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NONLINEAR DEFORMATION OF AN INFLATABLE ANISOTROPIC TOROIDAL MEMBRANE

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Abstract. *Nonlinear axisymmetric deformation of a toroidal shell under action of internal pressure is considered. The shell is made of neo-Hookean material and is reinforced by two system of threads located on parallels and meridians. The nonlinear theory of membranes is used. For the evaluating deformations and displacements of a membrane the system of the ordinary differential equations of the fourth order is obtained. The method of asymptotical integration in the case when the meridian radius is much smaller than the parallel one is elaborated. Comparison of asymptotic and numerical results is performed.*

1 INTRODUCTION

Textile composites and pneumatic structures have become increasingly popular for a variety of applications in a civil engineering, architecture, aerospace engineering, etc. [1]. Typical examples of inflatable toroidal shells include tires, pneumatic jacks and inflatable pools. In this paper the axisymmetric nonlinear deformation under internal pressure of the toroidal shell reinforced of fibers is studied. The bending of an inflatable cylindrical membrane was considered in the paper [2].

2 MAIN ASSUMPTIONS

It is supposed that the toroidal shell is made of a cylindrical textile composite pipe which contains two systems of threads located on parallels and meridians. The lengths of not deformed threads are equal accordingly L and l (see Figure 1a).

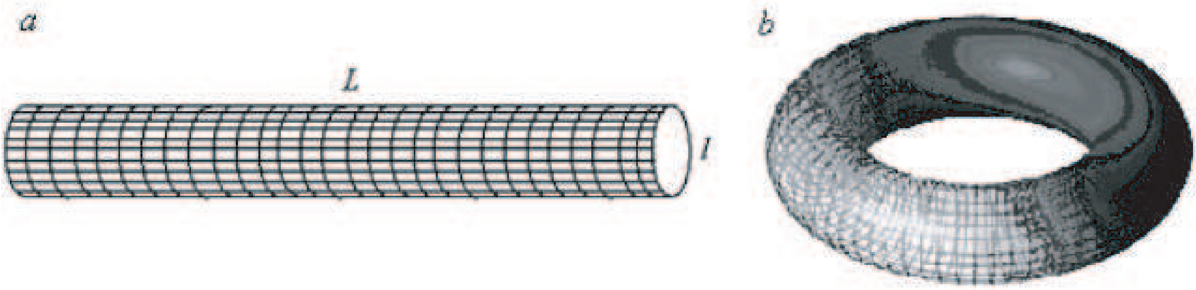

Figure 1: Transformation of the cylindrical pipe into toroidal membrane.

We assume that the fibers are disposed frequently enough. After averaging we get the two-dimensional elastic medium. This medium we consider as an anisotropic membrane. Let's close end faces of a pipe and create inside of it the pressure q. Connecting end faces of a pipe, we get a membrane like the toroidal one (see Figure 1b).

All fibers going on membrane meridians are stretched, but some fibers going on parallels are compressed if $q < q_*$, where q_* is the minimal value of pressure at which all parallels are stretched. A membrane can not hold the compression stresses. Therefore in case $q < q_*$ the part of its surface is covered by folds. If the internal pressure q increases then the area covered by folds decreases. At $q = q_*$ all shell will be stretched.

3 BASIC EQUATIONS

The theory of elastic membranes is a particular case of nonlinear theory of shells developed in the work of W.T. Koiter, W. Pietraszkiewicz, K.Z. Galimov, A. Libai and J.G. Simmonds [3]. The nonlinear membrane theory has the satisfactory accuracy for sufficiently thin shells.

For a toroidal membrane the following geometrical relations are valid:

$$\begin{gathered}\lambda_1 = \frac{ds}{ds_0}, \qquad \lambda_2 = \frac{r}{R}, \qquad R = \frac{L}{2\pi}, \\ \frac{dr}{ds} = -\sin\theta, \qquad \frac{d\hat{z}}{ds} = \cos\theta, \qquad \frac{1}{R_1} = \frac{d\theta}{ds}, \qquad \frac{1}{R_2} = \frac{\cos\theta}{r},\end{gathered} \tag{3.1}$$

where $s_0 \in [0, l]$ and $s(s_0)$ are the length of the meridian arch before and after deformation, $r(s_0)$ is the radius of a parallel, $\hat{z}(s_0)$ is the height of a parallel above a point O, $\lambda_1(s_0)$ and

$\lambda_2(s_0)$ are the stretch ratios of meridians and parallels, R_1 and R_2 are the radiuses of curvature of a surface, θ is the angle between a tangent to a meridian and the vertical direction (see Figure 2).

