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SEISMIC RESPONSE OF A STONE MASONRY SPIRE

Matthew J. DeJong[1], Christopher Vibert[1]

[1] University of Cambridge
Trumpington Street, Cambridge, CB2 1PZ, United Kingdom
mjd97@cam.ac.uk, cv257@cam.ac.uk

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Abstract. *Stone masonry spires are vulnerable to seismic loading. Computational methods are often used to predict the dynamic linear elastic response of masonry spires, but this approach is significantly limited to the point when the first masonry joint begins to open. In this paper, analytical and computational modeling methods are used to address the full dynamic response of masonry spires until collapse. An analytical framework is first presented, which addresses both the elastic oscillation response and the rigid rocking response of masonry spires. In this context, the seismic response of the spire of the church of St. Mary Magdalene in Waltham on the Wolds, United Kingdom, which was damaged in the 2008 Lincolnshire Earthquake, is addressed. Both analytical and computational discrete element modeling are applied to predict the response to a variety of base accelerations. Results of both methods are compared to evaluate their utility and to understand the seismic damage which occurred.*

1 INTRODUCTION

Large earthquakes are relatively rare in the United Kingdom, and ensuing damage is often limited to unreinforced masonry spires, chimneys and towers. As spires typically top significant heritage structures, there are numerous recorded examples of spire damage during historic seismic events [e.g. 1,2]. More recently, several stone masonry spires were badly damaged during the 2008 Lincolnshire Earthquake in the United Kingdom, despite the relatively small magnitude of earthquake ground motion. The damage to one such spire, which sat atop the 13^{th} century parish church of St. Mary Magdalene in Waltham on the Wolds, provides the impetus for this research, and will be used as a case study herein.

The stability of masonry spires under static dead load and wind load was addressed by Heyman [3] in the context of ultimate load theory, assuming i) infinite compressive strength, ii) zero tensile strength, and iii) no sliding between masonry units. Heyman [3] assumes that octagonal spires can be modeled as conical shells, and demonstrates the importance of a solid spire tip to resist overturning during high winds.

The dynamic response and collapse of masonry spires has been given relatively little attention. Previous studies concentrate on the onset of cracking due to elastic response, rather than the prediction of post-damage response and complete collapse. In this study, dynamic response is also tackled in the context of rocking structures, for which the literature is extensive. Housner [4] provides the fundamental formulation for investigating the rocking response. Zhang and Makris [5] provide a critical contribution regarding the response of rocking objects to cycloidal pulses which can dominate earthquake ground motions and govern overturning collapse.

The dynamic response of masonry structures can be computationally predicted using Discrete Element Modeling (DEM), which models the contact and separation between individual stones within a masonry structure. In particular, DEM was used to model the dynamic rocking response of the masonry arches by De Lorenzis *et al.* [6], with a more in-depth sensitivity study of modeling parameters provided by DeJong *et al.* [7]. These studies demonstrate the utility of DEM to evaluate analytical models which capture the governing dynamic response.

The aim of this paper is to provide a general approach for evaluating the dynamic response of masonry spires under seismic loading, and then to evaluate that approach by assessment of the damage to the spire of the parish church of St. Mary Magdalene. The general approach is presented first, and makes use of an analytical formulation for predicting damage and rocking response. Subsequently, DEM is used to evaluate the analytical approach through an in-depth computational investigation of the spire in question.

2 ANALYTICAL FORMULATION

The analytical approach for modeling the dynamics of masonry spires will be broken down into three aspects. First, static analysis will be used to obtain a reference point for lateral stability. Second, the linear elastic dynamic response will be approximated. Finally, the rigid rocking response will be considered. A conical shell is assumed to be representative of the octagonal spire, and the three assumptions of ultimate load theory are taken [3].

