g. in [17]): some of them try to take into account the synchronization by means of empirical coefficients and to establish a dependence of the force on the structural response, givig rise to non linear load models. A different and more recent strategy is inspired to the one adopted for flow/wind-structure interaction problems, that is, the pedestrians are considered as a dynamical system, which has its own governing rules and that interacts with the structure system. This approach results in coupled models characterized by non-linear, multi-physic and multi-scale features and therefore require a computational approach to be solved. Moreover, special attention must be paid to the modelling of the crowd and of the interaction terms because of the special features of the crowd subsystem already mentioned in the previous sections. These modelling issues are treated in the following sections.

3.1 Crowd models

Crowd dynamics modelling is quite recent and is mainly derived from vehicular traffic modelling, which has been widely analysed in the field of applied mathematics and transportation engineering since the pioneering work of Prigogine and Herman at beginning of the Seventies [72]. An up-to-date review and critical analysis of the traffic and crowd models so far proposed can be found in Bellomo and Dogbé [73]. The literature on crowd dynamics, which has been arguably initiated by the works of Henderson (e.g. [8]), has undergone a rapid development in recent years, being motivated by the engineering demand for dimensioning of large transportation facilities, such as underground stations or airport terminals, and by safety problems, such as evacuation under panic conditions in case of danger or structural collapses.

According to the observation scales introduced in §2.1, three different kinds of modelling framework can be derived, corresponding to microscopic, mesoscopic or macroscopic description. It should be pointed out that each type of representation is chacterised by advantages and disadvantages and, at the present state of the art, it is not possible to establish the validity of a class of models with respect to the others [73]. This section is mainly devoted to the macroscopic modelling framework, since it is the only one so far used in crowd-structure interaction literature. Nevertheless, a brief description of the mesoscopic and microscopic mathematical structures is given at the end of the section in the perspective of their future application to this kind of problems. It is worth pointing out that what follows refers to modelling in normal flow condition, that is, in absence of panic, coherently with the subject of this contribution.

The representation of crowds is usually performed in two space dimension. In the writers' opinion, the particular case of interest, namely dense crowds crossing footbridges, can be represented as a monodimensional and unidirectional flow, as in the case of vehicular traffic dynamics. The flow along footbridges is mainly monodimensional because of their line-like geometry (the footbridge length is one or two order of magnitude larger than the width of the walkway) and because pedestrians usually share the main objective of crossing the bridge with maximum efficiency and minimum time [74]. Unidirectional crowd flow frequently occurs on footbridges due to the occurrence of particular events (opening day, demonstrations) or due to their specific function and location (link to transport facilities, such as railway stations or bus terminals). Indeed, most of the SLE occurrences have taken place in one of the above described conditions: for instance, the Maori demonstration on the Auckland Harbour Bridge in 1975, the London Millenium Bridge opening day in 2000 [1], the evacuation from a boat race stadium to the bus terminal in the T-bridge in Japan in 1989 [65]. Neverthless, the more general description in two space dimension is reported in the following: the monodimensional one can be easily derived as a particular case. Therefore, let us consider the system in two space dimension and let D be the domain occupied by the crowd, that can be either bounded or unbounded (Fig. 13).

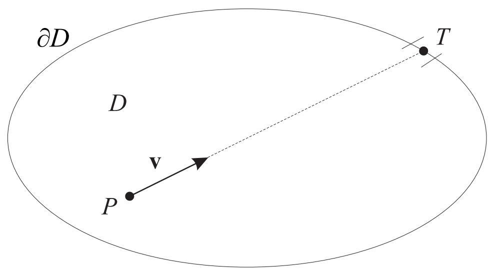


Figure 13: Geometry of the domain occupied by the crowd (P=position, T=target)

3.1.1 Macroscopic models

Macroscopic models, in analogy with the principles of fluid dynamics, refer to the derivation, on the basis of conservation equations and material models, of an evolution equation for the mass density and linear momentum, regarded as macroscopic observable quantities of the flow of pedestrians assumed to be continuous. Macroscopic models are based on the hypothesis that the crowd as a whole acts in a rational way and that all individuals have the same characteristics and share the same goal, called 'target' (T). In addition, the possibility of the presence of objects or persons that the pedestrians want to avoid should be considered in the model by introducing their repulsive effect. The general mathematical framework is given by the system of partial differential equations (PDEs), that express the conservation of mass and momentum, written in two space dimension:

$$\begin{cases} \partial_t \rho + \nabla_{\mathrm{x}} \cdot (\rho \mathbf{v}) = 0, \\ \partial_t \mathbf{v} + \mathbf{v} \cdot \nabla_{\mathrm{x}} \mathbf{v} = \mathbf{A}[\rho, \mathbf{v}], \end{cases} \tag{11}$$

where $\rho = \rho(\mathbf{x}, t)$ and $\mathbf{v} = \mathbf{v}(\mathbf{x}, t)$ are the crowd density and velocity, respectively; $\mathbf{x} = \{x, y\}$ and t are the space and time independent variables, respectively; $\mathbf{A}[\rho, \mathbf{v}]$ models the mean acceleration and square brackets indicate that it may be a fuctional of its arguments; the *dot-product* denotes inner-product of vectors. The functional dependence of $\mathbf{A}$ on its arguments is called a constitutive assumption. Different constitutive assumptions lead to different models, describing different actual situations. According to the specific constitutive assumption, models can be derived that involve only some of the two equations in (11), namely:

- First order models: only described by the first equation with a closure $\mathbf{v} = \mathbf{v}_e[\rho]$ that links the local velocity to the crowd density (e.g. [7]), where the subscript e refers to the equilibrium conditions;
- Second order models: obtained from both equations with the addition of a phenomenological relation that describes the psycho-mechanic action $\mathbf{A} = \mathbf{A}[\rho, \mathbf{v}]$ on the pedestrians, that is, the internal driving force or motivation of the pedestrian.

