COMPDYN 2011
III ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis, V. Plevris (eds.)
Corfu, Greece, 25–28 May 2011

A MODIFIED TRUNCATION METHOD FOR PRESSURE RECONSTRUCTION IN CASE OF NON PONCTUAL IMPACT ON AN ELASTIC PLATE

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Keywords: Inverse Problem, Identification, Impact, Regularization, Truncation Method, Plates

Abstract. *This work deals with reconstruction of distributed force signal resulting from a non punctual object impacting perpendicularly an elastic homogeneous and isotropic rectangular plate. The impacting force is assumed to be uniformly distributed over a rectangular patch of the plate. The direct problem was solved by using modal decomposition method with explicit analytical modes. A discrete problem was written for that by sampling the obtained convolution integral. To extract the pressure signal by deconvolution of the dynamic response measured at a given point of the plate, solution of an inverse problem had been considered. Since this type of problem is known to be ill-posed due to bad conditioning of the involved Toeplitz like matrix, regularization is needed to obtain a physically meaningful solution. A new regularization technique based on truncation filtering was examined. This technique uses as a first step the generalized decomposition of Toeplitz matrix on singular values. Then, regularization of the decomposed form through a truncation filter is performed. The truncation consists in eliminating the first low index terms up to an optimal rank representing the contribution of low amplitude generalized singular values. If the impact force signal has a half sine like standard form, the index corresponding to time instant where the maximum displacement response is obtained was found to be the optimal order of truncation. This technique has proved to be effective in reconstruction of impact pressures through various cases of study and the computational cost was found to be much lower than that of the classical truncation method based on L-curve criterion.*

1 INTRODUCTION

To perform structural health monitoring or reliability analysis of structures, it is essential to provide accurate characterization of input forces experienced during service operation. In common practice, the input force is measured by using a force transducer that is positioned in the load path. On many circumstances, such as a high-speed impact of an object onto a structure, it is difficult to apply this technique such as a bird impacting an aeroplane fuselage. Another technique that has been widely employed for the impact-force signal reconstruction is based on analysis of the inverse problem. This means that the dynamic force is recovered from the data of the measured elastic response. When the impact point is known, the problem is equivalent to operating deconvolution of two signals: the measured response and the transfer function characterizing the dynamics of the structure. In many cases, the deconvolution results in an ill-posed problem in which the data noise strongly affects the solution accuracy. Therefore, it is difficult to obtain an accurate solution for such problems, so that, regularization is needed to obtain a physically meaningful solution.

There are a number of publications which deal with the impact-force reconstruction. In [1, 2, 3] the impact force profile had been reconstructed by using spectral analysis. The proposed method had utilized the convolution theorem that expresses the time domain deconvolution as a simple division in the frequency domain. Later on, various authors [4, 5, 6, and 7] adopted a more systematic approach to regularize the deconvolution problem by using either the singular value decomposition method (SVD) or the generalized singular value decomposition method (GSVD). These authors had considered the problem of a localized impact where the object could be approximated as a single point. In many cases, however, the impacting object is massive and the impact zone could not be approximated as a single point and the impacting force takes the form of a distributed pressure over the impact zone. In more recent works [8, 9, 10, 11] the problem of reconstruction of distributed dynamic loads on structures like Euler beam, thin plates or cylindrical shells had been tackled. The authors had used either the modified modal method or the mode-selection method. Though these methods are robust in comparison with Tikhonov based methods some problems such as the improvement of the selection criterion and the relative high errors on boundaries are still open [11].

Certain researchers have indicated that a major drawback of the Tikhonov-GSVD method is the expensive computational cost associated to GSVD and consequently the method is only suitable for small scale problems.

