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PERIODIC SHEAR BEAM AS A MODEL OF A HIGH BUILDING UNDER FUZZY NON-STATIONARY EXCITATION

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Abstract. *In the paper the problem of the shear vibration of a finite periodic composite beam with uncertain parameters as the model of a high building under a stochastic excitation is considered. The solution of the problem was found using the random dynamic influence function which allows applying the perturbation method while the average tolerance approach allows passing from differential equations with periodic variable coefficients to differential equations with constant coefficients. Different types of uncertainty of the structure parameters and the excitation process have been considered, namely: fuzzy numbers, random variables, random functions, fuzzy random variables, fuzzy random functions and fuzzy stochastic processes. This allows a wide analysis of complex problems of the shear vibrations of periodic composite beams with fuzzy random parameters under fuzzy stochastic excitations. Much attention has been focused on for obtaining the solution in the most genera case.*

1 INTRODUCTION

The dynamics response of the sheared beams or sheared plates as models of multistorey buildings was subjects of research by some authors [1-4]. In all the cases the homogeneous models have been assumed which do not exactly describe the real structure. Each storey of the building consists of two parts with different stiffnesses and masses. For this reason as a model of the building we can consider a sheared periodic beam. In most cases it is assumed that the parameters of the structure are deterministic. On the other hand, the structural parameters like geometry characteristics, material and damping properties might be uncertain to some extent. Their uncertainty may have a strong influence on the reliability of the structure in the dynamic context and be the crucial factor which determines the safety of the structure. Dynamic analysis of structures often involves two kinds of uncertainty. One of them is the randomness and the other one is the fuzziness which describe imprecision. The random variability is described by use of probability theory and the imprecision by use of fuzzy sets. Very often sufficient statistical data are not available in this case a fuzzy function (fuzzy process) or fuzzy random variable (fuzzy stochastic process) is possible to employ for modeling purposes. The concept of fuzzy random variables allows to combine both randomness and imprecision.

In the paper the problem of the shear vibration of a finite periodic composite beam as the model of high building with uncertain parameters (fuzzy random variables) under a fuzzy stochastic excitation is considered. The solution of the problem was found based on the fuzzy random dynamic influence function while the average tolerance approach allows passing from differential equations with periodic variable coefficients into differential equations with constant coefficients. The tolerance averaging method proposed by Woźniak [5-7] has several advantages and may be used as an alternative to the well-known homogenization method. The idea of random dynamic influence function has been presented in [8-10]. The fuzzy set theory was initiated by Zadeh [11]. The concept of fuzzy random variables was introduced by Kwarkernaak [12], Puri and Ralescu [13] and combines both randomness and imprecision. The dynamic response of the system with deterministic parameters under fuzzy stochastic excitation has been considered among other in the papers [14-20]. The definition of the variance of fuzzy random variables can be found in the papers [21-23]. The application of the uncertain forecasting in engineering and computational mechanics based on fuzzy stochastic processes is presented in the monographs [24,25].

2 GENERAL SOLUTION

Let us consider stochastic vibrations of a periodic straight cantilever beam of length h with a varying cross-section as a model of the building. The differential equation of motion of the sheared beam has the form

$$-[K(\mathbf{b}_\alpha, x)u_{\alpha,x}(\mathbf{b}_\alpha, x,t)]_{,x} + c(\mathbf{b}_\alpha, x)\dot{u}_\alpha(\mathbf{b}_\alpha, x,t) + m(\mathbf{b}_\alpha, x)\ddot{u}_\alpha(\mathbf{b}_\alpha, x,t) = p_\alpha(x,t), \qquad (1)$$

where $u_\alpha(\mathbf{b}_\alpha, x,t)$ denotes the vertical displacement of the beam, $K(\mathbf{b}_\alpha, x) = (\kappa FG)_\alpha(x)$, $m(\mathbf{b}_\alpha, x)$, $c(\mathbf{b}_\alpha, x)$ are, respectively, the shear rigidity of the beam, mass of the beam per unit length and the damping coefficient, all of which are random functions of the spatial coordinate x, $x \in [0,h]$. Furthermore, G is the shear modulus of elasticity, κ is the shear stiffness factor that depends on the cross-sectional shape and F is the cross-sectional area. The function $p_\alpha(x,t)$ represents the excitation process of the beam in space and time. The symbol $(\cdot)_{,x}$ and the superimposed dot denote differentiation in space and time, respectively. The fuzzy random parameters of the beam are presented as a vector $\mathbf{b}_\alpha = [b_{\alpha 1}, b_{\alpha 2}, ... b_{\alpha r}]^T$, where the superscript T denotes the transposition operation. It is assumed that the expected value $E[\mathbf{b}_\alpha]$ and the covariance matrix $\mathbf{C}_{\mathbf{b}_\alpha \mathbf{b}_\alpha} = [\mathrm{cov}(b_{\alpha i}, b_{\alpha j})]_{rxr} = E\ [\mathbf{b}_\alpha \mathbf{b}_\alpha^T] - E\ [\mathbf{b}_\alpha]\, E\, [\mathbf{b}_\alpha^T]$ are known. The symbol $E[\,\cdot\,]$ is the expectation operation. Possible random beam parameters include: the Kirchhoff modulus, the damping coefficient and the dimensions of the beam cross-section. It is assumed that the excitation process of the structure is a fuzzy stochastic process. Additionally, we assume that the structural and load parameters are mutually independent. The solution will be found within the correlation theory; therefore exact knowledge of the probability distributions of these random variables is not required.

In particular cases we can consider different types of the uncertainties of the structure and load parameters. They are fuzzy numbers, random variables (random functions) or in most general cases fuzzy random variables (fuzzy random function - fuzzy stochastic processes).

The boundary conditions of the cantilever beam have the form

$$u_\alpha(\mathbf{b}_\alpha, 0, t) = 0, \quad u_{\alpha,x}(\mathbf{b}_\alpha, h, t) = 0. \tag{2}$$

In the particular case when $K(\mathbf{b}_\alpha, x) = K(\mathbf{b}_\alpha) = (\kappa GF)_\alpha = const,\ c(\mathbf{b}_\alpha, x) = c(\mathbf{b}_\alpha) = const$ and $m(\mathbf{b}_\alpha, x) = m(\mathbf{b}_\alpha) = const,$ Eq. (1) has the form

$$K(\mathbf{b}_\alpha) u_{\alpha,xx}(\mathbf{b}_\alpha, x, t) + c(\mathbf{b}_\alpha) \dot{u}_\alpha(\mathbf{b}_\alpha, x, t) + m(\mathbf{b}_\alpha) \ddot{u}_\alpha(\mathbf{b}_\alpha, x, t) = p_\alpha(x) f_\alpha(t). \tag{3}$$

The standard methods of analyzing the rod dynamics are effective only if the coefficients in Eq. (1) are constant. The quantities $K(\mathbf{b}_\alpha, x),\ c(\mathbf{b}_\alpha, x)$ and $m(\mathbf{b}_\alpha, x)$ in this study are modeled as periodic fields and are rapidly varying *l*-periodic functions

$$K(\mathbf{b}_\alpha, x) = K(\mathbf{b}_\alpha, x+l), \quad c(\mathbf{b}_\alpha, x) = c(\mathbf{b}_\alpha, x+l), \quad m(\mathbf{b}_\alpha, x) = m(\mathbf{b}_\alpha, x+l). \tag{4}$$

The length l is the height of a single storey of the building and is small as compared with the height h of the building $(l << h)$.

