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MATHEMATICAL MODELING OF IRREVERSIBLE DEFORMING, MICRO- AND MACROFRACTURE OF ROCK IN THE VICINITY OF A BOREHOLE IN ITS DYNAMICAL UNLOADING

Alexey B. Kiselev and Pavel P. Zacharov

Mechanics and Mathematics Faculty of Moscow M.V. Lomonosov State University
Leninskie Gory, 1, MSU, Main Bldg, Moscow 119992, Russia
e-mail: akis@mech.math.msu.su

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Abstract. *The rock formation at a depth of several thousand meters is exposed to a hydrostatic pressure. Drilling a borehole makes the drill experience same pressure from the walls of the borehole. On fast removing of the drill from the borehole the dynamical process of unloading borehole walls begins. The sharp decrease of normal pressure on the walls of the borehole brings to the increase of ring stresses. The waves of unloading propagate from the borehole, which could cause fragmentation of rock and blocking the borehole with fractured material. The goal of the present paper is to give the problem statement foe the dynamical process of unloading the internal walls of the borehole after removing the drill, and successive oil-bearing layer's fracturing. The layer is represented by the model of damageable thermoelastoplastic material with two parameters of damaging (by evolution of micropores and by shear microfracturing). The criterion of the beginning of new free surfaces within the material) uses the principle of the critical value of specific dissipated energy.The problem is treated as two-dimensional (plane deformed state). This task is solved numerical modeling on Lagrancian mesh by method similar to M.L. Wilkins one and on local reconstruction of the Lanrangian grid in the vicinity of the fracture origination.*

1 SETTING OF A PROBLEM

Description of process we will in cylindrical coordinate system which axis Oz is coincide with axis of a hole. Then all parameter of problem depend on space coordinate r,θ and the time t.

Write the mass, momentum and internal energy equations in cylindrical coordinate system r,θ:

$$\frac{\dot\rho}{\rho}=-\dot\varepsilon_r-\dot\varepsilon_\theta;\quad \rho\dot v_r=\frac{\partial\sigma_r}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta}+\frac{\sigma_r-\sigma_\theta}{r},\qquad \rho\dot v_\theta=\frac{\partial\sigma_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta}+2\frac{\sigma_{r\theta}}{r};\tag{1}$$

$$\rho c_\sigma\dot T+\alpha_v\dot\sigma T=S_r\dot\varepsilon_r^p+S_\theta\dot\varepsilon_\theta^p+S_z\dot\varepsilon_z^p+2S_{r\theta}\dot\varepsilon_{r\theta}^p+A\dot\alpha^2+\Lambda\dot\omega^2$$

Here and later point under symbol denote the material derivative with respect to time; ρ - density; v_r, v_θ - velocity components; σ_r, σ_θ, $\sigma_{r\theta}$ - stress tensor components (σ_θ – ring stress), which decompose on spherical $\sigma=(\sigma_r+\sigma_\theta+\sigma_z)/3$ and deviator parts $S_r,S_\theta,S_{r\theta}=\sigma_{r\theta};S_r+S_z+S_\theta=0;$ $\dot\varepsilon_r,\dot\varepsilon_\theta,\dot\varepsilon_{r\theta}$ - velocity strain tensor components; $\dot\varepsilon_r^p,\dot\varepsilon_\theta^p,\dot\varepsilon_z^p$, $\dot\varepsilon_{r\theta}^p$ - plastic components of velocity strain tensor; T - temperature; ω, α - scalar damage parameters; c_σ - is the heat conductivity at constant stress; α_v - is the coefficient of cubic expansion; Λ, A - constants of materials, connected with damage parameters ω and α; $\dot\varepsilon_r^p,\dot\varepsilon_\theta^p,\dot\varepsilon_z^p,\dot\varepsilon_{r\theta}^p$ - plastic components of velocity strain tensor; T - temperature; ω, α - scalar damage parameters of medium; c_σ - is the heat conductivity at constant stress; α_v - is the coefficient of cubic expansion; Λ, A - medium parameters connected thermal and damage processes.