2.1 Static Analysis

As a starting point, static analysis is useful to determine the minimum horizontal ground acceleration (if applied for infinite duration) necessary to cause overturning of a conical shell. For the geometry in Figure 1(a), the fraction (λ) of gravitational acceleration (g) required for overturning is:

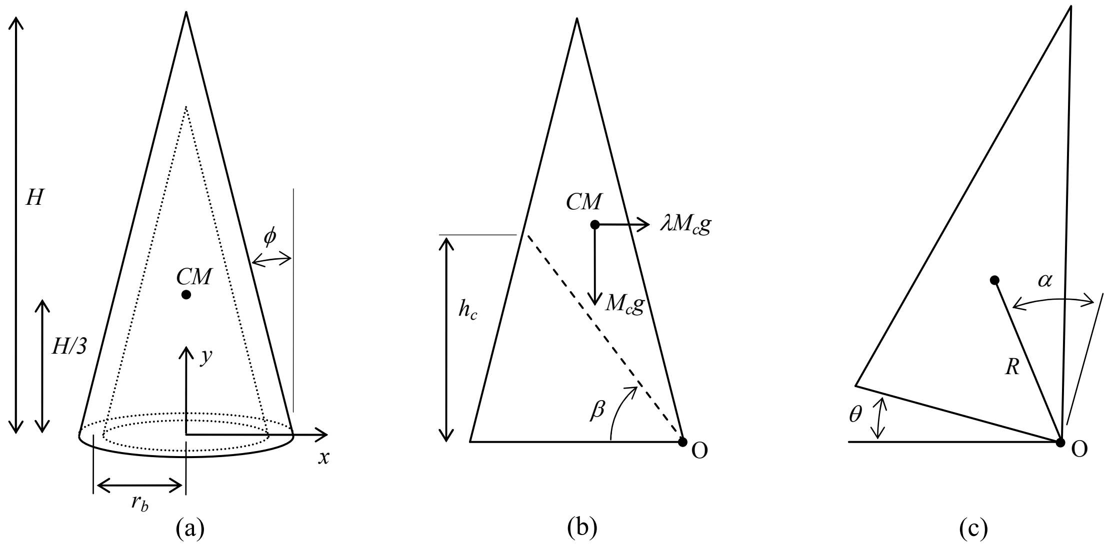


Figure 1: Definition of masonry spire geometry.

$$\lambda = \frac{3r_b}{H} \tag{1}$$

The conical shell has a relatively low center of gravity (H/3), and is therefore more resistant to overturning than a solid rectangular prism. However, assuming masonry structures have no tensile capacity, diagonal cracks may open when lateral loads are applied [8]. In reality, the location of these cracks may be limited by interlocking of blocks, but assuming that a diagonal crack can form at any angle β (Figure 1(b)), the fraction (λ) of gravitational acceleration (g) required for overturning is:

$$\lambda = \frac{3r_b}{H} \frac{\left(\frac{1}{\pi^2} h_c^{\ 3} - \left(\frac{2}{\pi^2} - \frac{2}{3} \right) h_c^{\ 2} H - 2h_c H^2 + 2H^3 \right)}{\left(h_c^{\ 3} - 2h_c^{\ 2} H + 2H^3 \right)} \tag{2}$$

where h_c is the crack height in Figure 1(b).

2.2 Dynamic Elastic Analysis

The dynamic response of masonry structures involves two stages: an initial elastic stage, during which the entire structure remains in compression, followed by a 'rocking' stage, during which masonry units separate and regain contact. The elastic stage will be addressed first, followed by the rocking stage in the next section.

Due to the slender nature of spires, Euler-Bernoulli beam theory can be used to estimate natural frequencies and mode shapes. The mass per unit height, *m(y)*, and bending stiffness, *EI*(y), of the spire can be written as:

$$\begin{aligned} m(y) &= \left(1 - \frac{y}{H}\right) m_b \\ EI(y) &= \left(1 - \frac{y}{H}\right)^3 EI_b \end{aligned} \tag{3}$$

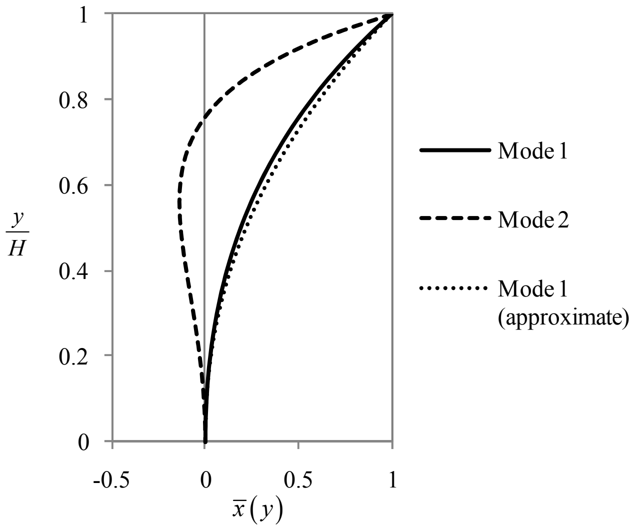


Figure 2: First two modes shapes of a conical shell [10], and the first mode shape from Equation (5) with k = 2.2.

where m_b is the mass per unit height at the base, I_b is the second moment of area at the base, and E is the Young's Modulus.