In the present work, we consider the impact pressure reconstruction problem in the case where a uniform distributed force is applied onto a homogeneous and isotropic elastic rectangular plate. The impacting zone is assumed to be a rectangular patch. For this purpose, the direct solution has been computed at first by using an analytical formula. Then, the regularization method based on GSVD method with truncation filtering was used. The truncation regularization method [12] is a particular case of the general filter factor regularization method and looks a lot like Tikhonov regularization [13], but it is simpler to implement. Here, a new technique of constructing the filter is examined. It is based on an a priori defined truncation order which reduces considerably the computational cost. This new method was tested on several case studies and the obtained results have shown that it is well suited in regularizing pressure reconstruction problems wherever the impact pressure profile is not too different from a half sine shape.

2 MATERIALS AND METHODS

We consider a rectangular plate as shown in figure 1 which has the dimensions a , b and e representing respectively the length, width and thickness. It is assumed to be simply supported on its ends. The plate is assumed to be made of a homogeneous and isotropic elastic material with Young's modulus E, Poisson's ratio υ and density ρ. The applied force modeling impact is assumed to be uniformly distributed over a rectangular patch of the plate. The dynamic response in terms of displacement, velocity, acceleration or strains is considered at a point which is located at a given distance away from the centre of the loading rectangle, figure 1.

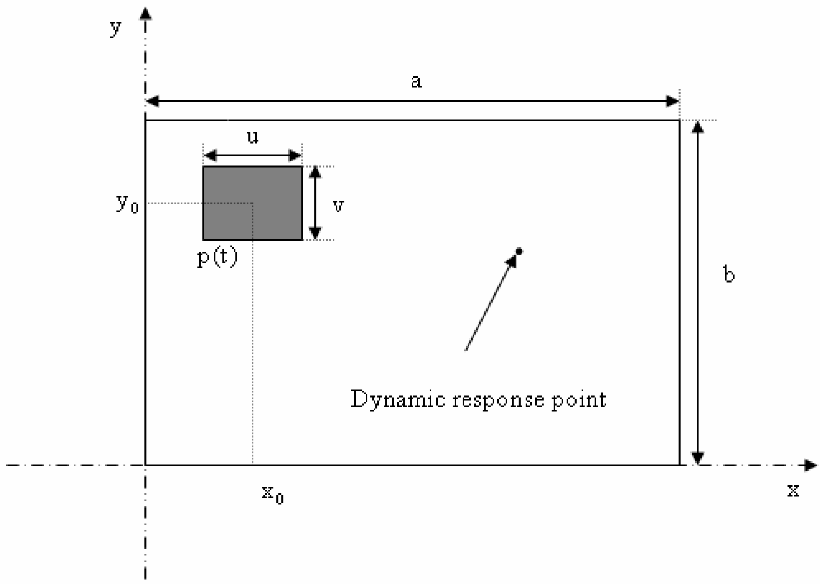

Figure 1: Simply supported rectangular plate showing the loading conditions and the point of response measurement

The equation of motion of a simply supported rectangular plate [14], can be expressed under the following form

$$D\Delta\Delta w(x,y,t) + c\dot{w}(x,y,t) + \rho h\ddot{w}(x,y,t) = q(x,y,t) \quad (1)$$

where x is the horizontal coordinate, y the vertical coordinate, t the time, $w(x,y,t)$ the transverse displacement, $q(x,y,t) = p(t)\Im_{[x_0-u/2,x_0+u/2]\times[y_0-v/2,y_0+v/2]}(x,y)$ the applied loading with $\Im$ the indicative function taking the value one on the domain shown in subscript and zero elsewhere, c the damping coefficient, $D = Ee^3/(12(1-\upsilon^2))$ the plate flexural rigidity modulus and $\Delta\Delta w(x,y,t) = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^2 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}$.