First order models First order models, which are reviewed in [75], need plugging experimental data into the model through the definition of a suitable closure equation. Different models can be proposed to describe the closure equation, which can be rewritten in the form:

$$\mathbf{v} = v[\rho]\mathbf{u}(\mathbf{x}), \tag{12}$$

where $\mathbf{u}(\mathbf{x})$ indicates a unit vector in the direction of $\mathbf{v}$. Three different models can be distinguished:

1. *Models of first kind.* They assume that the velocity is only a function of the local density, therefore $v(\rho)$ can be simply given by one of the speed-density relations discussed in §2.1. The pedestrians move from position P towards the target T, therefore the direction $\mathbf{u}(\mathbf{x})$ is given by:
$$\mathbf{u}(x, y) = \frac{\overrightarrow{T - P}}{\left|\overrightarrow{T - P}\right|}. \tag{13}$$
2. *Models of second kind.* They are based on the more realistic assumption that the pedestrians move towards the target with a speed that depends not only on the density but also on its gradient:
$$\mathbf{v} = v[\rho, \nabla\rho]\mathbf{u}(\mathbf{x}). \tag{14}$$
This can be achieved, for instance, by using an apparent density model [76] which is based on the concept that the pedestrians feel an apparent density ρ^*, which is larger than the real one, if the local density gradient is positive (trend to jam conditions), while it is smaller than the real one if the gradient is negative (trend to vacuum). The apparent density can be expressed in analogy with the De Angelis' proposal [77] for vehicular traffic as:
$$\rho^* = \rho\left[1 + \eta(1 - \rho)\nabla\rho \cdot \mathbf{u}\right], \tag{15}$$
where η is a positive parameter. Similar effects can be achieved through a non-local model, namely by introducing in the closure equation a space dislocation $\delta \geq 0$, as suggested in [76], so that $v = v[\rho(x + \delta, t)]$. From a phenomenological point of view, the dislocation length takes into account the non local and anisotropic pedestrian behaviour described in §2.1. A similar assumption has been made in [78] for a kinetic vehicular model, where δ is referred to as *visibility length*. A non-local crowd model has been

developed by the writers and coworkers in [18]. The space dislocation introduces in the mass conservation equation a diffusive term that has the effect of turning the hyperbolic equation into a parabolic one. The diffusive term prevents the occurrence of unrealistic shock wave phenomena, due to the fact that equilibrium conditions that correspond to a steady uniform flow are instantaneously imposed in unsteady out-of-equilibrium conditions.

3. *Models of third kind.* First-order models of crowds dynamics can be further improved observing that, in the reality, the pedestrians in a crowd do not follow a straight line towards their target. Rather, in their motion they try avoiding high density zones, in this way following minimum density paths. In this case, also the direction vector depends on the density ($\mathbf{u} = \mathbf{u}(\rho)$).

Second order models Second order models, which are reviewed in [24], need the description of the acceleration term $\mathbf{A}[\rho, \mathbf{v}]$. The acceleration can be viewed as the sum of two contributions:

$$\mathbf{A}[\rho, \mathbf{v}] = A_F[\rho, \mathbf{v}]\mathbf{u}(\mathbf{x}) + A_P[\rho, \mathbf{v}]\mathbf{u}(\mathbf{x}), \tag{16}$$

where A_F is the frictional acceleration, due to the adaptation to the mean flow velocity v_e in steady uniform flow condition, and A_P is the acceleration between pedestrians, due to the adaptation to local density gradients. For a complete review of the specific models that can be obtained from the above general expressions, the interested reader is addressed to [24, 73].

In conclusion, it should be kept in mind that the macroscopic description represents only a rough approximation of the physical reality, since the system under consideration does not satisfy the classical continuity assumption. The flow is in fact granular, which means that distances among pedestrians may not be negligible with respect to the length of the walkway, especially in low density regimes. Another drawback is that macroscopic models assume all pedestrians behaving in the same averaged way. On the other hand, these kinds of models allow a quite immediate application and are characterised by a lower computational complexity with respect to microscopic or kinetic models. Therefore, to the authors' opinion, they are suitable to be used when they have to be coupled with models describing mechanical systems, as in the case of crowd-structure interaction.

3.1.2 Mesoscopic or kinetic models

The so-called generalised kinetic models [73] describe the evolution of the probability distribution functions of the velocity and position of the pedestrians. The mathematical structure is given by a set of non-linear integro-differential equations of the type:

$$\partial_t f(\mathbf{x}, \mathbf{v}, t) + \mathbf{v} \cdot \nabla_{\mathbf{x}} f(\mathbf{x}, \mathbf{v}, t) = \mathcal{J}[f](\mathbf{x}, \mathbf{v}, t), \tag{17}$$

where $f(\mathbf{x}, \mathbf{v}, t)$ is the distribution function over the microscopic state, being $\mathbf{x}$ and $\mathbf{v}$ the position and velocity of the pedestrians at time t, and $\mathcal{J}[f]$ is an operator, in general non linear, which models the interactions among pedestrians.

The existing literature on the application of this approach to crowd dynamics is still in progress (e.g. in [80]). Guidelines and perspective ideas are reported in [73], where some kinetic models developed for vehicular traffic, such as the so-called *models with weighted binary interactions* [81] or *models with long range interaction*, are suggested as suitable to be adapted to pedestrian traffic since they include the concept of visibility zone.