Velocity strain tensor components are expressed over velocity components:

$$\dot\varepsilon_r=\frac{\partial v_r}{\partial r},\qquad \dot\varepsilon_\theta=\frac{1}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_r}{r},\qquad \dot\varepsilon_{r\theta}=\frac{1}{2}\left(\frac{\partial v_\theta}{\partial r}+\frac{1}{r}\frac{\partial v_r}{\partial\theta}-\frac{v_\theta}{r}\right),\tag{2}$$

and decompose on elastic and plastic components:

$$\dot\varepsilon_r=\dot\varepsilon_r^e+\dot\varepsilon_r^p,\ \ \dot\varepsilon_\theta=\dot\varepsilon_\theta^e+\dot\varepsilon_\theta^p,\ \ \dot\varepsilon_{r\theta}=\dot\varepsilon_{r\theta}^e+\dot\varepsilon_{r\theta}^p,\ \ \dot\varepsilon_z=\dot\varepsilon_z^e+\dot\varepsilon_z^p\equiv 0.\tag{3}$$

Plastic flow are incompressible: $\dot\varepsilon_r^p+\dot\varepsilon_\theta^p+\dot\varepsilon_z^p\equiv 0$.

The system of constitutive equation for a model of damageable thermoelastoviscoplastic medium is as follows [1-3]:

$$\begin{cases}\dot\sigma'=K_0\left(\dot\varepsilon_r+\dot\varepsilon_\theta-\alpha_v\dot T-\dfrac{\Lambda}{3}\dot\omega\dfrac{\partial\dot\omega}{\partial\sigma}\right)\\ \left(S'_{ij}\right)^\nabla+\lambda S'_{ij}=2\mu_0\dot e_{ij}-2A\dot\alpha\dfrac{\partial\dot\alpha}{\partial S_{ij}}\\ S'_{ij}S'_{ij}\le\dfrac{2}{3}Y_0^2(\sigma)\\ Y_0=c_1\sigma+c_2\end{cases}.\tag{4}$$

Here symbol ∇ - Yaumann derivative of deviator components stress tensor; $\dot{e}_{ij}$ - deviator of velocity strain tensor; $\sigma' = \dfrac{\sigma}{(1-\omega)}$ $\quad S'_{ij} = \dfrac{S_{ij}}{(1-\omega)(1-\alpha)}$; K_0 and μ_0 - volume and shear module for an undamaged material; ω_{ij} – tensor of rotation:

$$\omega_r = \omega_\theta = 0\,,\ \omega_{r\theta} = -\omega_{\theta r} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r} - \frac{\partial v_\theta}{\partial r}\right). \tag{5}$$

The last relation from (4) – Misses-Shlexer rule, connecting limit of elasticity under simple tension Y_0 and pressure in layer ($-\sigma$); c_1, c_2 – material constants.

System of equations (1) - (5) close by kinetic equations for damage parameters ω, α:

$$\frac{\dot{\omega}}{\omega} = B\left(\frac{\sigma}{1-\omega} - \sigma_*\right) H\left(\frac{\sigma}{1-\omega} - \sigma_*\right) + \frac{\sigma - \sigma^+}{4\eta_0} H(\sigma - \sigma^+) + \frac{\sigma - \sigma^-}{4\eta_0} H(\sigma^+ - \sigma)\,, \tag{6}$$

$$\sigma^+ = -\frac{2}{3} Y_0 \ln\omega - p_0\left(\frac{\omega_0}{\omega}\right)^\gamma, \quad \sigma^- = +\frac{2}{3} Y_0 \ln\omega - p_0\left(\frac{\omega_0}{\omega}\right)^\gamma,$$

$$\dot{\alpha} = C\left(\frac{S_u}{(1-\omega)(1-\alpha)} - S_u^*\right) H\left(\frac{S_u}{(1-\omega)(1-\alpha)} - S_u^*\right).$$

Here η_0 - dynamic viscosity for an undamaged material; p_0 – initial pressure in pore («горное» pressure); γ - index of adiabatic curve for medium, filling pore; ω_0 - initial porosity; $S_u = \sqrt{S_{ij} S_{ij}}$ - intensity of stress deviator; S_u^*, σ_*, B, C - constants of material; H(x) - Heaviside unit function.

From equations (6) we see that the first term describing the viscous growth in domains of tension of the material comes into play. The second term describes the viscoplastic flowing in pores when the material is compressed. Note that the equation for ω taken without the dynamical problem on a single spherical pore of inner radius a and other radius b in a viscoplastic incompressible material. Damage parameter α connect with intensity of stress deviator and describe fracture shear.