The above governing equation is assumed to be subjected to the following boundary conditions

$$w = 0 \quad \text{and} \quad \frac{\partial^2 w}{\partial x^2} = 0 \quad \text{for } x = 0 \text{ and } x = a$$

$$(2)$$

$$w = 0 \quad \text{and} \quad \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{for } y = 0 \text{ and } y = b$$

By applying the modal superposition technique, the displacement $w(x,y,t)$ can be shown to be expressed under the following form

$$w(x,y,t)=\int_0^t h(x_0,y_0,u,v,x,y,t-\tau)p(\tau)d\tau \tag{3}$$

where h is the transfer function given by

$$h(x_0,y_0,u,v,x,y,\tau)=\frac{16}{\rho h\pi^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{mn\gamma_{mn}}\sin(\frac{m\pi x_0}{a})\sin(\frac{n\pi y_0}{b})\sin(\frac{m\pi u}{a})\sin(\frac{n\pi v}{b})\sin(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b})\sin(\gamma_{mn}\tau)e^{-\xi_{mn}\omega_{mn}\tau} \tag{4}$$

in which ω_{mn}, γ_{mn} and ξ_{mn} are respectively the circular eigenfrequency, the damped circular eigenfrequency ($\gamma_{mn}=\omega_{mn}\sqrt{1-\xi_{mn}^2}$) and the damping ratio for a given eigenmode (m,n).

In many practical circumstances it is possible to represent realistically the impact-force such as a half-sine function, figure 2. Shape of real impact force signal is not too different from this standard profile.

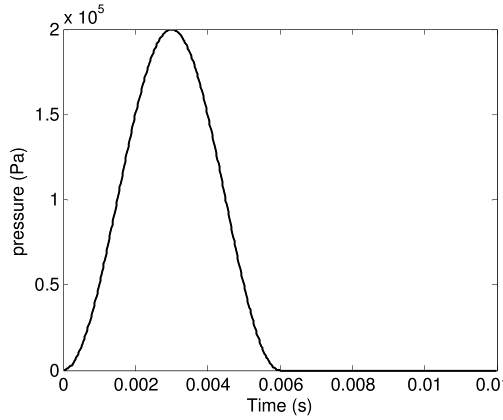

Figure 2: Pressure signal profile defined on the period time $T=0.006\,s$

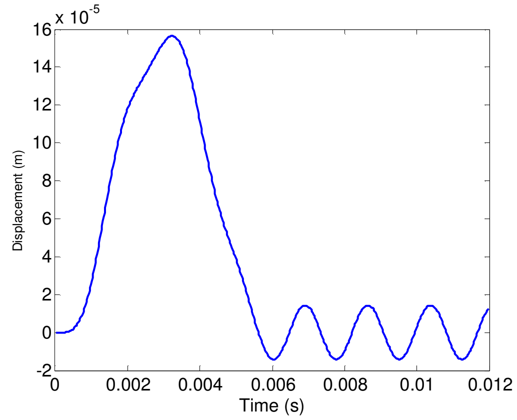

Figure 3: Calculated displacement at point $(x=0.041\,m, y=0.041\,m)$

The elastic response $w(x,y,t)$ can be computed over the considered time interval $[0,T_c]$ by integrating explicitly equation (3) where x_0, y_0, u, v , x, y take fixed values. In this work, the selected configuration of the impacted plate is defined by the following parameters values: $a=2.05$ m , $b=2.05$ m , $e=5\times10^{-3}$ m , $\xi_{mn}=0$, $x_0=y_0=0.1025$ m , $u=v=0.0342$m , $x=y=0.041$m and $T_c=0.012$s.

Figure 3 gives the displacement calculated at the point $(x=0.041\,m, y=0.041\,m)$. The direct elastic response in terms of displacement which is given in figure 3 is stored and will be used in the following to reconstruct the impact-force signal.

To identify the impact-force acting on the plate over the rectangular domain of impact, the transfer function based approach is used. In more general problems, transfer functions can be determined analytically [15], experimentally [16], or numerically. Here, the transfer function is evaluated analytically through time integration of equations (3) and (4).