Therefore, the mass and bending stiffness vary similarly to the solid wedge beam analyzed by Naguleswaran [9], who determined the corresponding natural frequencies:

$$\omega_n = \sqrt{\frac{\Omega_n^{\ 2} E I_b}{m_b H^4}} \tag{4}$$

where Ω_n = 5.315, 15.21, and 30.02 for the first three modes. The fundamental mode shapes for the first two modes are depicted in Figure 2.

Alternatively, Rayleigh's principle can be used to estimate the mode shape and compute the corresponding natural frequencies. Assume a mode shape of the form:

$$\bar{x}(y) = \left(\frac{y}{H}\right)^k \tag{5}$$

where $\bar{x}(y)$ is the modal translation at height y, and k is a constant. The fundamental natural frequency is approximated by:

$$\bar{\omega}_1 = \sqrt{\frac{\int_0^H EI(y)\left(\frac{d^2\bar{x}}{dy^2}\right)^2 dy}{\int_0^H m(y)\left(\bar{x}(y)\right)^2 dy}} \tag{6}$$

The minimum fundamental frequency $\bar{\omega}_1$ occurs for k = 2.2, and the corresponding mode shape compares reasonably well with the actual mode shape derived by Naguleswaran [9] (Figure 2). Modal analysis using equations (5) and (6) can now be applied to determine the point at which elastic oscillation would cause damage and initiate a rocking response.

2.3 Dynamic Rocking Analysis

If the earthquake loading induces a large enough response, the spire would begin to rock as it has no tensile capacity, and the elastic natural frequencies would be completely altered. To investigate whether rocking could cause collapse, consider a rigid conical shell on a rigid foundation.

The rigid conical shell will begin to rock (Figure 1(c)) when the overturning moment exceeds the resisting moment, which occurs at a maximum ground acceleration of $a_{crit} = \lambda g$, where λ is defined in Equation (1) above. Once rocking commences, the response can be treated in a similar fashion to Housner [4], assuming spinning of the cone about its vertical axis does not occur. The equations of motion are:

$$\begin{aligned} I_0\ddot{\theta} + MgR\sin\left(+\alpha-\theta\right) &= -M\ddot{u}_g R\cos\left(+\alpha-\theta\right) \quad \rightarrow \quad \theta > 0 \\ I_0\ddot{\theta} + MgR\sin\left(-\alpha-\theta\right) &= -M\ddot{u}_g R\cos\left(-\alpha-\theta\right) \quad \rightarrow \quad \theta < 0 \end{aligned} \tag{7}$$

where M is the total mass of the cone, R, α, and θ are defined in Figure 1(c), and I_O is the mass moment of inertia about point O in Figure 1(c):

$$I_0 = \frac{5}{4}Mr_b^{\,2} + \frac{1}{6}MH^2 \tag{8}$$

Assuming small angles, equation (7) can be rewritten in the form:

$$\begin{aligned} \ddot{\theta} - p^2\theta &= -p^2\left(\frac{\ddot{u}_g}{g}+\alpha\right) \quad \rightarrow \quad \theta > 0 \\ \ddot{\theta} - p^2\theta &= -p^2\left(\frac{\ddot{u}_g}{g}-\alpha\right) \quad \rightarrow \quad \theta < 0 \end{aligned} \tag{9}$$

where $p = \sqrt{MgR/I_O}$ is the frequency parameter of the block.

Still following the formulation of Housner [4], the impact can be modeled by a coefficient of restitution, c_v, defined as the ratio of the angular velocities before and after impact:

$$c_v = 1 - \frac{MR^2}{I_O}\left(1-\cos 2\alpha\right) \tag{10}$$

Equations (9) and (10) now describe the response of the conical shell to horizontal ground motion in general. However, as ground motion impulses often govern overturning collapse, impulse collapse diagrams similar to those presented by Zhang and Makris [5] could directly be plotted.