3.1.3 Microscopic models

Microscopic models describe the dynamics of each single pedestrian under the action of the surrounding people. The state of the system is described by the position and velocity of each pedestrian as a function of time. Models developed at a microscopic scale are generally described by a system of ordinary differential equations (ODEs). The structure, analogous to that of Newtonian dynamics, is as follows:

$$\begin{cases} \dfrac{d\mathbf{x}_i}{dt} = \mathbf{v}_i, \\ \dfrac{d\mathbf{v}_i}{dt} = \mathbf{F}_i(\mathbf{x}_i, \ldots, \mathbf{x}_N, \mathbf{v}_i, \ldots, \mathbf{v}_N), \end{cases} \tag{18}$$

where $\mathbf{x}_i(t) = \{x, y\}_i$ is, for $i = 1, \ldots, N$, the position in D of each ith individual of a crowd of N individuals; $\mathbf{v}_i(t) = \{v_x, v_y\}_i$ is the velocity of each person; t is the time independent variable; x and y are the space independent variables.

Different modelling approaches correspond to different ways of representing the acceleration term $\mathbf{F}_i$ on the basis of the interpretation of individual behaviours. Among the different microscopic models that have been developed (e.g. [11]), it is worth citing the *social force models* (e.g. [9, 10, 82], the *cellular automata models* (e.g. [83]) and the *magnetic force models* (e.g [84]).

The solution of Eq. (18) provides the time evolution of position and velocity of pedestrians. Macroscopic quantities, such as density and mass velocity, are then obtained by suitable averaging processes performed either at fixed time over a suitable space domain or at fixed space over a suitable time interval.

The main critical aspects related to microscopic modelling consist in dealing with a large number of equations and in transferring the microscopic information to macroscopic quantities that can be observed or measured. Another issue that should be carefully considered is the heterogeneous behaviour of pedestrians, due to a change in the environmental conditions, such as the transition from normal to panic conditions.

3.2 Force models

A suitable load model should be defined in order to describe the external action that the pedestrians exert on the structure. Force models are generally classified into two main categories [85]: time domain and frequency domain force models. Time domain force models usually describe the pedestrian action as a periodic force: they can be deterministic, when a general model is proposed for each human activity (i.e. walking, running, jumping), or probabilistic, when they take into account the fact that most of the parameters that influence the human force (like body weight or walking frequency) are random variables that should be described in terms of their PDF. Assuming that the structure is a linear or linearized system, frequency domain force models could be alternatively proposed: they are based on the more realistic assumption that pedestrian loads are random processes and walking forces are represented by their PSD. This approach, which is widely used in earthquake engineering, represents a topical research axis in the case of the pedestrian vertical action [86, 87] but, to the authors' knowledge, it has never been directly applied to the SLE.

In the following, the attention will be focused on the time domain force models that have been proposed to describe the action of several pedestrians. Specifically, this section deals with those models that are specifically addressed to the SLE or that try to account for one or both synchronisation phenomena (among the pedestrians and between the pedestrians and the structure), while a critical review extended also to vertical force models can be found in [61, 17].

Bearing in mind the observation scales introduced in 2.1, it is worth pointing out that most of the design-oriented compact force models come from empirical data obtained for the single pedestrian and hence the crowd force is obtained by multiplying the single pedestrian force by an equivalent number of pedestrians. The latter is usually estimated from observation of real crowd events and, according to the authors, it can be considered as an implicit way to tackle the scaling problem involved by passing from the microscopic to the macroscopic description.

Most of the load models proposed in literature are based on some common simplifying assumptions:

- the crowd-footbridge system is modelled as a structural oscillator to which some external load is applied, therefore the crowd is viewed as an imposed load, rather than as a dynamical system;
- the structural response is dominated by one mode, therefore the structure dynamics is described by the following equation of motion:

$$\ddot{p}_j(t) + 2\xi_j\omega_j\dot{p}_j(t) + \omega_j^2 p_j(t) = \frac{1}{M_{sj}}F_j(t), \tag{19}$$

  where $p_j(t)$ is the principal coordinate of the jth mode, ξ_j is the jth modal damping ratio, $\omega_j = 2\pi f_{sj}$ is the jth natural circular frequency being f_{sj} the natural frequency, M_{sj} is the modal mass and $F_j(t)$ the modal force;

- the crowd is uniformly distributed along the footbridge span;
- the force is periodic and represented by a Fourier series [88, 89]. Usually only a single harmonic, having the same frequency as the footbridge frequency, is retained.

In order to give a homogeneous description of the selected models, a common notation is introduced. The percentage of synchronised pedestrians is generally indicated as S, while the nomenclature S_{ps} and S_{pp} is introduced when the authors explicitly refer to the synchronisation between the pedestrians and the structure or among the pedestrians, respectively.

Before focusing on the lateral force models, it is worth citing the Matsumoto et al.'s [44] model, firstly derived for bridges vibrating in the vertical direction, since it represents one of the first attempts to model the action of several pedestrians. Assuming that the number of pedestrians per second that enter the bridge follows a Poisson distribution and that they walk with the same frequencies and random phases, the total structural response can be obtained by multiplying a single pedestrian's response by the multiplication factor $\sqrt{N}$, where N is the number of pedestrians on the bridge at any time instant. Therefore, the force exerted by N pedestrians can be expressed as:

$$F(t) = \sqrt{N}\alpha G \sin(2\pi f_p t), \tag{20}$$

where α is the Dynamic Load Factor (DLF) - i.e. the ratio of the force amplitude to the weight G of a single pedestrian ($\bar{G} = 700N$) and f_p is the walking frequency. This model considers all pedestrians as uncorrelated, therefore it is not suitable for use in the presence of synchronisation phenomena due to lateral vibrations. In fact, the application of the model to estimate the structural response on SLE test-cases [65] significantly underestimated the measured lateral vibration amplitude.