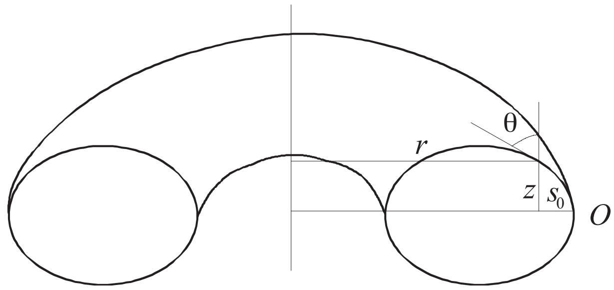

Figure 2: Toroidal membrane.

The equilibrium equations in a projection to the tangent to a meridian and the normal to the membrane give

$$\frac{d(rT_1)}{ds} + T_2 \sin\theta = 0, \qquad \frac{T_1}{R_1} + \frac{T_2}{R_2} = q, \tag{3.2}$$

where q is the internal pressure, T_1 and T_2 are stress resultants.

Assume that the shell and threads are made of incompressible homogeneous isotropic materials with elastic potentials

$$G_i \Phi_i(\lambda_1, \lambda_2, \lambda_3), \quad i = 1, 2, 3, \tag{3.3}$$

where G_i are constants of a material. For the threads going on meridians and parallels $i = 1$ and $i = 2$ accordingly, and $i = 3$ for a material of the shell. In case of small deformations G_i is the shear modulus.

Whereas the material is incompressible, equality $\lambda_1\lambda_2\lambda_3 = 1$ is valid. Besides for the threads going on meridians $\lambda_2 = \lambda_3$. Therefore the potential (3.3) becomes

$$G_1 \Phi_1(\lambda_1, \lambda_1^{-1/2}, \lambda_1^{-1/2}) = G_1 \Psi_1(\lambda_1).$$

For the threads going on parallels $\lambda_1 = \lambda_3$ and

$$G_2 \Phi_2(\lambda_2, \lambda_2^{-1/2}, \lambda_2^{-1/2}) = G_2 \Psi_2(\lambda_2).$$

For the shell potential (3.3) have the form

$$G_3 \Phi_3(\lambda_1, \lambda_2, (\lambda_1\lambda_2)^{-1}) = G_3 \Psi_3(\lambda_1, \lambda_2).$$

The stress resultants entering into equations (3.2) are the sums of the tensile thread forces and the stresses arising as result of a deformation of the shell:

$$T_1 = \frac{G_1 N_1 S_1}{\lambda_2}\frac{d\Psi_1}{d\lambda_1} + \frac{G_3 h_0}{\lambda_2}\frac{\partial \Psi_3}{\partial \lambda_1}, \qquad T_2 = \frac{G_2 N_2 S_2}{\lambda_1}\frac{d\Psi_2}{d\lambda_2} + \frac{G_3 h_0}{\lambda_1}\frac{\partial \Psi_3}{\partial \lambda_2}, \tag{3.4}$$

where N_1, N_2 and S_1, S_2 are numbers of threads on unit of length in a cross-sectional direction and the areas of cross-section of threads in the state before deformation for meridians and parallels correspondingly, h_0 is the thickness of the shell before deformation.

If at $\varphi \in [\varphi_*, \pi]$ formula (3.4) gives $T_2 < 0$ then according to membrane hypotheses it is necessary to put $T_2 = 0$ in system (3.2) when $\varphi \in [\varphi_*, \pi]$.

4 DIMENSIONLESS VARIABLES

Let's enter dimensionless variables by formulas

$$\begin{aligned} &z = \frac{\hat{z}}{R}, \quad s_0 = \rho\varphi, \quad \rho = \frac{l}{2\pi}, \quad \mu = \frac{\rho}{R}, \quad Q = \frac{qR}{G_0 h_0}, \\ &t_1 = \frac{T_1\lambda_2}{G_0 h_0}, \quad t_2 = \frac{T_2\lambda_1}{G_0 h_0}, \quad g_i = \frac{G_i N_i S_i}{G_0 h_0}, \quad i = 1,2, \quad g_3 = \frac{G_3}{G_0}, \end{aligned} \tag{4.1}$$

where G_0 is any number, $\mu < 0.5$, and parameters g_i characterize relative stiffness of treads in comparison with one of the shell. Due to a choice G_0 any of values g_i can be taken equal 1.