Finally, the fact that the spire has no tensile capacity must again be considered. Diagonal cracking could result in the rocking response of the ‘cracked cone’ in Figure 2(b) about point O, where point O need not be located at the bottom corner. The crack could occur further up the spire. In this case, equations (7) and (9) could still be used to predict the dynamic response, but the impact formulation must be reconsidered.

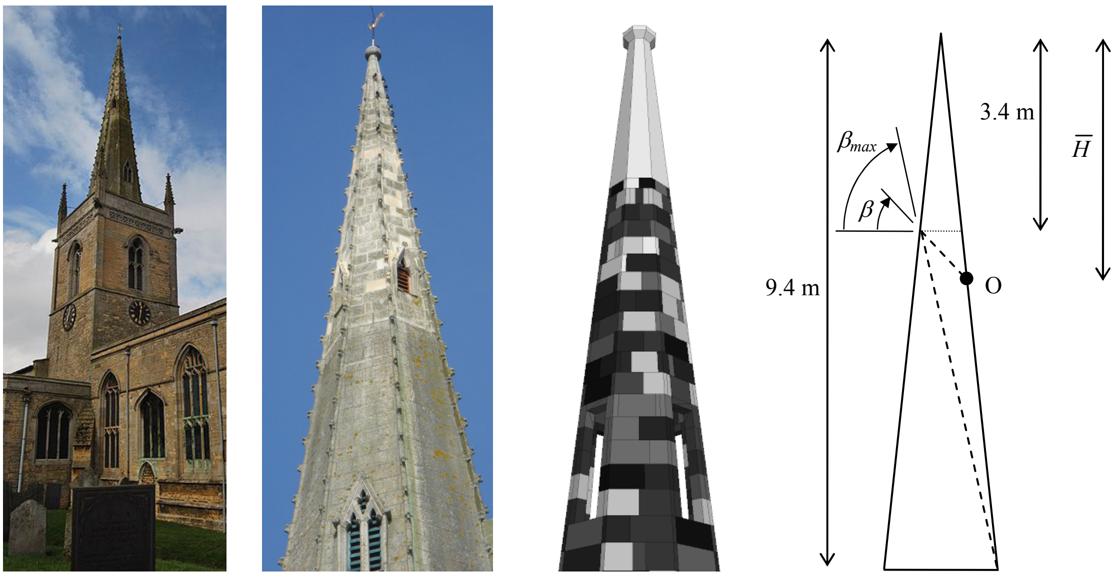
3 DISCRETE ELEMENT MODELING

Discrete Element Modeling (DEM) is a tool which can predict the more detailed response of a spire which is actually comprised of numerous separate masonry units. DEM will be used to predict levels of damage and collapse to the spire, rather than to predict precise displacements, which is impossible. In this section, the modeling assumptions are first explained, followed by simulation of static, impulse, and earthquake loading. All simulations were carried out using 3DEC [10].

3.1 Spire Characteristics and Modeling Assumptions

The spire of the church of St. Mary Magdalene in Waltham on the Wolds, UK (Figure 3(a)), was badly damaged during the earthquake, after which the top half of the spire was reconstructed (Figure 3(b)). Construction and survey documents from the dismantling of the spire were used to develop an accurate model of the pre-earthquake spire, in which each stone is individually considered (Figure 3(c)). All earthquake damage was concentrated above the height of the top windows, so only the reconstructed section was modeled. The entire spire is 19.1 meters tall; the model consists of the top 9.4 meters (Figure 3(d)). The top 3.4 meters of spire is solid, with each course tied together by an interior metal rod. This section was modeled as a single rigid block. The average height of each masonry course is 0.3 meters, with an assumed average hydraulic lime mortar joint thickness of 1 cm.

The modeling parameters used for DEM simulations are presented in Table 1. Rigid blocks were specified to limit computational time. The joint stiffness k_j was calculated by lumping all of the stone and mortar deformation in the joints [11]. Stiffness proportional damping was specified to approximate inelastic impact between blocks and to limit unrealistic high frequency vibrations [11]. Mass proportional damping was not used.



(a) (b) (c) (d)

Figure 3: (a) Church of St. Mary Magdalene, (b) repaired masonry spire, (c) DEM model of spire, and (d) model spire geometry.

