CONSTITUTIVE MODEL FOR FRP AND TIE – CONFINED CONCRETE

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Abstract. Confining wraps or jackets to rehabilitate and strengthen existing concrete columns has proven to be an efficient technique for seismic retrofit of structures. However, most of the compressive strength models of confined concrete only consider the increased strength and ductility provided by fiber reinforced polymers (FRPs), neglecting the contribution of the existing steel reinforcement inside the column’s section. Even if the existing steel stirrups in a reinforced concrete column are not sufficient to confine the concrete core, they must also contribute along with the FRP jacket in confining the section. Therefore, the FRP-confined concrete model proposal by fib has been enhanced to take into account the confining effect of the already existing steel reinforcement when retrofitting a reinforced concrete column with FRP jacketing. To this end, confining pressures contributed at each step of deformation by the case of existing transverse and longitudinal steel reinforcement have been evaluated considering the stress-strain law of the reinforcing steel. Moreover, compatibility of strain in the lateral direction between the jacketing system and the encased concrete is enforced. Finally, the bilinear stress-strain response of FRP-confined concrete is terminated by jacket rupture owing to hoop strains exceeding the strain capacity of the material or to interaction of the jacket with the buckled longitudinal bars. Correlation with three experimental studies gives promising results.
1 Modeling of Concrete Confined with Steel & FRP

The behavior of confined circular sections under axial load is characterized by the radial lateral dilation, which causes radial confining forces or else axisymmetric passive confining pressure that increases with the amount of lateral expansion. Considering this scheme for the case of confinement by means of FRP jacketing, in order to define the confining pressure acting on the section, it is necessary to define the jacket strain, or circumferential strain, parallel to the fibers orientation. Relating the circumferential strain to the strain in the radial direction, the following simple relationship is obtained (Figure 1):

\[ \varepsilon_c = \frac{\Delta C}{C} = \frac{2\pi R (1 + \varepsilon_r - 1)}{2\pi R} = \varepsilon_r \]  

(1)

Owing to the axisymmetry of the problem, the outcome is that the circumferential strain and the strain in the radial direction are equal. This property has been extensively used to calculate directly the radial confining forces based on experimental data by strain gages attached parallel to the fibers orientation in order to obtain the circumferential strains. Along this line, it seems useful to try and extend the simple calculation above to the case where steel stirrups and external FRP jacketing are simultaneously present. The steel ties divide the section into two parts: the first is the concrete core and the second is the concrete cover.

\[ \varepsilon_c = \frac{\Delta C}{C} = \frac{2\pi \left[ R_{\text{core}} (1 + \varepsilon_{r,\text{core}}) + c (1 + \varepsilon_{r,\text{cover}}) \right] - (R_{\text{core}} + c)}{2\pi (R_{\text{core}} + c)} \]  

(2)

However, for the concrete core the following assumption still holds:

\[ \varepsilon_{c,\text{core}} = \varepsilon_{r,\text{core}} \]  

(3)

As explicitly stated above, the equation of radial strains and jacket strains for the case of both FRP and steel confined concrete in circular sections is no longer valid. The circumferential strain of the external jacket is based on the radial strains of both concrete cover and concrete core, where in the latter the presence of the steel ties plays an important role.
2 NUMERICAL MODEL

The mechanical properties of concrete (strength, ductility, energy dissipation) are substantially enhanced under a triaxial stress state. In practice, in order to develop a similar stress state, closed stirrups or spiral reinforcement are used, so that, together with the longitudinal reinforcement, the lateral expansion of concrete is limited. This kind of (passive) confinement affects the behavior of the material favorably after the initiation of internal cracking, which gives rise to the initiation of expansion.

For low strain values, the stress state in the transverse steel reinforcement is very small and the concrete is basically unconfined. In this range, steel and FRP jacketing behave similarly. That is, the inward pressure as a reaction to the expansion of concrete increases continuously. Therefore, speaking in terms of variable confining pressures corresponding to the axial strain level in the section and active triaxial models defining axial stress-strain curves for concrete subject to constant lateral pressure, it can be stated that the stress-strain curve describing the stress state of the section has to cross all active confinement curves up to the curve with lateral pressure equal to the one applied by the stirrups at yielding. Beyond yielding of stirrups, the lateral pressure is still increasing only due to the FRP jacketing, while the steel lateral pressure remains constant. The corresponding stress-strain curve of the section throughout this procedure converges to a confined-concrete axial stress-strain curve that is associated with a lateral pressure magnitude equal to the tensile strength of the FRP jacket plus the yielding strength of ties (excluding the strain hardening behavior of steel, since ultimate strains of steel are usually much higher that those of FRP jackets). In order to model this behavior, an existing FRP-confined concrete model (Spoelstra and Monti 1999) has been enhanced to include the steel ties contribution and thus model in a more consistent way circular columns with transverse reinforcement and retrofitted with FRP jacketing. The above model was based on an iteration procedure that needed to be modified as Figure 2 shows.