It is difficult to find the solution of Eq. (1) because the coefficients are strongly periodic. We solve Eq. (1) basing on the concepts of the tolerance-averaged model [5-7]. Using this procedure it is possible to transform Eq. (1) to the form of a system of averaged differential equations with constant coefficients. This approximation describes the effect of the structural length parameter of the beam. We define $\Omega = (0,\ \mathrm{h})$, $\Delta(x) = (x - l/2, x + l/2)$, $l << h,$, $x \in \Omega^0$, $\Omega^0 = \{x \in \Omega^0 : \Delta x \in \Omega\}$. The periodic functions will be averaged by means of the formula

$$< g(x,t) >= \frac{1}{l} \int_{x-\frac{l}{2}}^{x+\frac{l}{2}} g(\xi, t) d\xi, \quad x \in \Omega^0, \tag{5}$$

where $g(x,t)$ is an arbitrary function defined on $\Omega = (0, h)$.

We base on Conformability Assumption [5-7] that the function $u(x,t)$ conforms to the *l*-periodic structure of the beam and together with all its derivatives it is periodic-like. Let us introduce the following decomposition of this function:

$$u_\alpha(\mathbf{b}_\alpha, x, t) = w_\alpha(\mathbf{b}_\alpha, x, t) + v_\alpha(\mathbf{b}_\alpha, x, t), \tag{6}$$

where $w(\mathbf{b}_\alpha, x,t)$ is the averaged part of the function $u(\mathbf{b}_\alpha, x,t)$ and $v(\mathbf{b}_\alpha, x,t)$ will be referred to as the fluctuating part of the function $u(\mathbf{b}_\alpha, x,t)$.
The modeling decomposition from Eq. (6) makes it possible to introduce two kinds of basic unknowns, namely function $w(\mathbf{b}_\alpha, x,t)$ which is a slowly varying function and $v(\mathbf{b}_\alpha, x,t)$ is an oscillating *l*-periodic-like function. Using Galerkin approximation we obtain the fluctuating function in the form

$$v_\alpha(\mathbf{b}_\alpha, x,t) = g^A(x) v_\alpha^A(\mathbf{b}_\alpha, x,t), \tag{7}$$

(the summation convention over A=1,2,... holds), where $g^A(x)$ are a priori known oscillating *l*-periodic-like functions and the new unknown amplitudes $v_\alpha^A(\mathbf{b}_\alpha, x,t)$ are sufficiently regular and slowly varying functions.
The functions $g^A(x)$ should satisfy conditions

$$< g^A(x) >= \frac{1}{l} \int_{x-\frac{l}{2}}^{x+\frac{l}{2}} g^A(x) dx = 0 \tag{8}$$

and

$$< m(\mathbf{b}_\alpha, x) g^A(x) >= \frac{1}{l} \int_{x-\frac{l}{2}}^{x+\frac{l}{2}} m(\mathbf{b}_\alpha, x) g^A(x) dx = 0 \tag{9}$$

Using the decomposition of Eqs. (6) and (7) and taking into account the Tolerance Averaging Approximation [5-7] after some manipulations we obtain the following system of $\mathrm{N}+1$ equations with constant coefficients for unknown functions $w(\mathbf{b}_\alpha, x,t)$ and $v^A(\mathbf{b}_\alpha, x,t)$, for $x \in \Omega_0$

$$\begin{aligned}
&- < K(\mathbf{b}_\alpha, x) > w_{\alpha,xx}(\mathbf{b}_\alpha, x,t) - < K(\mathbf{b}_\alpha, x) g^A_{,x}(x) > v^A_{\alpha,x}(\mathbf{b}_\alpha, x,t) \\
&+ < c(\mathbf{b}_\alpha, x) > \dot{w}_\alpha(\mathbf{b}_\alpha, x,t) + < m(\mathbf{b}_\alpha, x) > \ddot{w}_\alpha(\mathbf{b}_\alpha, x,t) =< p_\alpha(x,t) >, \\
&< K(\mathbf{b}_\alpha, x) g^B_{,x}(x) > w_{\alpha,x}(\mathbf{b}_\alpha, x,t) + < K(\mathbf{b}_\alpha, x) g^B_{,x}(x) g^A_{,x}(x) > v^A_\alpha(\mathbf{b}_\alpha, x,t) \\
&+ < c(\mathbf{b}_\alpha, x) g^B(x) g^A(x) > \dot{v}^A_\alpha(\mathbf{b}_\alpha, x,t) + \\
&< m(\mathbf{b}_\alpha, x) g^B(x) g^A(x) > \ddot{v}^A_\alpha(\mathbf{b}_\alpha, x,t) =< p_\alpha(x,t) g^A(x) >,
\end{aligned} \tag{10}$$

where $A,\ B = 1, 2, \ldots, N$.

It has been assumed that the damping coefficient fulfills $c(\mathbf{b}_\alpha, x) = 2\beta m(\mathbf{b}_\alpha, x)$, where β=*const* and hence $< c(\mathbf{b}_\alpha, x) g^A(x) >= 0$. The derivation of the beam equations (10) is analogous to the derivation of the rod equations [8].

2.1 Eigenvalue problem

Let us consider a deterministic eigenvalue problem. It this case we assume

$$< m(\mathbf{b}_\alpha, x) g^A(x) >= \frac{1}{l} \int_{x-\frac{l}{2}}^{x+\frac{l}{2}} m(\mathbf{b}_\alpha, x) g^A(x) dx = 0 \tag{11}$$

and

$$p_\alpha(x,t) = 0, \quad c(x) = 0 \tag{12}$$

After introducing (11) to equations (10) and taking into account (12) we obtain for $A = B = 1$

$$\begin{aligned} &< K(x) > W_{,xx}(x) + < K(x) g^1_{,x}(x) > V^1_{,x}(x) + \omega^2 < m(x) > W(x) = 0, \\ &< K(x) g^1_{,x}(x) > W_{,x}(x) + < K(x)[g^1_{,x}(x)]^2 > V^1(x) + \\ &-\omega^2 < m(x)[g^1(x)]^2 > V^1(x) = 0. \end{aligned} \tag{13}$$

From the second of Eqs. (13) one obtains

$$V^1(x) = \frac{< K(x) g^1_{,x}(x) >}{\omega^2 < m(x) g^1(x) g^1(x) > - < K(x) g^1_{,x}(x) g^1_{,x}(x) >} W_{,x}(x). \tag{14}$$

Introducing relationship (14) to the first of Eqs. (13) one obtains the differential equation in the following form

$$\begin{aligned} &[< K(x) > + \frac{[< K(x) g^1_{,x}(x) >]^2}{\omega^2 < m(x) g^1(x) g^1(x) > - < K(x) g^1_{,x}(x) g^1_{,x}(x) >}] W_{,xx}(x) \\ &+ \omega^2 < m(x) > W(x) = 0, \end{aligned} \tag{15}$$

which can be written shortly as

$$W_{,xx}(x) + \lambda^2 W(x) = 0, \tag{16}$$

where

$$\lambda^2 = \frac{\omega^4 < m(x) >< m(x)(g^1(x))^2 > - \omega^2 < m(x) >< K(x)(g^1_{,x}(x))^2 >}{\omega^2 < K(x) >< m(x)(g(x))^2 > - < K(x) >< K(x)(g^1_{,x}(x))^2 > + (< K(x) g^1_{,x}(x) >)^2} \tag{17}$$

The solution of the equation (16) has the form

$$W(x) = A \sin \lambda x + B \cos \lambda x \tag{18}$$

Boundary conditions for cantilever beam have the form

$$W(0) = 0, \quad W_{,x}(h) = 0 \tag{19}$$

Therefore the eigenfunctions and eigenvalues are given by

$$W_n(x) = \sin \lambda_n x\,, \quad \lambda_n = \frac{\pi}{h}(n - \frac{1}{2}), \quad n = 1, 2, 3, \tag{20}$$