E_{stone} [GPa]	E_{mortar} [GPa]	Density, ρ [kg/m^3]	k_{joint} [GPa/m]	Friction angle [degrees]	Stiffness proportional damping constant [-]
30	10	2600	98	30	2.1×10^{-5}

Table 1: Modeling parameters.

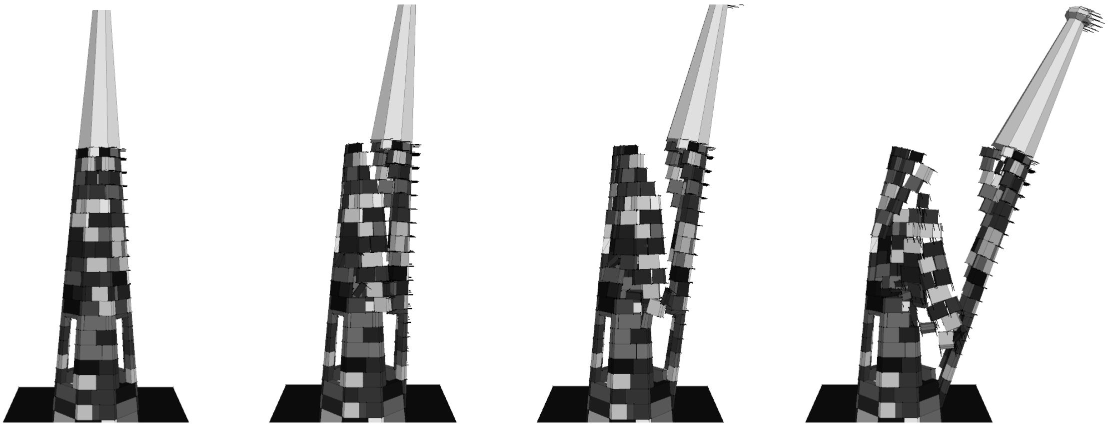


Figure 4: Progressive collapse under constant horizontal ground acceleration.

3.2 Constant acceleration results

To evaluate the minimum possible acceleration which could cause collapse, an increasing horizontal acceleration was applied until the overturning occurred. As expected, diagonal cracking of the spire occurred (Figure 4). The window openings and the lack of interlocking between blocks above the windows allowed a remarkably vertical crack to form. The collapse acceleration varied from 0.164g to 0.176g in the four horizontal directions, showing some effects of varying block interlock. In general, the spire collapsed with a crack angle of approximately $\beta_{\max}$ (Figure 3(d)). According to equation (2), λ = 0.192, which compares well considering the assumption of a perfect conical shell instead of a windowed octagonal spire with vertical joints. A refined analytical model including the solid tip and more precise geometry yields $\lambda = 0.168$.

It is worth noting that even this simple simulation indicates that the poor interlock between blocks limited the ability of the spire to withstand lateral acceleration. Friction due to better interlocking could significantly reduce β.

3.3 Single Impulse Results

The spire was also subjected to a suite of single cycle sinusoidal ground acceleration pulses of maximum amplitude a_p and of duration T_p. The response of the spire was repeatedly simulated to evaluate the pulse characteristics which cause damage (visible residual displacements) and complete collapse. The results are presented in Figure 5, in which the regions above the curves represent regions of damage and collapse respectively. A representative rocking response of the spire is depicted in Figure 6, where the ground moves to the right and the spire rocks to the left (Figure 6(a,b)), recovers and impacts (Figure 6(c)), and then collapses to the right (Figure 6(d)). This mode of collapse will be referred to as Mode I collapse; immediate collapse to the left without impact will be referred to as Mode II collapse (after [5]).

The analytical model presented in §2.3 enables the prediction of Mode I and Mode II rocking collapse. However, the response is sensitive to p. For two identical spires of different scale, the larger will be much more resistant to overturning. The collapse curve for the entire spire (9.4 meters tall), and for the solid spire tip alone (3.4 m tall), are also presented in Figure 5. The region inside the lower loop of the collapse curve represents Mode I collapse, while the region above the upper portion of the collapse curve represents mode II collapse. Clearly the tip alone is more vulnerable.