Using formulas (3.1), (3.2) and (4.1) we get the following system for unknown variables $\theta(\varphi)$, $t_1(\varphi)$, $\lambda_2(\varphi)$ and $z(\varphi)$

$$\begin{aligned} &\frac{d\theta}{d\varphi} = \frac{\mu}{t_1}(\lambda_1\lambda_2 Q - \hat{t}_2\cos\theta), \quad \frac{dt_1}{d\varphi} = -\mu\hat{t}_2\sin\theta, \\ &\frac{d\lambda_2}{d\varphi} = -\mu\lambda_1\sin\theta, \quad \frac{dz}{d\varphi} = \mu\lambda_1\cos\theta, \\ &\hat{t}_2 = \max\{t_2, 0\}, \quad 0 \le \varphi \le 2\pi. \end{aligned} \tag{4.2}$$

The function

$$\Phi(\lambda_1, \lambda_2, \lambda_3) = \frac{1}{2}(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3)$$

corresponds to neo-Hookean material. Assume that $\Phi_1 = \Phi_2 = \Phi_3 = \Phi$. Then after the introduction of dimensionless variables (4.1) formulas (3.4) take the form

$$t_1 = g_3\left(\lambda_1 - \frac{1}{\lambda_1^3\lambda_2^2}\right) + g_1\left(\lambda_1 - \frac{1}{\lambda_1^2}\right), \quad t_2 = g_3\left(\lambda_2 - \frac{1}{\lambda_1^2\lambda_2^3}\right) + g_2\left(\lambda_2 - \frac{1}{\lambda_2^2}\right). \tag{4.3}$$

Owing to the symmetry of the problem to the plane $z = 0$, to find its periodic solution it is enough to find the solution of system (4.2), (4.3) satisfying the boundary conditions

$$\theta(0) = 0, \quad z(0) = 0, \quad \theta(\pi) = \pi, \quad z(\pi) = 0. \tag{4.4}$$

5 THE APPROXIMATE SOLUTION FOR COMPLETELY STRETCHED MEMBRANE

Assume that $\mu \ll 1$, $t_2 \ge 0$ for $\varphi \in [0, \pi]$ and introduce the new variables

$$\alpha = \mu^{-1}t_1, \quad \beta = \mu^{-1}(\lambda_1 - 1), \quad \gamma = \mu^{-1}(\lambda_2 - 1), \quad \delta = \mu^{-1}t_2, \quad \zeta = \mu^{-1}z. \tag{5.1}$$

Equations (4.2), (4.3) and boundary conditions (4.4) take the form

$$\begin{aligned} &\frac{d\alpha}{d\varphi} = -\mu\delta\sin\theta, \quad \frac{d\theta}{d\varphi} = \frac{1}{\alpha}[(1+\mu\beta)(1+\mu\gamma)Q - \mu\delta\cos\theta], \\ &\frac{d\zeta}{d\varphi} = (1+\mu\beta)\cos\theta, \quad \frac{d\gamma}{d\varphi} = -(1+\mu\beta)\sin\theta. \end{aligned} \tag{5.2}$$

$$\begin{aligned} &\mu\alpha = (g_1 + g_3)(1+\mu\beta) - \frac{g_1}{(1+\mu\beta)^2} - \frac{g_3}{(1+\mu\beta)^3(1+\mu\gamma)^2} \\ &\mu\delta = (g_2 + g_3)(1+\mu\gamma) - \frac{g_2}{(1+\mu\gamma)^2} - \frac{g_3}{(1+\mu\gamma)^3(1+\mu\beta)^2} \end{aligned} \tag{5.3}$$

$$\theta(0) = \zeta(0) = \zeta(\pi) = 0, \quad \theta(\pi) = \pi. \tag{5.4}$$

Substitute into (5.2)–(5.4) the asymptotic expansions

$$\begin{aligned} &\alpha = \alpha_0 + \mu\alpha_1, \quad \beta = \beta_0 + \mu\beta_1, \quad \gamma = \gamma_0 + \mu\gamma_1, \\ &\delta = \delta_0 + \mu\delta_1, \quad \theta = \theta_0 + \mu\theta_1, \quad \zeta = \zeta_0 + \mu\zeta_1. \end{aligned} \tag{5.5}$$

In the zeroth approximation we get

$$\begin{aligned} &\alpha_0 = Q, \quad \theta_0 = \varphi, \quad \zeta_0 = \sin\varphi, \quad \gamma_0 = \cos\varphi + a_0, \\ &\beta_0 = A_1 - A_2\gamma_0, \quad \delta_0 = A_3\gamma_0 + A_4, \end{aligned} \tag{5.6}$$

where a_0 is the arbitrary constant which can be found at the construction of the first approximation,

$$A_1 = \frac{Q}{4g_3 + 3g_1}, \quad A_2 = \frac{2g_3A_1}{Q}, \quad A_3 = 4g_3 + 3g_2 - 2g_3A_2, \quad A_4 = QA_2.$$

It follows from formulas (4.1), (5.1) and (5.6) that in the zeroth approximation a cross-section of the membrane is the circumference of the radius ρ. The distance between the center of this circumference and the center of the torus is equal to $R + a_0\rho$.