From the relationship (14) one obtains

$$V^1_n(x) = \frac{< K(x) g^1_{,x}(x) > \lambda_n}{\omega_n^2 < m(x) g^1(x) g^1(x) > - < K(x) g^1_{,x}(x) g^1_{,x}(x) >} \cos \lambda_n x, \tag{21}$$

and the eigenfrequencies

$$\omega_{n1/2}^2 = \frac{1}{2}\{\frac{< K(x)(g_{,x}^1(x))^2 >}{< m(x)(g^1(x))^2 >} + \lambda_n^2 \frac{< K(x) >}{< m(x) >} \pm \\ \pm\sqrt{[\frac{< K(x)(g_{,x}^1(x))^2 >}{< m(x)(g^1(x))^2 >} - \lambda_n^2 \frac{< K(x) >}{< m(x) >}]^2 + 4\lambda_n^2 \frac{(< K(x) g_{,x}^1(x) >)^2}{< m(x) >< m(x)(g^1(x))^2 >}}\}. \tag{22}$$

In the case of a uniform beam ($K(x) = K = const.,\ m(x) = m = const.$) the eigenfrequencies have the form

$$\omega_n = \lambda_n \sqrt{\frac{K}{m}} = \lambda_n v_s. \tag{23}$$

The quantity $v_s = \sqrt{K/m}$ represents the shear wave velocity in the beam.

2.2 Forced vibration of the beam

The aim of this chapter is to find the general solution for probabilistic characteristics of the system response for arbitrary excitation stochastic process $p_\alpha(x,t) = q_\alpha(x) f_\alpha(t),$ where $q_\alpha(x)$ is a fuzzy deterministic function. The problem is being solved within the correlation theory. When the parameters of Eqs. (10) are random the problem can be solved only if the right sides of the equations (10) are deterministic. To overcome these difficulties we introduce the random dynamic influence function $H_\alpha(\mathbf{b}_\alpha, x,t) = H_{w\alpha}(\mathbf{b}_\alpha, x,t) + g^A(x) H_{v\alpha}^A(\mathbf{b}_\alpha, x,t)$ (RDIF) [8,9] which satisfies the following equations

$$\begin{aligned} &- < K(\mathbf{b}_\alpha, x) > H_{w\alpha,xx}(\mathbf{b}_\alpha, x,t) - < K(\mathbf{b}_\alpha, x) g_{,x}^A(x) > H_{v\alpha,x}^A(\mathbf{b}_\alpha, x,t) + \\ &+ < c(\mathbf{b}_\alpha, x) > \dot{H}_{w\alpha}(\mathbf{b}_\alpha, x,t) + < m(\mathbf{b}_\alpha, x) > \ddot{H}_{w\alpha}(\mathbf{b}_\alpha, x,t) =< q_\alpha(x) > \delta(t), \\ &< K(\mathbf{b}_\alpha, x) g_{,x}^B(x) > H_{w\alpha,x}(\mathbf{b}_\alpha, x,t) + < K(\mathbf{b}_\alpha, x) g_{,x}^B(x) g_{,x}^A(x) > H_{v\alpha}^A(\mathbf{b}, x,t) + \\ &+ < c(\mathbf{b}_\alpha, x) g^B(x) g'^A(x) > \dot{H}_{v\alpha}^A(\mathbf{b}_\alpha, x,t) + < m(\mathbf{b}_\alpha, x) g^B(x) g^A(x) > \ddot{H}_{v\alpha}^A(\mathbf{b}_\alpha, x,t) = 0, \end{aligned} \tag{24}$$

with initial conditions

$$H_{w\alpha}(\mathbf{b}_\alpha, x,0) = 0,\ \dot{H}_{w\alpha}(\mathbf{b}_\alpha, x,0) = 0,\ H_{v\alpha}^A(\mathbf{b}_\alpha, x,0) = 0,\ \dot{H}_{v\alpha}^A(\mathbf{b}_\alpha, x,0) = 0. \tag{25}$$

If the random dynamic influence function $H(\mathbf{b}, x,t) = H_w(\mathbf{b}, x,t) + g^A(x) H_v^A(\mathbf{b}, x,t)$ is known then the response of the beam to be found can be presented in the following form:

$$\begin{aligned} u_\alpha(\mathbf{b}_\alpha, x,t) &= w_\alpha(\mathbf{b}_\alpha, x,t) + g^A(x) v_\alpha^A(\mathbf{b}_\alpha, x,t) = \int_{t_0}^{t} H_\alpha(\mathbf{b}_\alpha, x,t-\tau) f_\alpha(\tau) d\tau = \\ &= \int_{t_0}^{t} H_{w\alpha}(\mathbf{b}_\alpha, x,t-\tau) f_\alpha(\tau) d\tau + g^A(x) \int_{t_0}^{t} H_{v\alpha}^A(\mathbf{b}_\alpha, x,t-\tau) f_\alpha(\tau) d\tau. \end{aligned} \tag{26}$$

where if $t_0 = 0$ then one considers transition vibrations and for $t_0 = -\infty$ one considers the steady-state vibration case.

On the basis of the relationship (26) we can obtain the expected value of the beam response

$$E[u_\alpha(\mathbf{b}_\alpha,x,t)]_\alpha = E[w_\alpha(\mathbf{b}_\alpha,x,t)]_\alpha + g^A(x)E[v_\alpha^A(\mathbf{b}_\alpha,x,t)]_\alpha =$$
$$=\int_{t_0}^{t} E[H_\alpha(\mathbf{b}_\alpha,x,t-\tau)]_\alpha E[f_\alpha(\tau)]_\alpha d\tau = \tag{27}$$
$$=\int_{t_0}^{t} E[H_{w\alpha}(\mathbf{b}_\alpha,x,t-\tau)]_\alpha E[f_\alpha(\tau)]_\alpha d\tau + g^A(x)\int_{t_0}^{t} E[H_{v\alpha}^A(\mathbf{b}_\alpha,x,t-\tau)]_\alpha E[f_\alpha(\tau)]_\alpha d\tau,$$

and the covariance of the displacement

$$Cov_{uu_\alpha}\left[x_1,x_2,t_1,t_2\right]_\alpha =$$
$$=\int_{t_0}^{t_1}\int_{t_0}^{t_2} E\left[H_\alpha\left(\mathbf{b}_\alpha,x_1,t_1-\tau_1\right)H_\alpha\left(\mathbf{b}_\alpha,x_2,t_2-\tau_2\right)\right]_\alpha Cov_{ff_\alpha}\left[\tau_1,\tau_2\right]_\alpha d\tau_1 d\tau_2 + \tag{28}$$
$$+\int_{t_0}^{t_1}\int_{t_0}^{t_2} Cov_{HH_\alpha}\left[x_1,x_2,t_1-\tau_1,t_2-\tau_2\right]_\alpha E\left[f_\alpha\left(\tau_1\right)\right]_\alpha E\left[f_\alpha\left(\tau_2\right)\right]_\alpha d\tau_2 d\tau_2,$$

where the covariance of RDIF can be estimated from

$$Cov_{HH_\alpha}\left[x_1,x_2,t_1,t_2\right]_\alpha = E\left[H_\alpha\left(\mathbf{b}_\alpha,x_1,t_1\right)H_\alpha\left(\mathbf{b}_\alpha,x_2,t_2\right)\right]_\alpha + \tag{29}$$
$$-E\left[H_\alpha\left(\mathbf{b}_\alpha,x_1,t_1\right)\right]_\alpha E\left[H_\alpha\left(\mathbf{b}_\alpha,x_2,t_2\right)\right]_\alpha$$

and $Cov_{ff_\alpha}\left[\tau_1,\tau_2\right]_\alpha$ denotes the time covariance of the excitation force.
The variance of the beam displacement is equal to

$$\sigma_{u_\alpha}^2(x,t) = Cov_{uu_\alpha}\left[x,x,t,t\right]_\alpha =$$
$$=\int_{t_0}^{t}\int_{t_0}^{t} E\left[H_\alpha\left(\mathbf{b}_\alpha,x,t-\tau_1\right)H_\alpha\left(\mathbf{b}_\alpha,x,t-\tau_2\right)\right]Cov_{ff_\alpha}\left[\tau_1,\tau_2\right]_\alpha d\tau_1 d\tau_2 + \tag{30}$$
$$+\int_{t_0}^{t}\int_{t_0}^{t} Cov_{HH_\alpha}\left[x,x,t-\tau_1,t-\tau_2\right]_\alpha E\left[f_\alpha\left(\tau_1\right)\right]_\alpha E\left[f_\alpha\left(\tau_2\right)\right]_\alpha d\tau_2 d\tau_2.$$