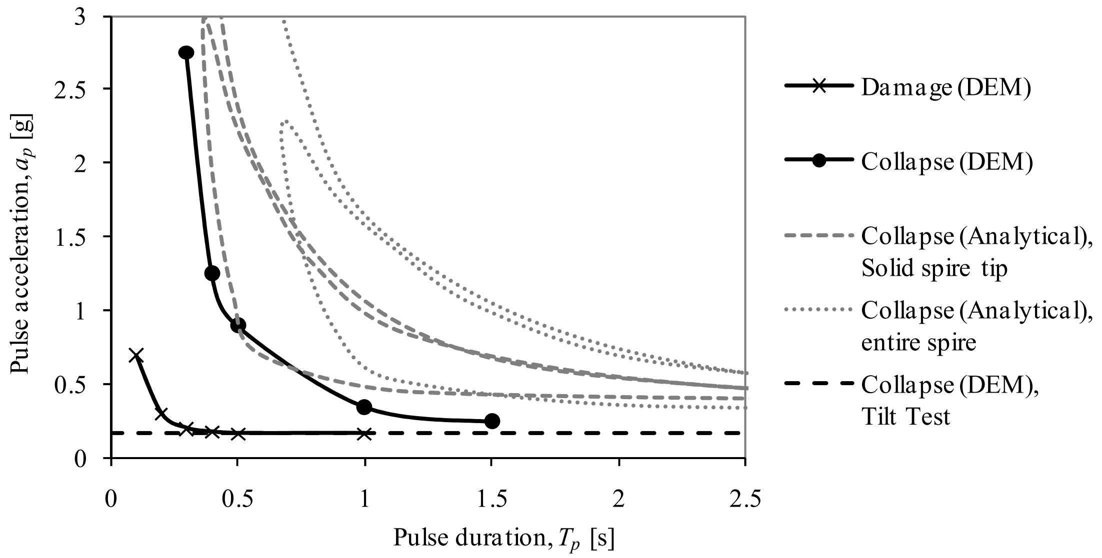


Figure 5: Damage and collapse diagram for a single sinusoidal base acceleration pulse.

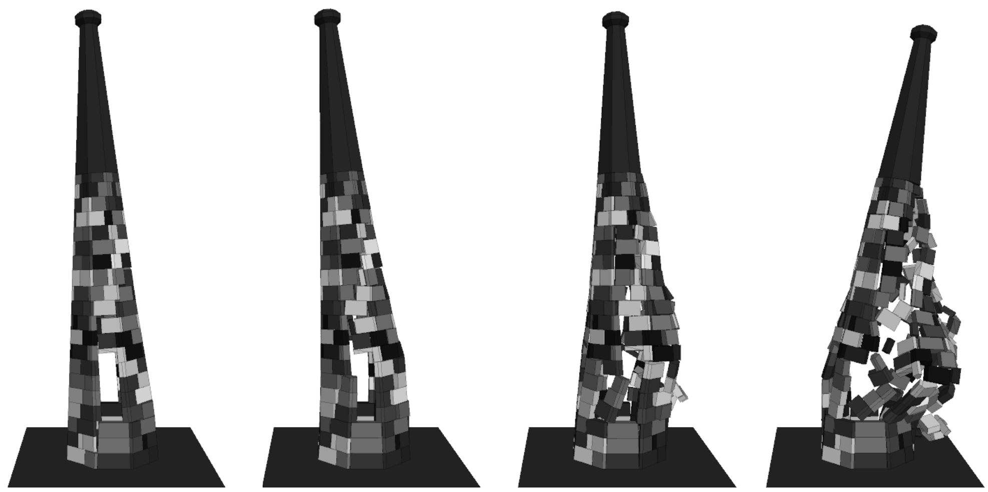
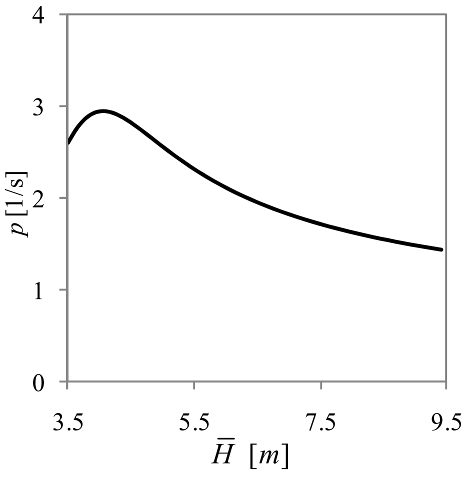