The solutions of equations of the first approximation

$$\begin{aligned} &\frac{d\alpha_1}{d\varphi} = -\delta_0\sin\varphi, \quad \frac{d\theta_1}{d\varphi} = \beta_0 + \gamma_0 - \frac{\alpha_1}{Q} - \frac{\delta_0}{Q}\cos\varphi, \\ &\frac{d\zeta_1}{d\varphi} = \beta_0\cos\varphi - \theta_1\sin\varphi, \quad \frac{d\gamma_1}{d\varphi} = -\beta_0\sin\varphi - \theta_1\cos\varphi, \end{aligned} \tag{5.7}$$

satisfy the boundary conditions

$$\theta_1(0) = \zeta_1(0) = \theta_1(\pi) = \zeta_1(\pi) = 0. \tag{5.8}$$

First equation (5.7) has solution

$$\alpha_1 = (A_3a_0 + A_4)\cos\varphi + \frac{A_3}{2}\cos^2\varphi + a_1Q, \tag{5.9}$$

where a_1 is the arbitrary constant. Substitution expressions (5.9) in second equation (5.7) and taking first condition (5.8) into account we get

$$\theta_1 = (A - 3c)\varphi + B\sin\varphi - \frac{c}{2}\sin 2\varphi,$$

where

$$A = A_1 + a_0(1 - A_2) - a_1, \quad B = 1 - A_2 - \frac{2(a_0A_3 + A_4)}{Q}, \quad c = \frac{A_3}{4Q}.$$

Equality $\theta_1(\pi) = 0$ holds if

$$A = 3c. \tag{5.10}$$

The solution of the third equation (5.7) satisfying the boundary condition $\zeta_1(0) = 0$ has the form

$$\zeta_1 = -\frac{1}{2}(B + A_2)\varphi + (A_1 - a_0A_2)\sin\varphi + \frac{B - A_2}{4}\sin 2\varphi + c\sin^3\varphi.$$

Taken into account the condition $\zeta_1(\pi) = 0$ we get equality $B + A_2 = 0$. Hence

$$a_0 = \frac{Q - 2A_4}{2A_3}.$$

The substitution of the expression for a_0 in formula (5.10) allows one to find a_1:

$$a_1 = A_1 + \frac{(1 - A_2)(Q - 2A_4)}{2A_3} - 3c.$$

The expression for the function γ_1 contains the constant term a_2 which can be found at the construction of the second approximation.

The condition $t_2 \geq 0$ at $\varphi \in [0, \pi]$ is necessary for a correctness of the obtained solution. After the substitution into the inequality $t_2 \geq 0$ the approximate expression $t_2 \simeq \mu\delta_0$, we get $Q \geq Q_0$, where $Q_0 = 2A_3$. The number Q_* for which by means of the asymptotic method the approximate expression Q_0 is obtained, represents a characteristic value of dimensionless pressure Q. In case of $Q < Q_*$ the part of the membrane is covered by folds, and at $Q > Q_*$ the membrane is completely stretched.

6 THE APPROXIMATE SOLUTION FOR PARTLY STRETCHED MEMBRANE

Assume that $t_2 > 0$ when $\varphi \in [0, \varphi_*]$ and $t_2 < 0$ when $\varphi \in [\varphi_*, \pi]$. Then in the area $\varphi \in [0, \varphi_*]$ all fibers going on parallels are streched and the part of the membrane surface $\varphi \in [\varphi_*, \pi]$ is covered by folds. At these assumptions equalities

$$t_2(\varphi_*) = 0, \quad \delta(\varphi_*) = 0 \tag{6.1}$$

take place.