The possibility of synchronisation among pedestrians due to high crowd density (0.6-1 ped/m^2) is envisaged by Grundmann et al. [90]. Piccardo and Tubino [91] report a force model which interpret this assumption:

$$F(x,t) = S_{pp}\alpha m_c(x) g \sin(2\pi f_p t), \tag{21}$$

where g is the gravity acceleration and $m_c(x)$ is the distribution of crowd mass along the bridge. In case of uniform crowd distribution, $m_c(x)g = NG/L$. Grundmann et al. explicitly ascribe the synchronisation phenomenon to the constrained movement of pedestrians due to high crowd density: for this reason the S_{pp} notation is introduced in Eq. (21). It is worth pointing out that Grundmann et al. disregard the occurrence of crowd-structure synchronisation in the case of perceptibly moving surface: on one hand, the latter phenomenon was described for the first time in the same year by Fujino and co-workers [65] and, on the other hand, the clear distinction between the two kinds of synchronisation has been introduced later [85, 58, 59].

Since the observation of the first SLE occurrences, several models have focused the attention on the synchronisation between the pedestrians and the structure. They generally express the lateral force as a function of the structural response and assume that the pedestrians are synchronised to the structure (i.e. walk with the same frequency as the structure, that is, $f_{pl} = f_s$). Fujino et al. [65] adopted a model similar to Eq. (21), where a guess value of the DLF α has been assumed higher than the one measured on a motionless platform, in order to recover a good estimate of the structural response measured on the T-bridge. After the closure of the London Millennium Bridge and the field tests performed on it, Dallard et al. [1] observed a linear relationship between the lateral force and the local lateral velocity of the deck $\dot{z}$ (Fig. 10), after the pedestrians had synchronised to the structure. Hence, the force exerted by N uniformely distributed pedestrians synchronised to the structure is empirically modelled as:

$$F(x,t) = k_1 \frac{N}{L} \dot{z}(x,t), \tag{22}$$

where the proportionality factor k_1 has to be determined experimentally and set equal to 300 Ns/m for the Millennium Bridge. Bearing in mind that the viscous damping force is proportional to the structural velocity as well as the pedestrian force, the moving pedestrians can be viewed as negative dampers (i.e. amplifiers), providing positive energy input in agreement with the phenomenological observation described in §2.2. If the magnitude of the structural damping force is lower than the one of the pedestrian force, the system is unstable, that is, the structural response tends to infinity for small perturbations. This is not in line with the actual self-limited nature of the force, discussed in §2.2. Another drawback of the model is that it describes the pedestrian action after the lock-in triggering, while it does not permit the pre-lock-in and triggering phases to be modelled. The Dallard et al.'s load model is well-suited to obtain a stability condition for the occurrence of SLE, based on the derivation of a critical number of pedestrians N_c that trigger the lock-in. The latter is derived by setting the equality between the structural modal damping force $2\xi_j\omega_j\dot{p}_j(t)M_{sj}$ and the pedestrian modal force $F_j(t)$, where $F_j(t) = \int_0^L F(x,t)\varphi_j(x)dx$, being L the span length and $\varphi_j(x)$ the mode shape. Other authors have proposed alternative stability criteria [56, 69, 92, 93], which are useful for design purposes, but out of the scope of this review.

Piccardo and Tubino [91] proposed a refinement of Eq. (21), by expressing the DLF α as a linear function of the footbridge lateral displacement z:

$$F(x,t) = S[\alpha + k_2 z(x,t)] m_c(x) g \sin(2\pi f_{pl} t), \tag{23}$$

where $k_2 \cong 2$ m^{-1} from the experimental data in [1]. Such a simple model allows to express the pedestrian force in both cases of motionless or moving platform accounting for the self-excitation mechanism. A single synchronisation coefficient S is introduced, which seems to include both kinds of synchronisation effects. This model shows the same shortcoming as the Dallard et al.'s model, that is, the linear dependence of the DLF on the structural response causes the force to have no upper limit when the pedestrian force is higher than the damping force.

Nakamura and Kawasaki [94] proposed an improvement of the Dallard at al.'s model, in order to account for the force self-limiting. The modal force is expressed as:

$$\begin{cases} F_j(t) = S_{ps} H(\dot{p}_j) k_3(f_{sj}) \alpha M_{pj} g, \\ H(\dot{p}_j) = \dfrac{\dot{p}_j(t)}{k_4 + |\dot{p}_j(t)|}, \end{cases} \tag{24}$$

where the DLF α is taken equal to 0.04 and M_{pj} is the modal mass of the pedestrians. The percentage of pedestrians synchronised to the structure S_{ps} is assumed equal to 0.2 from the laboratory tests of Dallard et al. [1]. The function $k_3(f_{sj})$ describes how pedestrians synchronise with the bridge natural frequency f_{sj}: the authors assume that "pedestrians are most likely to synchronise at the frequency around 1.0 Hz, but it is unknown how wide the bridge frequency range around 1.0 Hz affects the synchronisation nature", so that k_3 is set constant and equal to unit in absence of experimental data. The function $H(\dot{p}_j)$ describes the "pedestrians synchronisation nature": the authors assume that the pedestrians synchronise proportionally with the girder velocity at low velocity values, while, for higher values, the pedestrians feel uncomfortable or unsafe and they detune. The value of $k_4 = 0.01$ is determined by trial and error based on the T-bridge data. This model, in spite of its compact expression, provides a more accurate description of the pedestrian-structure synchronisation mechanism, which is recognised to be a function of both the deck velocity and the frequency detuning. A comparison between the force per person obtained with the models of Dallard at al. and Nakamura and Kawasaki is shown in Fig. 14.