The evolution of the intensive plastic flow and accumulation of microstructure damages may be considered as a process of prefracture of the material. The entropy criterion of limiting specific dissipation [1, 3]:

$$D = \int_0^{t_*} \frac{1}{\rho}(d_M + d_F + d_T)dt = D_*\,, \tag{7}$$

$$d_M = S_{ij}\dot{\varepsilon}_{ij}^p\,,\ d_F = \Lambda\dot{\omega}^2 + A\dot{\alpha}^2\,,\ d_T = \kappa\frac{(grad\,T)^2}{T}$$

is proposed as the criterion of the beginning of macrofracture (i.e., the beginning of formation of cracks (new free surfaces) in material). Here t_* is the time of the beginning of fracture; D_* is a constant of the material (the limiting specific dissipation); d_M, d_F and d_T are mechanical dissipation, dissipation of continuum fracture and thermal dissipation. Similar model used for decision problem of hydraulic fracturing of oil layer ([2] etc.).

2 INITIAL AND BOUNDARY CONDITIONS

Initial rock layer be in rest: $v_r = v_\theta = 0$ in time $t = 0$. Additionally we must define initial description of stresses in layer σ_r, σ_θ, $\sigma_{r\theta}$, value of damage parameters ω and α as functions of space coordinates r and θ.

As initial distribution for stresses in rock layer we use decision of next static elasticity problem: consider infinite cylindrical solid with circular cut; on infinitum in two mutually perpendicular directions applied contractive stresses Σ_1 and Σ_2 corresponding known as "rock pressure". In common case $\Sigma_1 \neq \Sigma_2$ that modeling no homogeneity stress state of rock layer. On surface of circular cut applied pressure $\Sigma_3 \geq 0$.

Distributions of initial stresses for this problem are [4]:

$$\begin{cases} \sigma_r = \dfrac{\Sigma_1+\Sigma_2}{2} + \dfrac{\Sigma_1-\Sigma_2}{2}\cos 2\theta + \dfrac{a^2}{r^2}\left(-\Sigma_3 - \dfrac{\Sigma_1+\Sigma_2}{2} - 2(\Sigma_1-\Sigma_2)\cos 2\theta\right) + \\ + \dfrac{a^4}{r^4}\left(\dfrac{3}{2}(\Sigma_1-\Sigma_2)\cos 2\theta\right) \\ \sigma_\theta = \dfrac{\Sigma_1+\Sigma_2}{2} - \dfrac{\Sigma_1-\Sigma_2}{2}\cos 2\theta + \dfrac{a^2}{r^2}\left(\Sigma_3 + \dfrac{\Sigma_1+\Sigma_2}{2}\right) + \dfrac{a^4}{r^4}\left(-\dfrac{3}{2}(\Sigma_1-\Sigma_2)\cos 2\theta\right) \\ \sigma_z = \nu(\sigma_r + \sigma_\theta) \\ \sigma_{r\theta} = -\dfrac{\Sigma_1-\Sigma_2}{2}\sin 2\theta - \dfrac{a^2}{r^2}\left(2(\Sigma_1-\Sigma_2)\sin 2\theta\right) + \dfrac{a^4}{r^4}\left(\dfrac{3}{2}(\Sigma_1-\Sigma_2)\sin 2\theta\right) \end{cases} \tag{8}$$

Here ν - Poisson coefficient, a - radius of hole.

At moment $t = 0$ happen sharply drop of pressure in hole from value Σ_3 to 0.

3 EXPLICIT SEPARATION OF MACRO FRACTURE ZONES

The entropy criterion of limiting specific dissipation is proposed as the criterion of the beginning of macro fracture (i.e., the beginning of formation of cracks (new free surfaces in material) (7). When criterion (7) is fulfilled at some point of material, a macro crack should be formed there, i.e., a new free surface that will spread over the body. In point where criterion of fracture fulfilled is realized explicit coasts of macro discontinuity. For this we construct separation of nodes of network on cells boundary – internal nodes and corresponding them edges of cells are boundary on this put condition of free surface or contact condition depending on situation [5]. Note that earlier we used procedure of bifurcation of Lagrangian network ([1, 3, 6, 7] et al).