At $\varphi \in [0, \varphi_*]$ the axisymmetric deformation of a toroidal membrane describe equation (5.2), (5.3). The function α', β', γ', θ' and ζ' define on the interval $\varphi \in [\varphi_*, \pi]$ satisfy the following equation

$$\begin{aligned} &\frac{d\alpha'}{d\varphi} = 0, \quad \frac{d\theta'}{d\varphi} = \frac{Q}{\alpha'}[(1 + \mu\beta')(1 + \mu\gamma'), \\ &\frac{d\zeta'}{d\varphi} = (1 + \mu\beta')\cos\theta', \quad \frac{d\gamma'}{d\varphi} = -(1 + \mu\beta')\sin\theta'. \end{aligned} \tag{6.2}$$

$$\mu\alpha' = (g_1 + g_3)(1 + \mu\beta') - \frac{g_1}{(1 + \mu\beta')^2} - \frac{g_3}{(1 + \mu\beta')^3(1 + \mu\gamma')^2} \tag{6.3}$$

We search the solutions of equations (5.2), (5.3) and (6.2), (6.3) satisfying boundary conditions

$$\theta(0) = \zeta(0) = 0, \quad \theta'(\pi) = \pi, \quad \zeta'(\pi) = 0, \tag{6.4}$$

$$\alpha(\varphi_*) = \alpha'(\varphi_*), \quad \theta(\varphi_*) = \theta'(\varphi_*), \quad \gamma(\varphi_*) = \gamma'(\varphi_*), \quad \zeta(\varphi_*) = \zeta'(\varphi_*), \tag{6.5}$$

in the form (5.5). To find the approximate value of the unknown number φ_* we use the asymptotic expansion $\varphi_* = \varphi_0 + \mu\varphi_1$.

In the zeroth approximation we get the same results, as in case when the toroidal membrane is completely stretched:

$$\begin{aligned} &\alpha_0 = \alpha_0' = Q, \quad \theta_0 = \theta_0' = \varphi, \quad \zeta_0 = \zeta_0' = \sin\varphi, \\ &\gamma_0 = \gamma_0' = \cos\varphi + a_0, \quad \beta_0 = \beta_0' = A_1 - A_2\gamma_0. \end{aligned}$$

It follows from second equality (6.1) that

$$a_0 = -\cos\varphi_0 - \frac{A_4}{A_3}.$$

The number φ_0 will be found at the construction of the first approximation.

If $\varphi \le \varphi_0$ then for the construction of the first approximation we can use equations (5.7). In case $\varphi \ge \varphi_0$ equations of the first approximation have the form

$$\begin{aligned} &\frac{d\alpha_1'}{d\varphi} = 0, \quad \frac{d\theta_1'}{d\varphi} = \beta_0 + \gamma_0 - \frac{\alpha_1'}{Q}, \\ &\frac{d\zeta_1'}{d\varphi} = \beta_0 \cos\varphi - \theta_1' \sin\varphi, \quad \frac{d\gamma_1'}{d\varphi} = -\beta_0 \sin\varphi - \theta_1' \cos\varphi. \end{aligned} \tag{6.6}$$

The solutions of equations (5.7) and (6.6) satisfy the boundary conditions

$$\theta_1(0) = \zeta_1(0) = \theta_1'(\pi) = \zeta_1'(\pi) = 0, \tag{6.7}$$

$$\alpha_1(\varphi_0) = \alpha_1'(\varphi_0), \quad \theta_1(\varphi_0) = \theta_1'(\varphi_0), \quad \zeta_1(\varphi_0) = \zeta_1'(\varphi_0), \quad \gamma_1(\varphi_0) = \gamma_1'(\varphi_0). \tag{6.8}$$

In a considered case formulas (5.7) and (5.8) suit for the definition of the functions $\alpha_1(\varphi)$ and $\theta_1(\varphi)$. Taken into account first equation (6.6) and first condition (6.8) we get

$$\alpha_1' = \alpha_1(\varphi_0) = -\frac{A_3}{2}\cos^2\varphi_0 + a_1 Q.$$

It follows from second equation (6.6) and third condition (6.7) that

$$\theta_1' = \left(A + 2c\cos^2\varphi_0\right)(\varphi - \pi) + (1 - A_2)\sin\varphi, \tag{6.9}$$

The substitution of expressions (5.8) and (6.9) into second condition (6.8) gives the equality

$$(A/c + 2\cos^2\varphi_0)\pi = 3\varphi_0 - 5\sin\varphi_0\cos\varphi_0 + 2\varphi_0\cos^2\varphi_0. \tag{6.10}$$