To solve the deconvolution problem associated to equation (3), a discrete problem must be written by sampling the convolution integral. This leads in the time domain to the following system of algebraic equations

$$W = HP \tag{5}$$

with

$$H = \begin{pmatrix} H(\Delta t) & 0 & & & 0 \\ H(2\Delta t) & H(\Delta t) & \ddots & & \\ H(3\Delta t) & H(2\Delta t) & \ddots & \ddots & \\ \vdots & \vdots & \ddots & \ddots & 0 \\ H(N\Delta t) & H((N-1)\Delta t) & \dots & \dots & H(\Delta t) \end{pmatrix} \quad \begin{aligned} W &= \left[w(\Delta t) \quad w(2\Delta t) \quad \cdots \quad w(N\Delta t)\right]^t \\ P &= \left[p(\Delta t) \quad p(2\Delta t) \quad \cdots \quad p(N\Delta t)\right]^t \end{aligned} \tag{6}$$

where H is the Toeplitz like transfer matrix, Δt is the sampling rate and N the total number of samples.

The sampling rate must be selected in order to recover a predefined cut-off frequency in the pressure signal. The matrix H is always ill-conditioned. This means that it can lead to an unstable solution which has no physical meaning. Therefore, to find a physically acceptable solution the deconvolution problem defined by equations (5) and (6) should be regularized.

Here, the regularization technique based on the generalized singular value decomposition (GSVD) is considered. It should be mentioned that the simpler SVD method has failed to regularize the actual problem. The GSVD-regularized solution of problem defined by equations (5) and (6) can be written as follows

$$[P] = [X][\Phi][\Delta]^{-1}[U]^t[W] = \left[H^*\right][W] \tag{7}$$

where (X, Δ, U) being the singular factors of H, $[\Phi]$ is the filter factor and $\left[H^*\right] = [X][\Phi][\Delta]^{-1}[U]^t$ is the regularized pseudo-inverse of H.

The filter factors goal is to minimize the influence of the low amplitude generalized singular values. Many techniques have been considered in the literature for that purpose. Among them, one finds the regularization techniques: Tikhonov method [13] and GSVD truncation method [5]. In the following the truncation based regularisation technique is used.

The truncation consists of eliminating the first low index terms up to the rank k. This index is called the regularization parameter. The index k should be selected in order to eliminate the small generalized singular values as well as the oscillating singular vectors.

The filter Φ defined by the truncation method writes:

$$\Phi_{ij} = f_i \, \delta_{ij} \quad i, j = 1, \dots, N \tag{8}$$

To build the filter Φ within the framework of truncation method, the rank k should be specified. The optimal rank should minimize the error between the identified pressure and the real pressure. Classically, the L-Curve method has been applied in order to determine the regularization parameter k by means of a graphical based method. This technique was developed in reference [5]. It is based on searching the optimum of a functional composed of two terms, a residue called $RN = \|W - HP\|_2$ (Residual Norm) and the norm of the solution, designated by $SN = \|P\|_2$ (Semi-Norm). When the k parametric curve defining SN versus RN is plotted, the optimal regularization parameter corresponds to the point of maximum curvature, the corner.

However, in practice, this method is often problematic; for measured data the L-curve is discrete and the determination of such point is delicate, because it is not distinguishable on the graph.

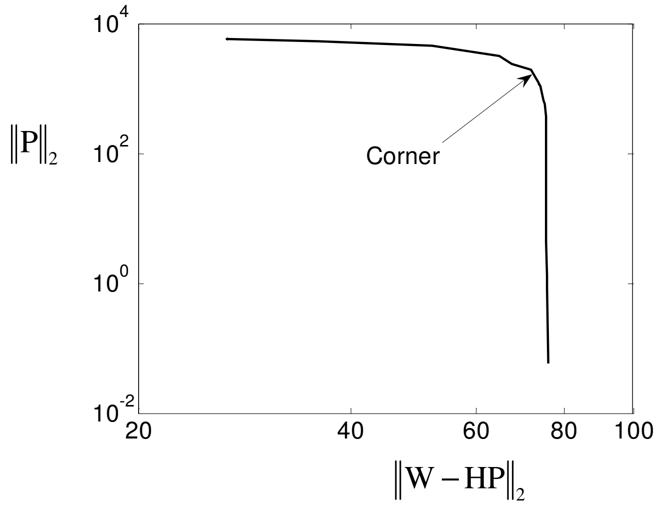

Figure 4. The L-curve associated to truncation filter design showing the corner point defining the optimal truncation order