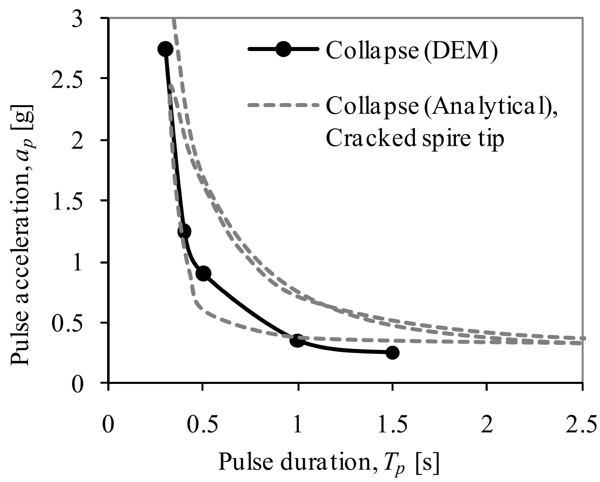
Figure 6: Progressive collapse under for a single sinusoidal base acceleration pulse with a_p = 1.0 g and T_p = 1.0 s (ground moves to the right).

In addition to considering the entire cracked cone and solely the cone tip, there may be a worse condition. Suppose the spire is allowed to crack, and the cracked portion begins rocking. For this case the maximum p value would shift the collapse curve furthest to the left. If the crack is assumed to initiate just below the solid tip, p is plotted as a function of the cracked height $\bar{H}$ in Figure 7(a). For the spire in question, the maximum value of p occurs for $\bar{H}$ = 4.1 meters, which would yield the collapse curve in Figure 7(b).

The results indicate that while rocking of the cracked spire tip is theoretically the worst condition, it may be slightly conservative. In fact, Mode I collapse due to this condition is unlikely due to the inability of the rocking mechanism to reflect without significant dissipation of energy. Instead, an alternate mode of failure may govern, in which the impulse is not large enough to cause Mode II collapse of the cracked spire, but the impulse is large enough to cause significant damage to the non-rocking portion of the spire, which then cannot sustain the impact resulting when the rocking portion of the spire recovers.



(a)



(b)

Figure 7: Progressive collapse under for a single sinusoidal horizontal base acceleration pulse.

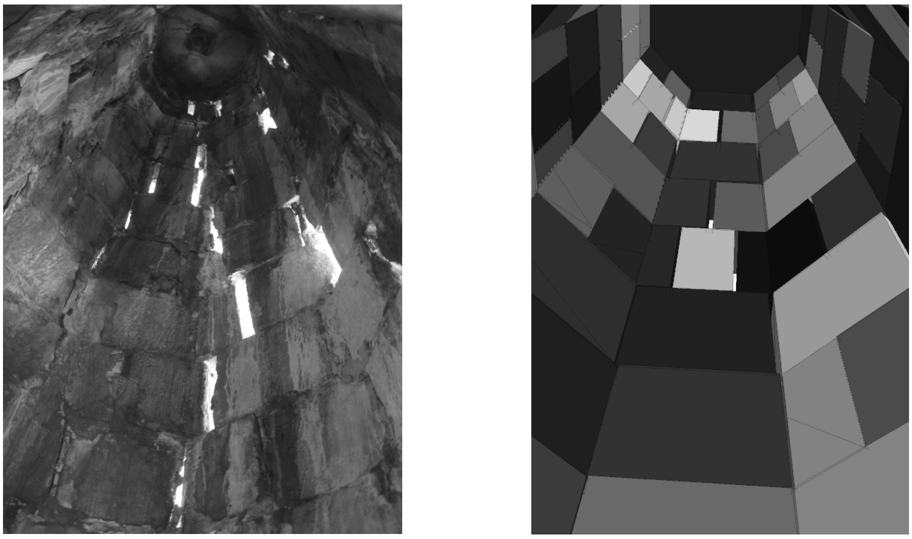
Regardless, Figures 5 and 7(b) indicate that the primary impulse within the earthquake ground motion experienced by the spire was way too small for rocking damage or overturning collapse to occur.