Whereas for partly stretched toroidal membrane equality (5.9) is not valid, the expression for the function $\zeta_1(\varphi)$ differs from the expression obtained in the previous section:

$$\zeta_1 = (A_1 - a_0 A_2)\sin\varphi + (A - 3c)(\varphi\cos\varphi - \sin\varphi) - \frac{1}{2}(B + A_2)\varphi + \frac{B - A_2}{4}\sin 2\varphi + c\sin^3\varphi. \tag{6.11}$$

The solution $\zeta_1'(\varphi)$ of the third equation (6.6) satisfying last boundary condition (6.7) has the form

$$\zeta_1 = (A_1 - a_0 A_2)\sin\varphi + \left(A - 2c\cos^2\varphi_0\right)[(\varphi - \pi)\cos\varphi - \sin\varphi) + \frac{1}{2}(\pi - \varphi) + \frac{1 - 2A_2}{4}\sin 2\varphi. \tag{6.12}$$

Substitute equalities (6.11) and (6.12) into third condition (6.8). Taking in account formula (6.10) after transformations we obtain the following equation for the evaluating φ_0:

$$\sin\varphi_0 - \varphi_0\cos\varphi_0 = \pi p, \quad p = \frac{Q}{Q_0}. \tag{6.13}$$

Equation (6.13) has no an explicit solution, however its root $\varphi_0 \in [0, \pi]$ depends only on one parameter $p \in [0, 1]$. Therefore for the estimation of the value of this root it is possible to use Figure 3.

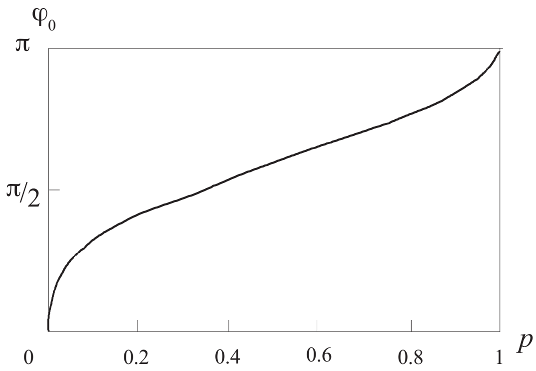

Figure 3: Plot of the functions $\varphi_0(p)$.

7 RESULTS

Consider three cases:

$$1) \quad g_1 = g_2 = 1,\ g_3 = 0; \quad 2) \quad g_1 = g_2 = g_3 = 1/2; \quad 3) \quad g_1 = g_2 = 0,\ g_3 = 1.$$

In the first case the stiffness of the threads only is taken into account, influence of the stiffness of the threads and the shell in the second case are equal and in the third case the threads are absent.

Table 1 lists the values of the dimensionless characteristic pressure Q_* for these cases. The second and third columns contain the values calculated by a numerical solution of equations (4.2) and (4.3) for $\mu = 0.1$ and $\mu = 0.01$. The last column contains the values $Q_0 = 2A_3$ found by the asymptotic approach. The error of the asymptotic formula $Q_* \simeq Q_0$ decreases with the parameter μ.

Case	Q_*		
	$\mu = 0.1$	$\mu = 0.01$	Asymptotics
1)	4.12	5.71	6.00
2)	4.59	6.15	6.43
3)	4.71	5.78	6.00

Table 1: The values of Q_* for three cases.

The boundaries φ_* of the area covered by folds for the case $g_1 = g_2 = g_3 = 1/2$ and for the different values of the dimensionless pressure Q are given in Table 2. The second and third columns contain the values calculated by a numerical solution of equations (4.2) and (4.3). The last column contains the root φ_0 of equation (6.13). The error of the asymptotic results decreases with the μ and Q.

8 CONCLUSIONS

The application of the asymptotic approach to the problem of the toroidal membrane deformation under internal pressure permits to obtain the simple approximate solution. In particular, the explicit expression Q_0 for the minimal dimensionless pressure Q_* at which folds on the shell are not formed is found. It is shown that the relative stiffness of treads influences value of the characteristic pressure Q_* a little.

Q	φ_*		
	$\mu = 0.1$	$\mu = 0.01$	Asymptotics
1	1.28	1.20	1.19
2	1.67	1.58	1.56
3	2.04	1.87	1.85
4	2.51	2.16	2.12

Table 2: The values of φ_* for the different Q.

The equation for the evaluating the boundary of the membrane area covered by folds φ_* is derived. This equation contains only one non-dimensional parameter Q/Q_0. For $\mu = 0.1$ and $Q < 3$ the error of the asymptotic results for the boundary φ_* in comparison with numerical ones is less than 10%.

9 ACKNOWLEDGEMENTS

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