Here a new heuristic method is proposed to determine the optimal rank truncation. Through various numerical tests conducted on pressure signals having half sine form, the rank defined as the index of the maximum value of the calculated displacement was found to yield a closer form of the real pressure input signal.

3 RESULTS AND DISCUSSION

Figure 5 presents the superposition of the real impact pressure with the pressure profile as obtained by the inverse problem solution for the impact centre zone given by $(x_0 = 0.0683\,\text{m}, y_0 = 0.0683\,\text{m})$ and the point of measurement located at $(x = 0.041\,\text{m}, y = 0.041\,\text{m})$. The pulse period considered is $T = 6\text{ms}$. The truncation order which is defined as the index of the maximum value of the calculated displacement is found to be 160. The associated CPU time is 68.94.

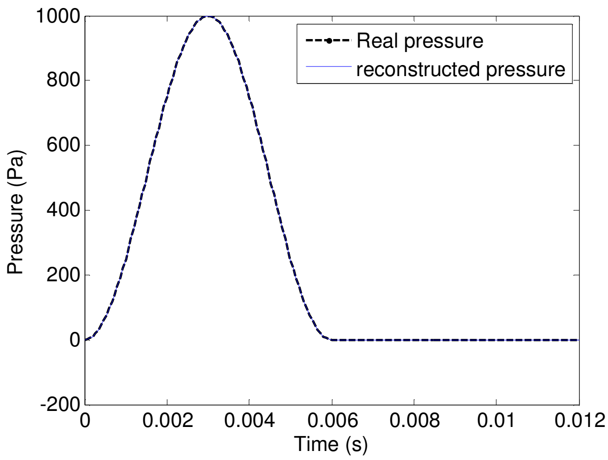

Figure 5: Comparison of the reconstructed pressure profile with the real input pressure for the input signal shown in figure 2 with period $T = 0.006\,\text{s}$

Figure 6 presents the superposition of the real impact pressure with the pressure profile as obtained by the inverse problem solution for the case where $(x_0 = 0.0683\,\text{m}, y_0 = 0.0683\,\text{m})$, $(x = 0.041\,\text{m}, y = 0.041\,\text{m})$ and a pulse period $T = 4\text{ms}$. The associated truncation order is 179.

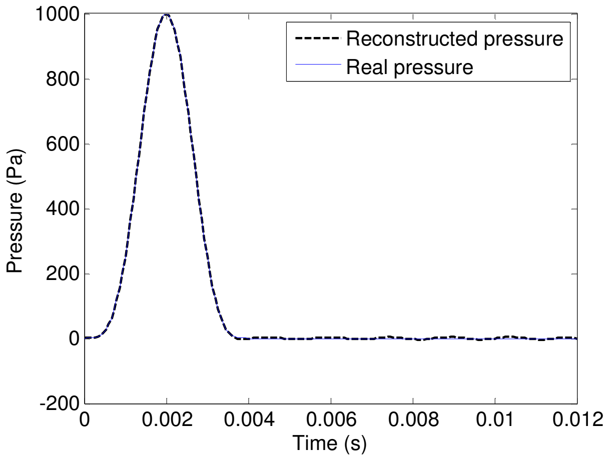

Figure 6: Comparison of the reconstructed pressure profile with the real input pressure having the form of figure 2 but with a period $T = 0.004\,\text{s}$

It is clear that the new proposed method and the L-curve based method permit the exact reconstruction of the signal. However in the new heuristic method for which the order of truncation is defined a priori as the index of the maximum value of the calculated displacement, a lower computational cost is reached. The gain is about 47%. This methodology was proven to yield results that are independent from the impact location, the measurement point and the pulse period. But, it is necessary that the profile of the impacting force should have a half sine like form. This is not a real limitation since most of the impacting force signals have this general form in practice.