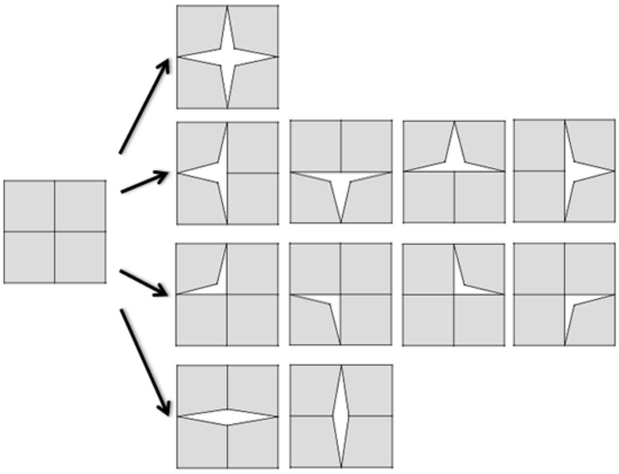

Figure 1: Types of “primary” cracks.

Procedure of decomposition for nodes of network consists of construction “primary” cracks for each node and their combination with already existing cracks. Interaction between “primary” cracks formation cracks biggest scale. On figure 1 show variants of construction “primary” cracks for internal node. Type of “primary” crack determine from analysis strain state on adjacent edges for present node.

4 RESULTS OF COMPUTATIONAL SIMULATION

This task is solved by numerical modeling on Lagrangian mesh by method similar to Wilkins one [18]. In calculation we used next value of parameters: ρ_0 =2000 kg/m^3; K_0=14000 MPa; μ_0=8400 MPA; η_0=100 Pa*c; Λ=1500 Pa*c; c_1=-0,09; c_2=40 MPa; γ=1,4; D^*=334,4 kJ/kg^3; $\omega_0 = 0{,}05$; A = 250 Pa*c; C = 0,00022(Pa*c)$^{-1}$; $B = 0$; $S_u^* = 32{,}5$ MPa. Radius of borehole is $a = 1m$, initial “rock” pressure, depending from coordinates r, θ, defined from formula (8) - $p_0 = -(\sigma_r + \sigma_\theta + \sigma_z)/3$.

Value of stresses, depending initial strain state are: $\Sigma_1 = -50$ MPa or $\Sigma_1 = -65$ MPa; $\Sigma_2 = -75$ MPa and $\Sigma_3 = 35$ MPa.

On figure 2 present dependence of velocities different borehole points, which position depended angle θ, from time t.

On figure 3 present process of origin and growth of cracks for different value of Σ_1. In left part of figure present sequential moments of time for $\Sigma_1 = -50$ MPa, in right part - for $\Sigma_1 = -65$ MPa. Intensity of grey color defines level of energy dissipation, white color correspondence limited dissipation in criteria of fracture (7) - $D_* = 334{,}4$ kJ/kg^3.

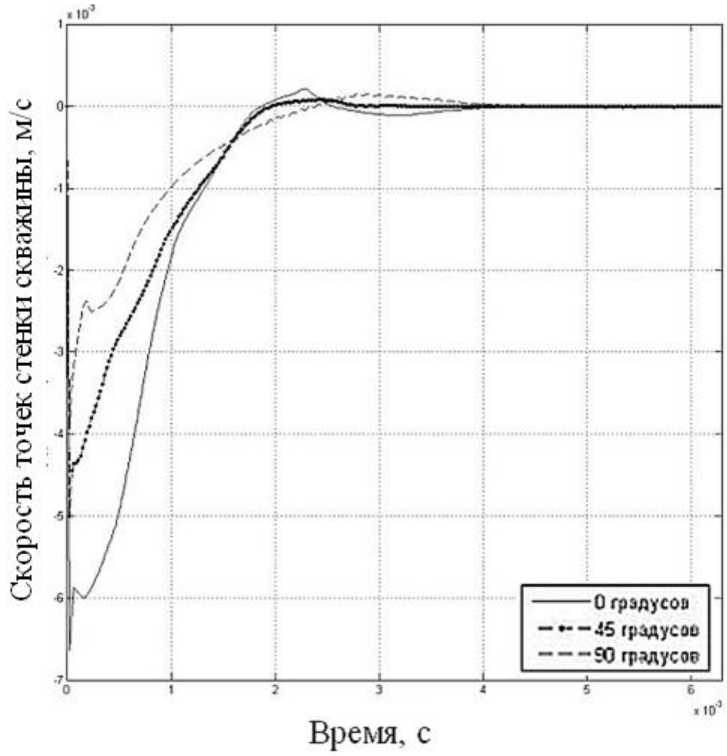

Figure 2: Velocity of different points of borehole.