Due to the relationship (27) one obtains

$$E[u_\alpha(\mathbf{b}_\alpha,x,t)]_{\alpha l} = \min\{\int_{t_0}^{t} E[H_{w\alpha}(\mathbf{b}_\alpha,x,t-\tau)]_\alpha E[f_\alpha(\tau)]_\alpha d\tau + \tag{31}$$
$$+g^A(x)\int_{t_0}^{t} E[H_{v\alpha}^A(\mathbf{b}_\alpha,x,t-\tau)]_\alpha E[f_\alpha(\tau)]_\alpha d\tau\},$$

and

$$E[u_\alpha(\mathbf{b}_\alpha,x,t)]_{\alpha r} = \max\{\int_{t_0}^{t} E[H_{w\alpha}(\mathbf{b}_\alpha,x,t-\tau)]_\alpha E[f_\alpha(\tau)]_\alpha d\tau + \tag{32}$$
$$+g^A(x)\int_{t_0}^{t} E[H_{v\alpha}^A(\mathbf{b}_\alpha,x,t-\tau)]_\alpha E[f_\alpha(\tau)]_\alpha d\tau\}.$$

Using α-*level* optimization procedure [24] for arbitrary $\alpha = \alpha_k \in [0,1]$ or the max-min in the extension principle [11], the smallest and the largest expected values at an established point x and time t can be found.
The lower and upper endpoints of the covariance could be defined using (28) as

$$Cov_{uu_\alpha}\left[x_1,x_2,t_1,t_2\right]_{\alpha l} =$$
$$= \min\{\int_{t_0}^{t_1}\int_{t_0}^{t_2} E\left[H_\alpha\left(\mathbf{b}_\alpha,x_1,t_1-\tau_1\right)H_\alpha\left(\mathbf{b}_\alpha,x_2,t_2-\tau_2\right)\right]_\alpha Cov_{ff_\alpha}\left[\tau_1,\tau_2\right]_\alpha d\tau_1 d\tau_2 + \tag{33}$$
$$+\int_{t_0}^{t_1}\int_{t_0}^{t_2} Cov_{HH_\alpha}\left[x_1,x_2,t_1-\tau_1,t_2-\tau_2\right]_\alpha E\left[f_\alpha\left(\tau_1\right)\right]_\alpha E\left[f_\alpha\left(\tau_2\right)\right]_\alpha d\tau_2 d\tau_2\},$$

and

$$Cov_{uu_\alpha}\left[x_1,x_2,t_1,t_2\right]_{\alpha r} =$$
$$= \max\{\int_{t_0}^{t_1}\int_{t_0}^{t_2} E\left[H_\alpha\left(\mathbf{b}_\alpha,x_1,t_1-\tau_1\right)H_\alpha\left(\mathbf{b}_\alpha,x_2,t_2-\tau_2\right)\right]_\alpha Cov_{ff_\alpha}\left[\tau_1,\tau_2\right]_\alpha d\tau_1 d\tau_2 + \tag{34}$$
$$+\int_{t_0}^{t_1}\int_{t_0}^{t_2} Cov_{HH_\alpha}\left[x_1,x_2,t_1-\tau_1,t_2-\tau_2\right]_\alpha E\left[f_\alpha\left(\tau_1\right)\right]_\alpha E\left[f_\alpha\left(\tau_2\right)\right]_\alpha d\tau_2 d\tau_2\}.$$

In the particular case if the load of the beam is a "white-noise" stationary stochastic process then $C_{ff_\alpha}(\tau_1-\tau_2)_\alpha = \sigma^2_{f\alpha}\delta(\tau_1-\tau_2)$ and the variance is given by the formula

$$\sigma^2_{u_\alpha}(x,t)_\alpha = \sigma^2_{f\alpha}\int_{t_0}^{t} E[H^2_\alpha(\mathbf{b}_\alpha,x,t-\tau)]_\alpha d\tau +$$
$$+E[f^2_\alpha]_\alpha \int_{t_0}^{t}\int_{t_0}^{t} Cov_{HH_\alpha}(x,x,t-\tau_1,t-\tau_2)_\alpha d\tau_1 d\tau_2. \tag{35}$$

Accordingly, we have obtained the formulas for the second-order probabilistic moments of the response of the structure.
We look for solutions of the system of Eqs. (24) in the form

$$H_{w\alpha}(\mathbf{b}_\alpha,x,t) = \sum_{n=1}^{\infty} y_{n\alpha}(\mathbf{b}_\alpha,t)\sin\lambda_n x, \tag{36}$$

and

$$H^A_{v\alpha}(\mathbf{b}_\alpha,x,t) = \sum_{n=1}^{\infty} z^A_{n\alpha}(\mathbf{b}_\alpha,t)\cos\lambda_n x \tag{37}$$

where $\lambda_n = \pi(n-1/2)/h, \quad n=1,2,3,.$
In the particular case of $A=1$ one obtains from Eqs. (24):

$$\begin{aligned}
&\ddot{y}_{n\alpha}(\mathbf{b}_\alpha,t)+2\beta_\alpha(\mathbf{b}_\alpha)\dot{y}_{n\alpha}(\mathbf{b}_\alpha,t)+\omega_{n\alpha}^2(\mathbf{b}_\alpha)y_{n\alpha}(\mathbf{b}_\alpha,t)+\\
&+\lambda_n\frac{<K(\mathbf{b}_\alpha,x)g_{,x}^1(x)>}{<m(\mathbf{b}_\alpha,x)>}z_{n\alpha}^1(\mathbf{b}_\alpha,t)=\frac{2}{H\lambda_n}\delta(t),\\
&\ddot{z}_{n\alpha}^1(\mathbf{b}_\alpha,t)+2\beta_{v\alpha}(\mathbf{b}_\alpha)\dot{z}_{n\alpha}^1(\mathbf{b}_\alpha,t)+\omega_{v\alpha}^2(\mathbf{b}_\alpha)z_{n\alpha}^1(\mathbf{b}_\alpha,t)+\\
&+\lambda_n\frac{<K(\mathbf{b}_\alpha,x)g_{,x}^1(x)>}{<m(\mathbf{b}_\alpha,x)(g^1(x))^2>}y_{n\alpha}(\mathbf{b}_\alpha,t)=0.
\end{aligned}\tag{38}$$

where $2\beta_\alpha(\mathbf{b}_\alpha)=\dfrac{<c(\mathbf{b}_\alpha,x)>}{<m(\mathbf{b}_\alpha,x)>},\quad \omega_{n\alpha}^2(\mathbf{b}_\alpha)=\lambda_n^2\dfrac{<K(\mathbf{b}_\alpha,x)>}{<m(\mathbf{b}_\alpha,x)>}=\lambda_n^2\overline{v}_{s\alpha}^2(\mathbf{b}_\alpha),$