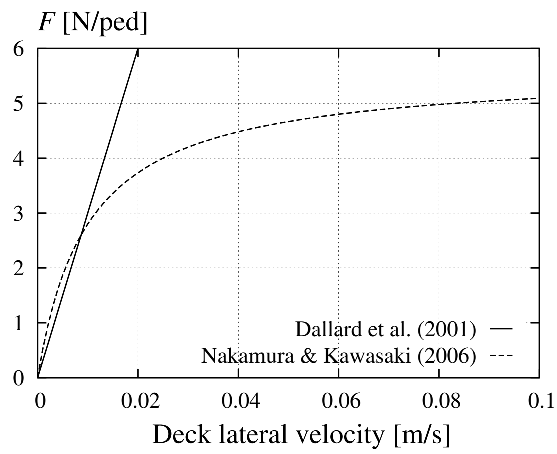


Figure 14: Comparison between the models in [1] and [94]

The models described so far explicitly take into account only one of the two synchronisation phenomena. A more advanced model was proposed by Newland [69], who expressed the lateral load as the sum of two terms, the first representing the force exerted by pedestrians on a fixed ground and the second being the component due to the deck lateral motion, under the hypothesis of small amplitudes of the deck motion ($z < 10$ mm):

$$F(x,t) = S_{pp} m_c(x) \ddot{z}_p(x,t) + S_{ps} \alpha_{ps} m_c(x) \ddot{z}(x, t-\tau), \tag{25}$$

where $z_p(x,t)$ is the displacement of the pedestrian's centre of mass on a fixed ground, α_{ps} is the ratio between the motion amplitude of the pedestrian's centre of mass and the platform (taken equal to 2/3) and τ is the time lag between the motion of the pedestrian's centre of mass and pavement. Because of the lack of data, Newland assumes $S_{ps} = S_{pp} = S$ constant in time and space. The model has the merit of recognising the different contribution of the two kinds of synchronisation and of considering the time delay of the pedestrian reaction with respect to the structural response. On the other hand, Newland assumes that the presence of the pedestrians does not modify the structural modal properties, which is questionable in the case of high crowd to structure mass ratio.

A quite different approach to account for synchronisation phenomena in the force model has been proposed by Strogatz et al. [56]. The model is explicitly formulated in a microscopic framework, where the force exerted by N pedestrians is espressed as:

$$F(t) = \sum_{i=1}^{N} F_i = \alpha G \sum_{i=1}^{N} \sin \Theta_i, \tag{26}$$

where Θ_i is the phase of the ith pedestrian, viewed as a weakly-coupled limit-cycle oscillator, according to the Kuramoto model [55]. This model expresses the Winfree's intuition about collective synchronisation (§2.1) in the phase equation

$$\frac{d\Theta_i}{dt} = \omega_i + Kr \sin(\Psi - \Theta_i), \ i = 1, \dots, N, \tag{27}$$

where Θ_i and ω_i are the phase and natural frequency of the ith oscillator, respectively; Ψ is the mean phase; the product Kr is the effective coupling, where K is the coupling strength and r the coherence. As the population becomes more coherent, the effective coupling increases and more oscillators are involved in the synchronisation process, that is, their phases Θ_i tend to the mean phase Ψ. In the Strogatz model, the bridge motion is assumed to alter the pedestrian's gait according to a modified version of Eq. (27):

$$\frac{d\Theta_i(t)}{dt} = \omega_{pi} + k_5 z(t) \sin(\Psi_s(t) - \Theta_i(t) + k_6), \tag{28}$$

where the walking circular frequencies ω_{pi} are distributed with a Gaussian PDF; k_5 models the pedestrian sensitivity to the bridge motion analogously to the coupling strength in the Kuramoto equation; $\Psi_s(t)$ is the phase of the footbridge vibration; k_6 is a phase lag parameter. The most valuable aspect of the model lies in its reference to a well-established research field about collective synchronisation, which recognises the SLE phenomenon among other well-known examples of synchronisation in physics of life. The main shortcoming is due to the difficulty in measuring the parameters k_5, tuned according to the Millennium Bridge case study $k_5 \approx 16$ $\mathrm{m}^{-1}\mathrm{s}^{-1}$, and k_6, assumed equal to $\pi/2$. Moreover, this model does not account for the synchronisation among pedestrians induced by dense crowd. The same model has been rewritten by Bodgi et al. [95] in a macroscopic form and neglecting the phase lag parameter.

The load model proposed by the writing authors [96], unlike the ones previously described, has been conceived within the framework of a crowd-structure interaction model (§3.3). In this perspective, the scaling problem has been explicitly considered in order to assure a consistent modelling scale. Specifically, the force model is based on a macroscopic description of crowd dynamics, which means that the crowd is characterised by its density, averaged walking velocity and step frequency. Both the two types of synchronisation and the contribution of the uncorrelated pedestrians are considered. Hence, the force per unit length exerted by the crowd walking along the bridge span is given by the sum of three components:

$$F(x,t) = F_{ps}(x,t) + F_{pp}(x,t) + F_s(x,t), \tag{29}$$

where F_{ps} is the term due to the synchronisation between the pedestrians and the structure, F_{pp} is due to the synchronisation among pedestrians and F_s is the part due to uncorrelated pedestrians. F_{ps} has the same frequency f_s as the excited lateral structural mode, while the other two terms have the same frequency f_{pl} as the lateral pedestrian footstep. f_{pl} is assumed to vary as a function of the walking velocity v according to Eq. (10). It is worth stressing that the pedestrians who walk with a step frequency equal to f_{pl} are the ones not synchronised to the structure, that is, they are not made sensitive to the deck lateral motion.

Each term of the overall force is weighted on the basis of phenomenological considerations, by means of three weights, N_{ps}, N_{pp} and N_s:

$$\begin{aligned} N_{ps} &= \rho B S_{ps}, \\ N_{pp} &= \rho B S_{pp}(1 - S_{ps}), \\ N_s &= \rho B - N_{ps} - N_{pp}, \end{aligned} \tag{30}$$

where the crowd density $\rho = \rho(x,t)$ is one of the state variables of the crowd system, B is the width of the footbridge walking path, S_{ps} and S_{pp} are the synchronisation coefficients, which both vary in the [0 1] range. It is worth pointing out that these weights can be viewed, at the microscopic scale, as the number of pedestrians that are synchronised with the structure, synchronised to each other and uncorrelated, respectively. Thanks to the distinction of pedestrians in three populations, the model is able to describe the triggering of lock-in: even though no one is synchronised to the structure, the presence of a high crowd density results in a lateral force that triggers the lateral vibration of the bridge.