4 CONCLUSION

A new heuristic method for reconstruction of distributed force in case of non punctual object impacting an elastic rectangular plate was proposed. This method is based on generalized singular value decomposition of Toeplitz like matrix obtained for the discrete convolution problem relating the displacement dynamical response at a given point and the impact pressure signal. This last was assumed to be uniform over a rectangular patch of the plate and to have a half sine profile. To build the filter needed for regularization of the inverse problem, the truncation based method was used. The order of truncation corresponding to the index of time associated to the maximum measured displacement was found to yield good results. This was proven to be the case independently from the impact location, the measurement point and the pulse period. Even when the signal is not a half sine one, good results are also obtained. The computational cost of this method is lower than that of the classical truncation method, which makes it as a relevant alternative to better build the truncation filter needed for regularising the deconvolution in inverse problems.

5 REFERENCES

[1] J.F. Doyle, Further developments in determining the dynamic contact law. *Experimental Mechanics* **24**, 265–270, 1984.

[2] J.F Doyle, Experimentally determining the contact force during the transverse impact of an orthotropic plate. *Journal of Sound and Vibration* **118**, 441–448, 1987.

[3] M.T. Martin, J.F. Doyle, Impact force identification from wave propagation responses. *International journal of Impact Engineering* **18**, 65-77, 1996.
[4] R. Adams, J.F. Doyle, Multiple force identification for complex structures, *Experimental Mechanics* **42**, 25-37, 2000.
[5]P.C. Hansen, *Rank-Deficient and Discrete Ill-posed Problems*, SIAM, Philadelphia, PA, 1998.
[6)] E Jacquelin, A. Bennani, P. Hamelin, Force reconstruction analysis and regularization of a deconvolution problem. *Journal of Sound and Vibration* **265**, 81-107, 2003.
[7] Y. Liu, S. Shepard, Dynamic force identification based on enhanced least- squares schemes in the frequency domain. *Journal of Sound and Vibration* **282**, 37-60, 2005.
[8]Y. Liu, W.S. Shepard Jr. An improved method for the reconstruction of a distributed force acting on a vibrating structure. *Journal of Sound and Vibration* **291**, 369-387, 2006.
[9]M.C. Djamaa, N. Ouelaa, C. Pezerat, J.L. Guyader, Reconstruction of a distributed force applied on a thin cylindrical shell by an inverse method and spatial filtering. *Journal of Sound and Vibration* **301**, 560-575, 2007.
[10] X.Q. Jiang, H.Y. Hu, Reconstruction of distributed dynamic loads on an Euler beam via mode-selection and consistent spatial expression, *Journal of Sound and Vibration* **316**, 122-136, 2009.
[11] X.Q. Jiang, H.Y.Hu, Reconstruction of distributed dynamic loads on a thin plate via mode-selection and consistent spatial expression. *Journal of Sound and Vibration* **323**, 626-644, 2009.
[12] B.A Mair, Tikhonov regularization for finitely and infinitely smoothing operators. SIAM *Journal of Mathematical Analysis* **25**, 135–147, 1994.
[13] A.N. Tikhonov, V. Y. Arsenin, *Solutions of ill-posed problems*, Wiley, New York, 1977.
[14] S. Timoshenko, Woinovsky-Krieger, *Theory of plates and shells*, McGraw-Hill Book Co., Eng. Mech. Series collection, 1940 (reprinted 1959), 580 p.
[15] L. Meirovitch, *Dynamics and Control of Structures.* John Wiley, New York, 1990.
[16] P.C. Hansen, *The L-curve and its use in the numerical treatment of inverse problems*, Tech. Report, IMM-REP 99-15, Dept. of Math. Model. Tech. Univ. of Denmark, 1999.

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