$2\beta_{v\alpha}(\mathbf{b})=\dfrac{<c(\mathbf{b}_\alpha,x)(g^1(x))^2>}{<m(\mathbf{b}_\alpha,x)(g^1(x))^2>},\quad \omega_{v\alpha}^2(\mathbf{b}_\alpha)=\dfrac{<K(\mathbf{b}_\alpha,x)(g_{,x}^1(x))^2>}{<m(\mathbf{b}_\alpha,x)(g^1(x))^2>}.$

The initial conditions have the form

$$y_{n\alpha}(\mathbf{b}_\alpha,0)=0,\quad \dot{y}_{n\alpha}(\mathbf{b}_\alpha,0)=0,\quad z_{n\alpha}^1(\mathbf{b}_\alpha,0)=0,\quad \dot{z}_{n\alpha}^1(\mathbf{b}_\alpha,0)=0\tag{39}$$

Let us consider the steady-state vibration ($t_0=-\infty$) of the beam under stationary, stochastic excitation. In this case the solutions (26-30) have the following form: for the expected value

$$\begin{aligned}
&E[u_\alpha(\mathbf{b}_\alpha,x,\infty)]_\alpha=E[w_\alpha(\mathbf{b}_\alpha,x,\infty)]_\alpha+g^A(x)E[v_\alpha^A(\mathbf{b}_\alpha,x,\infty)]_\alpha=\\
&=E[f_\alpha]_\alpha\int_0^\infty E[H_\alpha(\mathbf{b}_\alpha,x,\tau)]_\alpha d\tau=\\
&=E[f_\alpha]\int_0^\infty E[H_{w\alpha}(\mathbf{b}_\alpha,x,\tau)]_\alpha d\tau+g^A(x)E[f_\alpha]\int_0^\infty E[H_{v\alpha}^A(\mathbf{b}_\alpha,x,\tau)]_\alpha d\tau,
\end{aligned}\tag{40}$$

for the covariance:

$$\begin{aligned}
&Cov_{uu_\alpha}\left[x_1,x_2,\infty,\infty\right]_\alpha=\\
&=\int_0^\infty\int_0^\infty E\left[H_\alpha\left(\mathbf{b}_\alpha,x_1,\tau_1\right)H_\alpha\left(\mathbf{b}_\alpha,x_2,\tau_2\right)\right]_\alpha Cov_{ff_\alpha}\left[\tau_1-\tau_2\right]_\alpha d\tau_1 d\tau_2+\\
&+E^2[f_\alpha]_\alpha\int_0^\infty\int_0^\infty Cov_{HH_\alpha}\left[x_1,x_2,\tau_1,\tau_2\right]_\alpha d\tau_2 d\tau_2,
\end{aligned}\tag{41}$$

and for the variance:

$$\begin{aligned}
&\sigma_{u_\alpha}^2(x,\infty)_\alpha=Cov_{uu_\alpha}\left[x,x,\infty,\infty\right]_\alpha=\\
&=\int_0^\infty\int_0^\infty E\left[H_\alpha\left(\mathbf{b}_\alpha,x,\tau_1\right)H_\alpha\left(\mathbf{b}_\alpha,x,\tau_2\right)\right]_\alpha Cov_{ff_\alpha}\left[\tau_1-\tau_2\right]_\alpha d\tau_1 d\tau_2+\\
&+E^2[f_\alpha]_\alpha\int_0^\infty\int_0^\infty Cov_{HH_\alpha}\left[x,x,\tau_1,\tau_2\right]_\alpha d\tau_2 d\tau_2.
\end{aligned}\tag{42}$$

The lower and upper endpoints of the expected value and variance can be found based on the relationships (40) and (42)

$$E[u_\alpha(\mathbf{b}_\alpha,x,\infty)]_{\alpha l}=\min\{E[f_\alpha]_\alpha\int_0^\infty E[H_\alpha(\mathbf{b}_\alpha,x,\tau)]_\alpha d\tau\}=$$
$$=\min\{E[f_\alpha]\int_0^\infty E[H_{w\alpha}(\mathbf{b}_\alpha,x,\tau)]_\alpha d\tau+g^A(x)E[f_\alpha]\int_0^\infty E[H^A_{v\alpha}(\mathbf{b}_\alpha,x,\tau)]_\alpha d\tau\},\tag{43}$$

$$E[u_\alpha(\mathbf{b}_\alpha,x,\infty)]_{\alpha r}=\max\{E[f_\alpha]_\alpha\int_0^\infty E[H_\alpha(\mathbf{b}_\alpha,x,\tau)]_\alpha d\tau\}=$$
$$=\max\{E[f_\alpha]\int_0^\infty E[H_{w\alpha}(\mathbf{b}_\alpha,x,\tau)]_\alpha d\tau+g^A(x)E[f_\alpha]\int_0^\infty E[H^A_{v\alpha}(\mathbf{b}_\alpha,x,\tau)]_\alpha d\tau\},\tag{44}$$

and

$$\sigma^2_{u_\alpha}(x,\infty)_{\alpha l}=\min\{\int_0^\infty\int_0^\infty E\left[H_\alpha(\mathbf{b}_\alpha,x,\tau_1)H_\alpha(\mathbf{b}_\alpha,x,\tau_2)\right]_\alpha Cov_{ff_\alpha}\left[\tau_1-\tau_2\right]_\alpha d\tau_1 d\tau_2+$$
$$+E^2[f_\alpha]_\alpha\int_0^\infty\int_0^\infty Cov_{HH_\alpha}\left[x,x,\tau_1,\tau_2\right]_\alpha d\tau_2 d\tau_2\},\tag{45}$$

$$\sigma^2_{u_\alpha}(x,\infty)_{\alpha r}=\max\{\int_0^\infty\int_0^\infty E\left[H_\alpha(\mathbf{b}_\alpha,x,\tau_1)H_\alpha(\mathbf{b}_\alpha,x,\tau_2)\right]_\alpha Cov_{ff_\alpha}\left[\tau_1-\tau_2\right]_\alpha d\tau_1 d\tau_2+$$
$$+E^2[f_\alpha]_\alpha\int_0^\infty\int_0^\infty Cov_{HH_\alpha}\left[x,x,\tau_1,\tau_2\right]_\alpha d\tau_2 d\tau_2\}.\tag{46}$$

In the case of the “white noise” excitation the variance has the form

$$\sigma^2_{u_\alpha}(x,\infty)_\alpha=\sigma^2_{f\alpha}\int_0^\infty E[H^2_\alpha(\mathbf{b}_\alpha,x,\tau]_\alpha d\tau+E^2[f_\alpha]_\alpha\int_0^\infty\int_0^\infty Cov_{HH_\alpha}(x,x,\tau_1,\tau_2)_\alpha d\tau_1 d\tau_2.\tag{47}$$

The randomness of the structural parameters is included in the random dynamic influence function $H_\alpha(\mathbf{b}_\alpha,x,t)=H_{w\alpha}(\mathbf{b}_\alpha,x,t)+g^A(x)H^A_{v\alpha}(\mathbf{b}_\alpha,x,t)$ which depends on the uncertain parameter vector $\mathbf{b}$. Here, another difficulty arises in the determination of the expected values and the second moment (covariance) of the RDIF which are in Eqs. (27-35) and (40-47). This problem can be solved using the perturbation method [8,9] or the Monte Carlo Method.