As it show from figure 3, cracks in material origin and growth on internal surface of borehole and propagate deep into rock under angle near 45° to tangent of borehole contour. Further development of fracture process takes place on two scenarios: either crack continues your growth in initial direction or after some time moments it turns under angle near 90° to your initial moving. As it followed wait, in case $\Sigma_1 = \Sigma_2$ observed symmetrical not depended from angle θ situation of fracture shear type near surface of borehole with dispersion of rock fragments.

5 CONCLUSIONS

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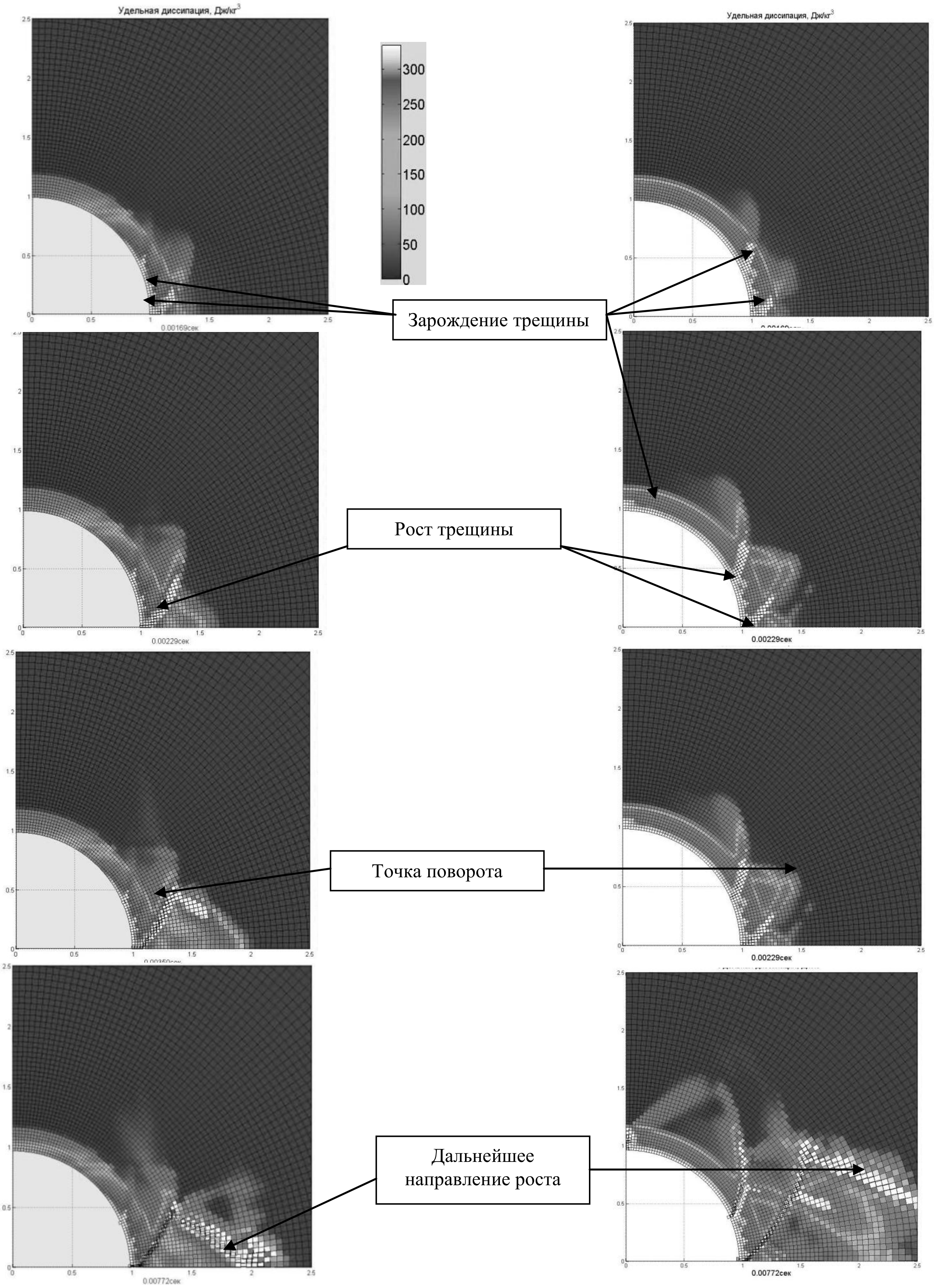

Figure 3: Process of cracks origin and growth.

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