3.4 Seismic Response Results

Unfortunately, ground motion acceleration data from the location of the earthquake was not available near the location of the spire. Instead, ground motion data from a similar epicentral distance was used. Unfortunately, this completely alters the effects of seismic wave directionality and local soil amplification. To compensate, the ground motion was scaled by a range of 2-6 times to account for local site amplification due to differences in soil conditions.

To determine the response of the spire, it is also necessary to consider the amplification of the ground motion by the structure itself. Using a procedure similar to that presented in §2.2, the natural frequency of the combined tower and spire was estimated to be ~ 6.8 Hz. The spectral earthquake data reached its maximum in a similar frequency range, so the fundamental mode was determined to dominate the response, and was used to approximate the horizontal motion at the base of the DEM model.

The DEM model was then used to simulate the seismic response. While assumptions are too numerous to be able to expect to exactly predict the spire response, a few conclusions can be drawn. Because the input acceleration was larger than 0.17g for very short durations, overturning mechanisms were repeatedly formed and closed almost immediately, causing the spire to very slightly walk apart, and leaving vertical gaps between stones (Figure 8(b)). Increased amplification of the ground motion input led to increased 'walking' and larger vertical gaps between stones. These same vertical gaps appeared above the windows of the actual spire (Figure 8(a)), and led to the need for reconstruction. The duration of the strong motion portion of the earthquake also appears to be critical to evaluate the level of 'walking'. Generally, it is evident that earthquake ground accelerations must have been amplified locally, by both the soil and the structure, to have caused the observed damage.



(a) (b)

Figure 8: Interior view of spire: (a) actual damage, (b) DEM simulation of damage.

4 CONCLUSIONS

The seismic collapse of unreinforced masonry spires is difficult to predict. There is clearly an initial elastic response which must be considered, followed by a rocking response which completely alters the dynamics of the system. The analytical approach presented herein provides a context for dealing with both of these aspects of the dynamic response. However, due to the number of simplifying assumptions involved in the analytical formulation, DEM is critical to evaluate the actual spire response.

Both the analytical and computational results indicate that the damage to the spire of the church of St. Mary Magdalene could not have been caused by the acceleration magnitudes recorded at a similar epicentral distance, or even the ground acceleration scaled due to local soil conditions. Additionally, it was not likely that the damage was caused by a single impulse within the ground motion. Instead, structural amplification seems to have caused large enough accelerations at the top of the spire to cause the spire to walk apart over a longer duration of time, producing the observed damage.

REFERENCES

[1] P. Hanning, *The Great English Earthquake*, Robert Hale Limited, 1976.

[2] S. Sargeant, R.M.W. Musson, Rotational earthquake effects in the United Kingdom. *Bulletin of the Seismological Society of America*, **99**(2B), 1475–1479, 2009.

[3] J. Heyman, *The stone skeleton,* Cambridge University Press, 1995.

[4] G.W. Housner, The behavior of inverted pendulum structures during earthquakes, *Bulletin of the Seismological Society of America*, **53**(2), 403-417.

[5] J. Zhang, N. Makris, Rocking response of free-standing blocks under cycloidal pulses, *ASCE Journal of Engineering Mechanics*, **127**(5), 473-483, 2001.

[6] L. De Lorenzis, M.J. DeJong, J. Ochsendorf, Failure of masonry arches under impulse base motion, *Earthquake Engineering and Structural Dynamics*, **36**(14), 2119-2136, 2007.

[7] M.J. DeJong, L. De Lorenzis, J. Ochsendorf, Numerical modeling of masonry arch stability under impulse base motion, M. Papadrakakis *et al.* (eds.), *Computational Methods in Structural Dynamics and Earthquake Engineering (COMPDYN2007)*, Rethymno, Crete, Greece, June 13-16, 2007.

[8] J.A. Ochsendorf, J.I. Hernando, S. Huerta, Collapse of masonry buttresses, *ASCE Journal of Architectural Engineering*, **10**(3), 2004.

[9] S. Naguleswaran, A direct solutions for the transverse vibration of Euler-Bernoulli wedge beams and cones. *Journal of Sound and Vibration*, **172**(3), 289-304, 1992.

[10] Itasca Consulting Group, *3DEC Version 3.0: User's Guide*, Minneapolis, Minnesota, 2003.

[11] M.J. DeJong, *Seismic assessment strategies for masonry structures*, Ph.D. Dissertation, Massachusetts Institute of Technology, Cambridge, Massachusetts, 2009.