If the excitation process is of the type of non-stationary kinematic excitation following relationships should be introduced in the above general solution $<q_\alpha(x)>=<m_\alpha(x)>$, $f_\alpha(t)=-\ddot{z}_\alpha(t)=e_\alpha(t)X_\alpha(t)$, where $e_\alpha(t)$ is a fuzzy deterministic envelope and $X_\alpha(t)$ is a fuzzy stationary stochastic process.

3 MODEL OF THE BUILDING

Each storey of the building consists of two parts with different stiffnesses and masses. For this reason as a model of the buiding we can consider a beam composed of a periodic array of two linearly elastic, homogeneous and isotropic constituents with perfect interfaces. Let us assume that the Kirchhoff moduli $G(x)$ are fuzzy random variables and are equal to $b_{1\alpha}=G_{1\alpha}$ on $(0,a)$ and $b_{2\alpha}=G_{2\alpha}$ on (a,l). The fuzzy random variables $G_{1\alpha}$ and $G_{2\alpha}$ are assumed to be mutually independent. The other variables are deterministic and are equal to, respectively,

F_1, κ_1, ρ_1 on $(0,a)$ and F_2, κ_2, ρ_2 on (a,l) where ρ_1 and ρ_2 are denote mass density. One introduces only one ($N=1$) shape function $g^1(x)$, which is piecewise linear, Fig.1.

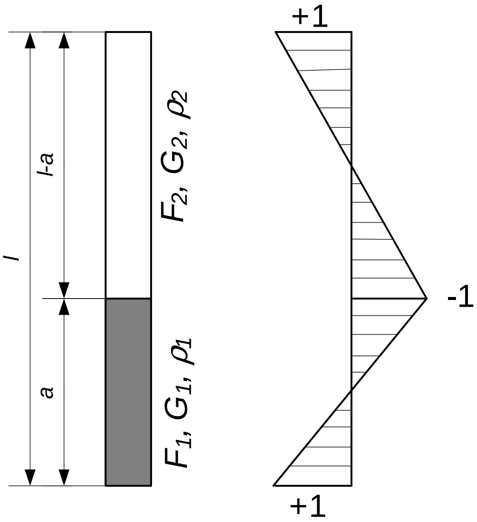

Figure 1:

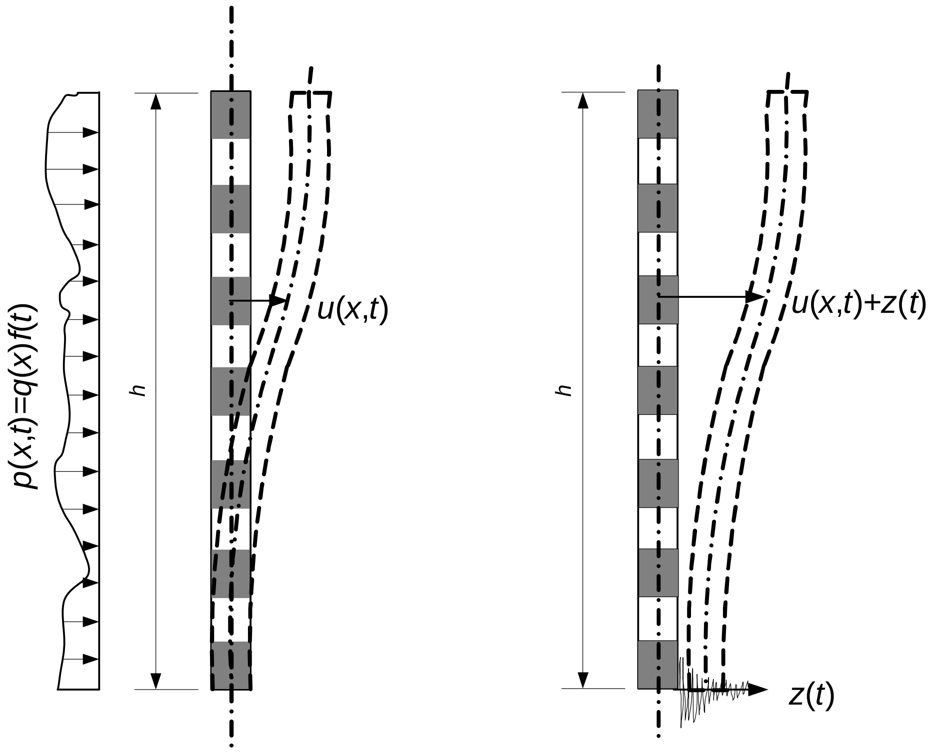

Figure 2:

In this case we have

$$
\begin{aligned}
&< K(x) >_{\alpha} = G_{1\alpha}\kappa_1 F_1 \frac{a}{l} + G_{2\alpha}\kappa_2 F_2 (1-\frac{a}{l}),\\
&< K(x) g^1_{,x}(x) >_{\alpha} = G_{1\alpha}\kappa_1 F_1 \frac{a}{l} - G_{2\alpha}\kappa_2 F_2 (1-\frac{a}{l}),\\
&< K(x) g^1_{,x}(x) g^1_{,x}(x)) >_{\alpha} = \frac{1}{3}[G_{1\alpha}\kappa_1 F_1 \frac{a}{l} - G_{2\alpha}\kappa_2 F_2 (1-\frac{a}{l})],\\
&< m(x) > = F_1\rho_1 \frac{a}{l} + F_2\rho_2(1-\frac{a}{l}),\\
&< m(x) g^1(x) g^1(x) > = \frac{1}{3}[F_1\rho_1 \frac{a}{l} + F_2\rho_2(1-\frac{a}{l})],\\
&< c(x) > = 2\beta[F_1\rho_1 \frac{a}{l} + F_2\rho_2(1-\frac{a}{l})],\\
&< c(x) g^1(x) g^1(x) > = \frac{2\beta}{3}[F_1\rho_1 \frac{a}{l} + F_2\rho_2(1-\frac{a}{l})].
\end{aligned}
\tag{48}
$$

For the particular case $a = l/2$ it is

$$
\begin{aligned}
&< K(x) >_{\alpha} = \frac{1}{2}(G_{1\alpha}\kappa_1 F_1 + G_{2\alpha}\kappa_2 F_2),\\
&< K(x) g^1_{,x}(x) >_{\alpha} = \frac{1}{2}(G_{1\alpha}\kappa_1 F_1 - G_{2\alpha}\kappa_2 F_2),\\
&< K(x) g^1_{,x}(x) g^1_{,x}(x)) >_{\alpha} = \frac{1}{6}(G_{1\alpha}\kappa_1 F_1 - G_{2\alpha}\kappa_2 F_2),\\
&< m(x) > = \frac{1}{2}(F_1\rho_1 + F_2\rho_2),\\
&< m(x) g^1(x) g^1(x) > = \frac{1}{6}(F_1\rho_1 + F_2\rho_2),\\
&< c(x) > = \beta(F_1\rho_1 + F_2\rho_2),\\
&< c(x) g^1(x) g^1(x) > = \frac{\beta}{3}(F_1\rho_1 + F_2\rho_2).
\end{aligned}
\tag{49}
$$