The pedestrian-structure coefficient S_{ps} is a function of two variables: the envelope of the deck lateral acceleration time history $\tilde{\ddot{z}} = \tilde{\ddot{z}}(x,t)$ and the frequency ratio defined as $f_r = f_{pl}/f_s$. The variation of S_{ps} versus $\tilde{\ddot{z}}$ is given by means of the piecewise function:

$$S_{ps}(\tilde{\ddot{z}}) = \begin{cases} 0 & \tilde{\ddot{z}} \le \ddot{z}_c, \\ 1 - e^{-b(\tilde{\ddot{z}} - \ddot{z}_c)} & \tilde{\ddot{z}} > \ddot{z}_c, \end{cases} \tag{31}$$

where the second branch is obtained from a fitting of the Dallard at al.'s experimental data [1], with $b = 2.68$. Pedestrians start to synchronise with the structure for values of $\tilde{\ddot{z}}$ higher than a critical acceleration value $\ddot{z}_c$, and everyone is synchronised when $\tilde{\ddot{z}}$ reaches the maximum value $\ddot{z}_M$. $S_{ps}(f_r)$ is supposed to have a Gaussian distribution, with a variance that grows when $\tilde{\ddot{z}}$ increases:

$$\begin{aligned} S_{ps}(f_r) &= e^{[-\eta(f_r-1)^2]}, \\ \eta(\tilde{\ddot{z}}) &= 50 e^{(-20\tilde{\ddot{z}}/\pi)}. \end{aligned} \tag{32}$$

This means that, for increasing values of the deck vibration, the pedestrians who walk with a step frequency that is different from f_s gradually become involved in the synchronisation phenomenon. For $\tilde{\ddot{z}} = \ddot{z}_M$, everyone is synchronised to the structure, whatever the value of f_r. This assumption is in agreement with Fig. 12: in case of large frequency detuning, pedestrians synchronise only for high values of the driving force amplitude, that is, for high values of the deck lateral acceleration. The synchronisation coefficient $S_{ps}(\tilde{\ddot{z}}, f_r)$ is given by the product of Eq.s (31) and (32). It is worth pointing out that the proposed form of S_{ps} can be compared to the product $S_{ps}H(\dot{p_j})k_3(f_{sj})$ in the Nakamura and Kawasaki's model (Eq. (24)).

The coefficient S_{pp} represents the degree of synchronisation among pedestrians and has been derived through a fitting of the experimental data in [5, 52], concerning standard deviation of walking frequencies as a function of the crowd density [97]. The fitting function is inspired to the trend of the coherence against the coupling strength in the Kuramoto model [55] (see Fig. 7): the coherence (i.e. S_{pp} herein) is null until the coupling strength (i.e. ρ herein) reaches a threshold value (ρ_c), which corresponds to a phase transition; for $\rho < \rho_c$ the coherence grows towards perfect synchronisation. S_{pp} is therefore expressed as:

$$S_{pp}(\rho) = \begin{cases} 0 & \rho \leq \rho_c, \\ 1 - e^{a(\rho-\rho_c)} & \rho > \rho_c, \end{cases} \tag{33}$$

where a=8.868 and ρ_c is set equal to 0.6 ped/m^2 [97]. The synchronisation coefficients S_{ps} and S_{pp} can be viewed, in a statistical framework, as cumulative density functions of an exponential and a Gaussian PDF, respectively.

Hence, the components of the total force are expressed as follows:

$$F_{ps} = N_{ps}G[\alpha(\tilde{\ddot{z}})\sin(2\pi f_s t + \pi) + \alpha(\tilde{\dot{z}})\cos(2\pi f_s t)], \tag{34}$$

$$F_{pp} = N_{pp}\alpha G \sin(2\pi f_{pl} t), \tag{35}$$

$$F_s = \sqrt{N_s}\alpha G \sin(2\pi f_{pl} t). \tag{36}$$

The F_{ps} component is written, according to Pizzimenti [57], as the sum of a component 180° out-phase of the acceleration and another in-phase with the lateral velocity. The DLF of the two components are expressed as piecewise functions of the envelopes of the deck lateral acceleration and velocity time history $\tilde{\ddot{z}} = \tilde{\ddot{z}}(x,t)$ and $\tilde{\dot{z}} = \tilde{\dot{z}}(x,t)$, respectively. Their detailed description can be found in [98]: herein it is worth pointing out that their trend comes from a fitting to experimental data for moderate deck vibration and guarantees that the amplitude of F_{ps} is self-limited for higher values of the deck motion. The last feature aims at reaching the same objective as in Nakamura and Kawasaki's model, even though through a different modelling approach. The expressions of F_{pp} and F_s are inspired to the models of Grundmann et al. (Eq. 21) and Matsumoto et al. (Eq. 20), respectively. Neverthless, it is worth recalling that these two force components vanish in congested traffic condition, due to the dependence of the walking frequency on the crowd velocity v, which is in turn dependent on the crowd density ρ (Eq. 4).

With respect to the previously described force models, the last one is based on the phenomenological description of the components of the coupled crowd-structure system in their fundamental constitutive laws, rather than being empirically derived from a single case study. This feature is expected to assure a more general applicability, as shown in [96], where the model is applied to two real cases. Moreover, this model allows several features of the SLE to be accounted for: the dependence of the pedestrian force on the state variables of crowd and structure systems, namely the crowd density and footbridge lateral response; the possibility of a inhomogeneous distribution of the crowd along the deck; the existence of two kinds of synchronisation; the presence of different frequency components in the overall force; triggering of the lock-in phenomena and the resulting self-limited oscillations. On the other hand, some shortcomings should be highlighted: the validity of the basic modelling assumptions expressed in Eq.s (29,30) should be demonstrated; the model is less design-oriented than the previous ones, which have the merit of being expressed by very compact formulas; the introduction of a greater amount of constitutive laws, with respect to the previous compact models, makes it more expensive its validation since several parameters have to be experimentally measured; many of the introduced fundamental laws are expressed in a qualitative way, due to the uncomplete knowledge of the synchronisation mechanisms that drive the SLE and the scarce availability of experimental data. This latter drawback is common to all the reviewed models and makes it difficult both the modelling and the tuning of the introduced parameters.