In the procedure below (Figure 2), after imposing an axial strain on the section, a pressure coming from the FRP jacket is assumed. Then, the Poisson’s coefficient until yielding of steel stirrups and the pressure coming from the steel ties is calculated based on the BGL model (Braga et al 2006). Here, also the longitudinal bars’ contribution and the arching action between two adjacent stirrups along the column are taken into account according to that model (see Table 1). The confining pressure in the concrete core is simply the summation of...
the lateral pressures contributed by the two confining systems (FRP and Steel). The fib’s model proposal (Spoelstra and Monti 1999) beyond this point is basically used to define the remainder of the parameters declared above, applying that model for the two different regions already mentioned. The focal point of the procedure is in the last step where the confining pressure of the jacket is defined based on the circumferential strain of Equation 2. Finally, at this point, cases where partial wrapping is applied have been included too (14th fib bulletin 2001, Table 1). Such an approach permits also in cases of repair and retrofit two different concrete strengths to be considered, one for the new layer of concrete applied externally and the other for the old concrete in the concrete core which may also be cracked due to former seismic loading.

![Iterative Procedure Diagram](image)

Figure 2. Iterative Procedure

3 FAILURE CRITERION

3.1 FRP Confined Circular RC Sections

It has been well established in recent studies that the rupture strains/strengths measured in tests on FRP confined cylindrical specimens fall substantially below those from flat coupon tensile tests. Several reasons have been suggested for the observed lower rupture strains in place, among which are [(Carey and Harries 2005), (Lam and Teng 2004), (Matthys et. al. 2005)]:

- Misalignment or damage to jacket fibers during handling and lay-up.
- The radius of curvature in FRP jackets on cylinders as opposed to flat tensile coupons.
- Near failure the concrete is internally cracked resulting in no homogeneous deformations. Due to this non-homogeneity of deformations and the high loads exerted on the cracked concrete, local stress concentrations may occur in the FRP reinforcement.
- The existence of a lap-splice zone in which the measured strains are much lower than strains measured elsewhere.
Accounting for these effects an ultimate tensile coupon FRP strain reduced by a k factor (ranging in literature between 50-80 %) is compared to the circumferential strain of concrete (Equation. 2) and the ultimate compressive axial strain of concrete is considered to be attained when:

\[ \varepsilon_c \leq k \cdot \varepsilon_{j,rup,coup} \]  \hspace{1cm} (4)

### 3.2 FRP&Steel Confined Circular RC Sections

In old-type circular columns with inadequate transversal reinforcing details (where FRP jacketing are a commonly used remedy), the unsupported length of longitudinal bars (between 2 successive stirrups) is often much greater than 6D_b. Therefore, the risk for buckling of longitudinal bars under compressive loads soon after yielding is higher. A dire implication is reduced effectiveness of the FRP wraps due to interaction between buckled longitudinal bars and the jacket which may cause premature failure by rupture of the jacket (Sheikh S.A. & Yao G. 2002), (Tastani et al. 2006). This is an additional source of error contributing to overestimation of strength of FRP confined concrete in addition to that generated by the difference between nominal and in-situ strain capacity of the wraps as detailed above. It is the objective of the present paper to study the interaction between wraps and compression reinforcement in FRP-encased reinforced concrete columns, with particular emphasis on the occurrence of instability conditions and the dependable compressive strain of the column prior to actual buckling of the rebars.

In this model, the dilation of concrete core and concrete cover are described through the following equation (Equation. 5) of the model by Spoelstra and Monti (1999).

\[ \varepsilon_r(\varepsilon_{\text{con}}, f_i) = \frac{E_{\text{con}}\varepsilon_{\text{con}} - f_c(\varepsilon_{\text{con}}, f_i)}{2\beta f_c(\varepsilon_{\text{con}}, f_i)} \]  \hspace{1cm} (5)  
\[ \beta = \frac{5700}{\sqrt{f_{co}}} - 500 \]

Thus, the lateral pressure of the FRP jacket confining the concrete cover has been taken into account and by relating the critical buckling conditions with the onset of significant strength loss of the concrete cover, the effect of the confining pressure exerted by the jacket in delaying the occurrence of buckling of the longitudinal bars can be evaluated. Therefore, the critical buckling conditions are delayed depending on how axially stiff is the jacket, which accordingly delays the failure of the concrete cover (that laterally supports the longitudinal bar). This onset of loss of resistance in concrete has been proved to be the point when the net volumetric strain of the material becomes equal to zero (Pantazopoulou and Mills 1995). In circular sections this occurs when:

\[ \varepsilon_v = 0 \Rightarrow 2 \cdot \varepsilon_r = \varepsilon_{\text{con}} \Rightarrow \nu = 0.5 \]  \hspace{1cm} (6)
However, another condition that should be valid for the attainment of critical instability conditions of the longitudinal bars in the high confining stress states under consideration is the occurrence of compression yielding of the longitudinal bar. Regarding this step, it is interesting to note with reference to Figure 3 for a given concrete strength, the point where the volumetric ratio becomes zero moves forward into higher axial compression strain values with increasing confining pressures. Thus, as shown by the two curves in Figure 3 corresponding to different confining pressures, the difference in the lateral behavior of the concrete cover (confined with the jacket’s pressure) and the concrete core (confined by both the steel’s and FRP’s pressure) should also be considered.