Using the perturbation method [8, 9] we obtain the following set of differential equations:
- Zeroth order equations

$$
\begin{aligned}
&-\frac{(\bar{G}_{1\alpha}\kappa_1 F_1 + \bar{G}_{2\alpha}\kappa_2 F_2)}{F_1\rho_1 + F_2\rho_2} H^0_{w\alpha,xx}(x,t) - \frac{4(\bar{G}_{2\alpha}\kappa_2 F_2 - \bar{G}_{1\alpha}\kappa_1 F_1)}{F_1\rho_1 + F_2\rho_2} H^{10}_{v\alpha,x}(x,t) +\\
&+2\beta \dot{H}^0_{w\alpha}(x,t) + \ddot{H}^0_{w\alpha}(x,t) = \frac{2 < q(x) >}{F_1\rho_1 + F_2\rho_2}\delta(t),\\
&\frac{12(\bar{G}_{2\alpha}\kappa_2 F_2 - \bar{G}_{1\alpha}\kappa_1 F_1)}{l^2(F_1\rho_1 + F_2\rho_2)} H^0_{w\alpha,x}(x,t) + \frac{48(\bar{G}_{1\alpha}\kappa_1 F_1 + \bar{G}_{2\alpha}\kappa_2 F_2)}{l^2(F_1\rho_1 + F_2\rho_2)} H^{10}_{v\alpha}(x,t) +\\
&+2\beta \dot{H}^{10}_{v\alpha}(x,t) + \ddot{H}^{10}_{v\alpha}(x,t) = 0.
\end{aligned}
\tag{50}
$$

- First order equations (for i=1,2.)

$$
\begin{aligned}
&-\frac{(\bar{G}_{1\alpha}\kappa_1F_1+\bar{G}_{2\alpha}\kappa_2F_2)}{F_1\rho_1+F_2\rho_2}H^{I}_{wi\alpha,xx}(x,t)-\frac{4(\bar{G}_{2\alpha}\kappa_2F_2-\bar{G}_{1\alpha}\kappa_1F_1)}{F_1\rho_1+F_2\rho_2}H^{1I}_{vi\alpha,x}(x,t)+\\
&+2\beta\dot{H}^{I}_{wi\alpha}(x,t)+\ddot{H}^{I}_{wi\alpha}(x,t)=\frac{1}{F_1\rho_1+F_2\rho_2}R_{1i\alpha}(x,t),\\
&\frac{12(\bar{G}_{2\alpha}\kappa_2F_2-\bar{G}_{1\alpha}\kappa_1F_1)}{l^2(F_1\rho_1+F_2\rho_2)}H^{I}_{wi\alpha,x}(x,t)+\frac{48(\bar{G}_{1\alpha}\kappa_1F_1+\bar{G}_{2\alpha}\kappa_2F_2)}{l^2(F_1\rho_1+F_2\rho_2)}H^{1I}_{vi\alpha}(x,t)+\\
&+2\beta\dot{H}^{1I}_{vi\alpha}(x,t)+\ddot{H}^{1I}_{vi\alpha}(x,t)=\frac{1}{F_1\rho_1+F_2\rho_2}R_{2i\alpha}(x,t),
\end{aligned}
\tag{51}
$$

where

$$
\begin{aligned}
&R_{11\alpha}(x,t)=H^{0}_{w\alpha,xx}(x,t)-4H^{10}_{v\alpha,x}(x,t),\\
&R_{21\alpha}(x,t)=\frac{12}{l^2}H^{0}_{w\alpha,x}(x,t)-\frac{48}{l^2}H^{10}_{v\alpha}(x,t),\\
&R_{12\alpha}(x,t)=H^{0}_{w\alpha,xx}(x,t)+4H^{10}_{v\alpha,x}(x,t),\\
&R_{22\alpha}(x,t)=-\frac{12}{l^2}H^{0}_{w\alpha,x}(x,t)-\frac{48}{l^2}H^{10}_{v\alpha}(x,t).
\end{aligned}
\tag{52}
$$

In particular case if $G_{1\alpha}=G_{2\alpha}=G_\alpha$ than the equations 47-50 have the forms

$$
\begin{aligned}
&<K(x)>_\alpha=G_\alpha[\kappa_1F_{1\alpha}\frac{a}{l}+\kappa_2F_{2\alpha}(1-\frac{a}{l})],\\
&<K(x)g^1_{,x}(x)>_\alpha=G_\alpha[\kappa_1F_{1\alpha}\frac{a}{l}-\kappa_2F_{2\alpha}(1-\frac{a}{l})],\\
&<K(x)g^1_{,x}(x)g^1_{,x}(x))>_\alpha=\frac{1}{3}G_\alpha[\kappa_1F_{1\alpha}\frac{a}{l}-\kappa_2F_{2\alpha}(1-\frac{a}{l})].
\end{aligned}
\tag{47a}
$$

for the particular case $a=l/2$

$$
\begin{aligned}
&<K(x)>_\alpha=\frac{G_\alpha}{2}(\kappa_1F_{1\alpha}+\kappa_2F_{2\alpha}),\\
&<K(x)g^1_{,x}(x)>_\alpha=\frac{1}{2}G_\alpha(\kappa_1F_{1\alpha}-\kappa_2F_{2\alpha}),\\
&<K(x)g^1_{,x}(x)g^1_{,x}(x))>_\alpha=\frac{1}{6}G_\alpha(\kappa_1F_{1\alpha}-\kappa_2F_{2\alpha}).
\end{aligned}
\tag{48a}
$$

- Zeroth order equations

$$-\frac{\bar{G}_\alpha(\kappa_1F_{1\alpha}+\kappa_2F_{2\alpha})}{F_{1\alpha}\rho_1+F_{2\alpha}\rho_2}H^0_{w\alpha,xx}(x,t)-\frac{4\bar{G}_\alpha(\kappa_2F_{2\alpha}-\kappa_1F_{1\alpha})}{F_{1\alpha}\rho_1+F_{2\alpha}\rho_2}H^{10}_{v\alpha,x}(x,t)+$$
$$+2\beta\dot{H}^0_{w\alpha}(x,t)+\ddot{H}^0_{w\alpha}(x,t)=\frac{2<q(x)>}{F_{1\alpha}\rho_1+F_{2\alpha}\rho_2}\delta(t), \tag{49a}$$
$$\frac{12\bar{G}_\alpha(\kappa_2F_{2\alpha}-\kappa_1F_{1\alpha})}{l^2(F_{1\alpha}\rho_1+F_{2\alpha}\rho_2)}H^0_{w\alpha,x}(x,t)+\frac{48\bar{G}_\alpha(\kappa_1F_{1\alpha}+\kappa_2F_{2\alpha})}{l^2(F_{1\alpha}\rho_1+F_{2\alpha}\rho_2)}H^{10}_{v\alpha}(x,t)+$$
$$+2\beta\dot{H}^{10}_{v\alpha}(x,t)+\ddot{H}^{10}_{v\alpha}(x,t)=0.$$

- *First order equations (for i=1,2.)*

$$-\frac{\bar{G}_\alpha(\kappa_1F_{1\alpha}+\kappa_2F_{2\alpha})}{F_{1\alpha}\rho_1+F_{2\alpha}\rho_2}H^I_{w\alpha,xx}(x,t)-\frac{4\bar{G}_\alpha(\kappa_2F_{2\alpha}-\kappa_1F_{1\alpha})}{F_{1\alpha}\rho_1+F_{2\alpha}\rho_2}H^{1I}_{v\alpha,x}(x,t)+$$
$$+2\beta\dot{H}^I_{w\alpha}(x,t)+\ddot{H}^I_{w\alpha}(x,t)=\frac{1}{F_{1\alpha}\rho_1+F_{2\alpha}\rho_2}R_{1\alpha}(x,t), \tag{50a}$$
$$\frac{12\bar{G}_\alpha(\kappa_2F_{2\alpha}-\kappa_1F_{1\alpha})}{l^2(F_{1\alpha}\rho_1+F_{2\alpha}\rho_2)}H^I_{w\alpha,x}(x,t)+\frac{48\bar{G}_\alpha(\kappa_1F_{1\alpha}+\kappa_2F_{2\alpha})}{l^2(F_{1\alpha}\rho_1+F_{2\alpha}\rho_2)}H^{1I}_{v\alpha}(x,t)+$$
$$+2\beta\dot{H}^{1I}_{v\alpha}(x,t)+\ddot{H}^{1I}_{v\alpha}(x,t)=\frac{1}{F_{1\alpha}\rho_1+F_{2\alpha}\rho_2}R_{2\alpha}(x,t),$$