Finally, it should be reminded that all the reviewed models neglect both the inter-subject and intra-subject variability [99]: this means that, on one hand, all the pedestrians are supposed to exert the same dynamic force and, on the other, that each feet of a single pedestrian produce exactly the same periodic force (deterministic models). These two features could be taken into account by introducing a probabilistic approach, analogously to what proposed in recent codes of practice [5, 4, 100] for both vertical and lateral action of pedestrians not synchronised to the structure: in other words, the parameters that mainly affect the human load (walking frequency, step length, dynamic load factors) are random variables and should be expressed in probabilistic terms.

3.3 Crowd-Structure Interaction models

To the authors' knowledge, the first attempt to propose a crowd-structure interaction model was made by the writing authors in [59, 101], successively developed in [42, 79] and recently adopted by other authors [102, 95] with slightly different formulation of the subsystem models. The framework is based on the so-called *partitioned approach*, which was first proposed by Park and Felippa [103, 104] and is widely used to model multi-physic coupled systems in the aerospace, mechanical and civil engineering fields, e.g. in fluid-structure interaction. According to this approach, systems are analysed through decomposition or partitioning, which is the process of spatial separation of a system into interacting components, called *partitions* or *fields*. One of the main advantages of the partitioned approach is the possibility to separately model each part of the system and to solve it with the most suitable numerical procedures. In such a way, each system component can undergo successive improvements as soon as new experimental data and modelling strategies become available. Furthermore, new components that account for emerging features of the phenomenon can be easily added to the original framework. In the case of crowd-structure interaction, the coupled system is decomposed into two physical subsystems, the Structure (S) and the Crowd (C), which interact between each other by means of forcing terms. In the following, each part of the model is described referring to the framework schematised in Fig. 15.

The Structure subsystem is modelled as a non-linear 3D damped dynamical system, whose equation of motion can be written as:

$$[m_s + m_c(\rho)]\,\partial_{tt}\mathbf{d} + \mathscr{C}\,[\partial_t\mathbf{d}] + \mathscr{L}\,[\mathbf{d}] = F(\rho, \tilde{\ddot{z}}), \tag{37}$$

where $\mathbf{d} = \mathbf{d}(\mathbf{x}, t)$ is the structural displacement, $\mathbf{x} = \{x, y, z\}$ and t are the space and time variables, m_s is the structural mass, m_c is the crowd mass, $\mathscr{C}$ and $\mathscr{L}$ are the damping and stiffness operators, respectively, and $\tilde{\ddot{z}} = \tilde{\ddot{z}}(x, t)$ is the envelope of the lateral acceleration of the deck. Eq. (37) is non-linear for two reasons: first, the forcing term F is a function of both the crowd density and the lateral acceleration of the deck; second, the overall mass m is given by the sum of the structure and the crowd mass. The latter derives from the solution of the PDE

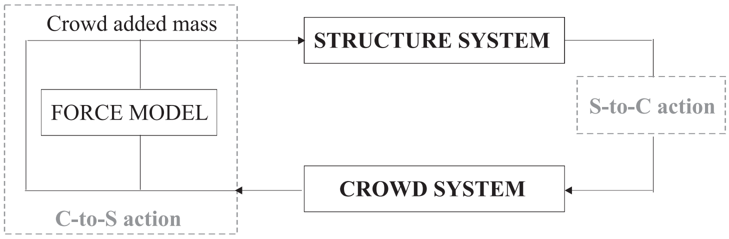


Figure 15: Framework of the time-domain coupled model

that governs the Crowd subsystem, in turn dependent on the solution of Eq. (37).

The crowd is described through a first order hydrodynamic model in the one-dimensional (1D) spatial domain. Even though crowd modelling is usually developed in 2D spatial domain, the mono-dimensional representation is more suitable to describe the phenomenon of interest, that is, dense crowd crossing footbridges, as already highlighted in §2.1. In addition, in spite of the coarse approximation of physical reality implied by the use of first order models, a relatively simple model is preferable to study the complexity of the crowd-structure coupled system. It follows that the reference framework describing the crowd dynamics is given by the 1D mass conservation equation in its Eulerian dimensional form, closed by a phenomenological relation (the closure equation) that links the mean velocity v to the mass density ρ and the lateral acceleration of the deck $\tilde{\ddot{z}}$:

$$\begin{cases} \partial_t \rho + \partial_x (\rho v) = 0, \\ v = v[\rho, \tilde{\ddot{z}}]. \end{cases} \tag{38}$$

The Structure-to-Crowd action is, therefore, expressed through the dependence of the walking velocity on the platform acceleration. Specifically, the pedestrians are assumed to adjust their step to the deck motion with a synchronization time delay τ, which is expected to be greater than the time interval between two succeeding steps ($\tau \geq 1/f_{pl}$). Therefore, bearing in mind that the pedestrians react to what they see in a suitable stretch of walkway in front of them (§3.1.1), both a space dislocation δ and a time delay τ are introduced in the closure equation:

$$v = v[\rho(x+\delta, t)]\; g[\tilde{\ddot{z}}(x, t-\tau)], \tag{39}$$

where the first term $v(\rho)$ is the speed-density relation of Eq. (4), and the second term $g(\tilde{\ddot{z}})$ is a slowing function that takes into account the sensitivity of v to the deck acceleration. The latter has a qualitative trend because of the lack of experimental data and is based on the following hypotheses:

- below the threshold of motion perception $\ddot{z}_c$, the pedestrians are not affected by the platform acceleration;
- the lateral motion of the deck reduces the walking velocity;
- after the pedestrians have stopped because of excessive lateral acceleration $\ddot{z}_M$ at time t_s, a stop-and-go time interval Δt_r should elapse before they start walking again.

It follows:

$$g(\tilde{\ddot{z}}) = \begin{cases} 1 & \tilde{\ddot{z}} \leq \ddot{z}_c \,\cap\, t \geq t_s + \Delta t_r, \\ (\ddot{z}_M - \tilde{\ddot{z}}(x, t-\tau))/(\ddot{z}_M - \ddot{z}_c) & \ddot{z}_c < \tilde{\ddot{z}} < \ddot{z}_M \,\cap\, t \geq t_s + \Delta t_r, \\ 0 & \tilde{\ddot{z}} \geq \ddot{z}_M \,\cap\, t_s < t < t_s + \Delta t_r, \end{cases} \tag{40}$$

where $\ddot{z}_c \cong 0.2$ m/s^2 [105] and $\ddot{z}_M = 2.1$ m/s^2 [62].

As far as the Crowd-to-Structure action is concerned, the scheme in Fig. 15 shows that it takes place in two ways. On one hand, the mass m is constantly updated by adding the pedestrian mass m_c to the structural mass m_s; on the other hand, a force model is proposed to determine the lateral force exerted by pedestrians on the footbridge deck [96]. The force model is the one described in §3.2 from Eq.s (29) to (36), with the following peculiarities: the space distribution of the crowd density along the bridge in any time instant is determined through the solution of Eq. (38); all the variables describing the deck motion that are introduced in the force model equations refer to the time $t - \tau$.

Further developments to this approach to crowd-structure interaction, together with proper numerical solution techniques, could provide a complementary tool to the experimental approach and a useful research tool to validate simplified load models, to reproduce the conditions of in situ tests with lower costs, to simulate scenarios which are difficult to reproduce in full scale, to simulate expected real events and to highlight emergent phenomena.

4 CONCLUSIONS

In this paper a review of the state of the art concerning the phenomenological analysis and modelling of the SLE has been proposed. Due to the multi-scale and multi-physic features of the involved complex phenomena, several scientific fields have been investigated, from biomechanics to structural engineering, from applied mathematics to transportation engineering.

The review has highlighted the still uncomplete knowledge of the mechanisms that drive the SLE, namely the synchronisation phenomena, the dependence of the force exerted by the pedestrians on the structural response, the triggering of the lock-in and the force self-limitation. The importance of a multidisciplinary approach has been outlined in this review, since many interesting suggestions can come from different research fields. Moreover, further experimental campaigns are needed in order to improve the comprehension of the phenomenon, to develop reliable models and to tune the existing ones.

As for the modelling of the parts of the complex crowd-structure system, the structural dynamic models are well-established in literature and widely applied in practice. The situation is different for the crowd models. Three modelling frameworks can be found in literature on the basis of the different observation scales (macro, meso, microscopic), but none of them have so far proved to be the most suitable to describe pedestrian traffic dynamics. The main drawback related to macroscopic models lies in the continuity assumption, which does not hold in case of very low density. On the other hand, microscopic models require handling a large number of model free parameters and equations. A further possibility for future research developments could be the proposal of a hybrid multi-scale approach: the latter could be based on the space and time combination of a microscopic description of the pedestrians in low density regime, through discrete models, with a macroscopic description of the dense crowd, through continuous first or second order models. In this framework, the hybrid approach would be dynamic, in the sense that the switch from one description to the other depends on the time and space evolution of the crowd density, that is, one of the crowd subsystem state variables. In general terms, this approach could be addressed to cope with the difficulty to describe the transition from individual to collective behaviour. Some suggestions in this direction can be found in the approach proposed in [106].

As far as the force models are concerned, the main deterministic time-domain load models specifically addressed to SLE have been reviewed. The difficulty in coupling the need for synthetic formulas to be used in design practice and the accuracy in the description of all the involved phenomena emerges from the review. On one hand, the compact models are not able to account for many of the SLE main features and are often tuned on a specific as-built structure; on the other hand, the load models proposed to tackle the above mentioned shortcomings are more demanding to be validated, due to the large number of constitutive laws introduced, and to be applied for engineering purposes. The difficulty in proposing a reliable load model for SLE is mainly related to the still uncomplete knowledge of the phenomenon. Most of the presented models can be ascribed to a macroscopic description, where inter-subject and intra-subject variability are neglected and all pedestrians are assumed to behave in the same averaged way. This assumption, which is very practical for engineering purposes, is now considered by several authors as not suitable to represent the actual nature of the pedestrian load. Therefore, a probabilistic approach to SLE could be developed.

Finally, the modelling framework proposed by the authors to develop a crowd-structure interaction model on the basis of the partitioned approach has been presented. The approach main advantage lies in the possibility to separately model each system components, which can be characterised to solve a particular problem. In the specific case dealt with in this paper, the framework has been adapted to describe the SLE phenomenon on footbridges. Both the crowd system and the interacting terms have been modelled within a macroscopic description. Even if a macroscopic approach seems more suitable in view of its practical application since it provides synthetic results on the crowd state, the coupling between the structural dynamics and a microscopic crowd modelling could be envisaged for future research perspectives. As for the interacting terms, the general framework could be improved by including new components that should account for some features so far not considered. Among others, the effects of the deck slope on the crowd dynamics is of interest due to the frequent use in civil engineering of structural types (arch structures, cable structures), which imply a deck with variable slope. The effects due to the presence of obstacles could also be modelled, since obstacles can be included in the footbridge design for architectural reasons (such as benches or lighting systems) or their presence could be used to allow control strategies of the pedestrian flow and, in turn, of the structural response.

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