Let us consider the shear vibration of a homogeneous beam. The random influence function has the form

$$H_\alpha(x,t)=\frac{2}{hm_\alpha}\sum_{n=1}^{\infty}\frac{q_n}{\Omega_{n\alpha}}e^{-\beta t}\sin\Omega_{n\alpha}t\sin\lambda_n x, \tag{53}$$

where $2\beta=\frac{c}{m}$, $(\Omega_{n\alpha})^2=\lambda_n^2\frac{G_\alpha F}{m}-\beta^2=(\omega_{n\alpha})^2-\beta^2$, $\lambda_n=\frac{\pi}{h}(n-\frac{1}{2})$, $\omega_n^2=\lambda_n^2\frac{K}{m}=\lambda_n^2v_s^2$,
$q_n=\int_0^h q(x)\sin\lambda_n x dx.$

We assume that the Kirchhoff modulus G_α is a fuzzy random variable. The other beam parameters are assumed to be deterministic. The random functions describing the load were assumed to be fuzzy weakly stationary stochastic processes, $E[f_\alpha(t)]_\alpha=E[f_\alpha]_\alpha=const.$, $C_{ff_\alpha}(t_1,t_2)=C_{ff_\alpha}(t_1-t_2)=C_{ff_\alpha}(t).$ Let us assume that the time and space correlation of the load process is of the "white noise" type, namely that the covariance functions has the form $C_{ff_\alpha}=\sigma^2_{f_\alpha}\delta(t).$ The solution will be found for the steady-state, i.e. $t_0=-\infty.$ The expected value is equal to

$$E[u_{\alpha\beta}(\mathbf{b}_\alpha,x,\infty)]_{\alpha\beta}=E[f_\alpha]_\alpha\int_{-\infty}^{t}E[H_\alpha(x,t-\tau)]\,d\tau=\frac{2E[f_\alpha]}{Fh}E[\frac{1}{G_\alpha}]_\alpha\sum_{n=1}^{\infty}\frac{q_n}{\lambda_n^2}\sin\lambda_n x. \tag{54}$$

In order to find the probabilistic characteristics the function of the random variables has been expanded into Taylor series around the mean value and restricted to three first items (components) of the expansion. The expected value of the response has form

$$E[u_\alpha(\mathbf{b}_\alpha, x, \infty)]_\alpha = \frac{2E[f_\alpha]_\alpha}{Fh}\frac{(1+v_{G\alpha}^2)}{E[G_\alpha]_\alpha}\sum_{n=1}^{\infty}\frac{q_n}{\lambda_n^2}\sin\lambda_n x, \tag{55}$$

where v_{G_α} is standard deviation of the Kirchhoff modulus.

The variance of the beam displacement for the steady-state vibrations (t→∞) can be shown in the form

$$\sigma_{u_\alpha}^2(x,\infty)_\alpha = \sigma_{f\alpha}^2\int_0^\infty E[h_\alpha^2(\mathbf{b}_\alpha, x, \tau]_\alpha d\tau + E^2[f_\alpha]_\alpha\int_0^\infty\int_0^\infty Cov_{hh_\alpha}(x,x,\tau_1,\tau_2)_\alpha d\tau_1 d\tau_2. \tag{56}$$

After calculating the integrals in the equation (56) one obtains

$$\begin{aligned}\sigma_{u_\alpha}^2(x,\infty)_\alpha &= \frac{\sigma_{f_\alpha}^2(1+v_{G_\alpha}^2)}{Fh^2 m\beta E[G_\alpha]_\alpha}\sum_{n=1}^{\infty}\frac{q_n^2}{\lambda_n^2}\sin^2\lambda_n x + \\ &+\frac{16\sigma_{f_\alpha}^2\beta}{h^2m^2}\sum_{n=1}^{\infty}\sum_{\substack{k=1\\ n\neq k}}^{\infty} q_n q_k E[\frac{1}{(2\beta^2+\omega_{n\alpha}^2+\omega_{k\alpha}^2)^2-4\Omega_{n\alpha}^2\Omega_{k\alpha}^2}]_\alpha \sin\lambda_n x\sin\lambda_k x.\end{aligned} \tag{57}$$

4 NUMERICAL EXAMPLE

For the testing the approach presented the shape functions of the expected value and standard deviation of the displacements on a top of multistorey tall building with averaged structure parameters have been calculated. The calculations have been done assuming that the expected value of the shear stiffness is a fuzzy number of triangular shape functions. Other quantities are being treated as deterministic parameters. The results obtained for the coefficient of variation of the shear stiffness equal to 0.1 are shown in the Figures 3 and 4.

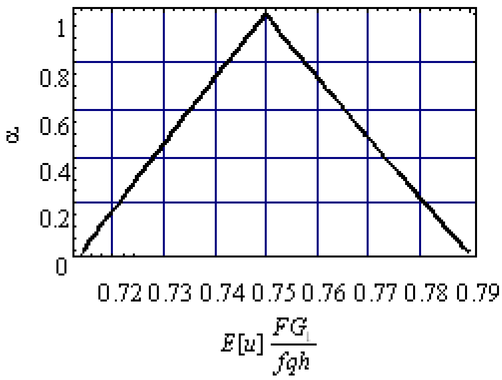

Figure 3. The shape function of the expected value of the displacement

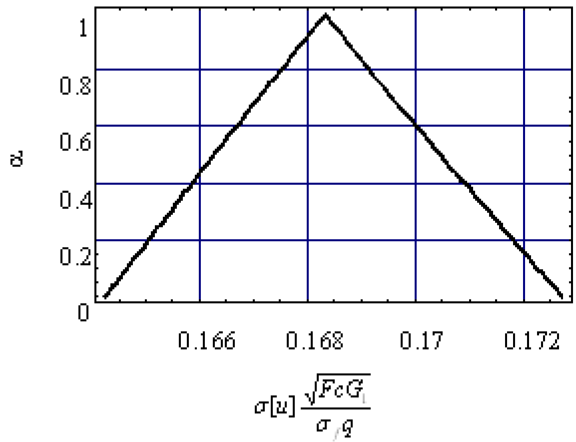

Figure 4. The shape function of the standard deviation of the displacement

5 SUMMARY AND CONCLUSIONS

- The model of a vibrating shear beam has been generalized on the finite periodic composite beam with uncertain parameters. Different types of uncertainty of the structure parameters and the excitation process have been considered, namely: fuzzy numbers, random variables, random functions, fuzzy random variables, fuzzy random functions and fuzzy stochastic processes. This allows for investigating a wide analysis of complex problems of the shear vibrations of periodic composite beams with fuzzy random parameters under fuzzy stochastic excitations. Much attention has been focused on for obtaining the solution in the most general case. Presented model of the finite periodic composite beam with uncertain parameters has been proposed as a model of multistory building. It has been assumed, that each storey has two different stiffnesses: one for the part without windows and doors and another one with windows and doors.
- For obtaining the solution within the correlation theory the fuzzy random dynamic influence function has been introduced, which allows for applying the perturbation method or Monte Carlo simulation.
- The difficulty connected with solving the differential equations with periodic variable coefficients has been overcame by applying the average tolerance approach, which transforms the differential equations with periodic variable coefficients into the averaged differential equations with constant coefficients.
- For the steady-state vibration of the beam with fuzzy random parameters under stationary stochastic excitation the expressions for calculating the expected value and variance of the beam displacements are given.

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