Figure 3: Volumetric Strain versus Axial Compressive Strain (Pantazopoulou and Mills 1995)

As it is shown in previous studies (Monti and Nuti 1992), (Bae et al. 2005), the buckling length and the $L_{buck}/D_b$ ratio are critical parameters for the post buckling behavior of longitudinal bars under compression. In cases of columns constructed with obsolete codes with the spacing of the stirrups ranging between 200-500 mm (buckling length) and bar diameters from 12-20 mm, the $L_{buck}/D_b$ ratio is ranging between 10-42. However, apart from old type columns the assumption that the buckling length is equal to the spacing of the stirrups in a RC column does not hold true in all cases (Dhakal and Maekawa 2002) and it may extend over more than a single tie spacing. In order to take into account this behavior (cases of reinforcement repair and FRP retrofit) the following procedure is suggested.

The longitudinal bar is modeled as a pin-ended bar supported along its length by an elastic foundation as shown in Figure 4. The foundation modulus is $k$ (N/mm$^2$) and it is such that when the bar deflects by an amount $u$, a restoring force $ku$ (N/mm) is exerted by the foundation normal to the bar.
Figure 4: A pin-ended bar on elastic foundation.

The governing homogeneous differential equation and the associated eigenvalue problem are:

\[ EI \psi'''' + P \psi'' + k \psi = 0 \]  

\[ P_{cr} = \frac{\pi^2 EI_{red}}{L_{buck}^2} \left[ \frac{m^2}{m^2} + \frac{1}{m^2} \left( \frac{kL_{buck}^4}{\pi^4 EI_{red}} \right) \right] \]

Note that if \( k = 0 \) (which occurs upon yielding of the stirrups), the minimum value of \( P_{cr} \) becomes the classical Euler buckling load. In order to determine the critical load, the buckling mode \( m \) equal to 1 should be used. The stiffness \( k \) representing the supporting system of stirrups could be calculated as follows:

\[ k = n \cdot \frac{E_s \cdot A_{sh}}{L_{buck} \cdot \pi \cdot D_{core}} \]

\[ L_{buck} = (n + 1) \cdot S \]

\[ EI_{red} = 0.5 \cdot E_s \cdot I_b \cdot \frac{f_{yf}}{400} \]  

(Dhakal and Maekawa 2002)

Solution of the problem above is obtained by setting the critical load of the bar equal to its yield force; in this case the only unknown is the number \( n \) of the stirrups, \( n \) over the buckling length. Therefore, by solving Equation 7 for \( n \), the buckling length is determined. The value of \( n \) may be rounded to the nearest integer owing to that the pin-ended bar segment engaged in buckling is assumed to span between successive inflection points of the real deformed shape. If convergence is not possible for \( n > 1 \), the buckling length is taken equal to the spacing of stirrups.

Summing up, after the critical conditions of a longitudinal compressive bar have been attained (this is assumed to coincide upon compression yielding of the bar and the Poisson’s coefficient at the concrete cover exceeds the value of 0.5) the buckling length of the bar is determined. Then, based on the model by Bae et al. 2005 who have related the axial strain with the transversal displacement of the buckled longitudinal bar for a given \( L_{buck}/D_b \) ratio the transversal displacement of the bar is calculated (Table 1). Since, for a longitudinal bar embedded in a RC member axial shortening of the bar means the same amount of shortening for the surrounding concrete mass (Pantazopoulou 1998), the axial strain in the bar is taken as equal with the axial strain of concrete. Finally, the jacket’s circumferential strain due to buckling is determined as follows:
\[
\varepsilon_{e,buck} = \frac{2 \cdot \pi \cdot |w-c|}{\pi \cdot D} \leq \varepsilon_{j,\text{rup,coup}} \quad \text{(Full wrapping)}
\]

\[
\varepsilon_{e,buck} = \frac{2 \cdot \pi \cdot |w-c|}{2 \cdot \pi \cdot D} \leq \varepsilon_{j,\text{rup,coup}} \quad \text{(Partial wrapping)}
\]

It follows from the above equation (Equation. 9) that a tolerance equal to the concrete cover for full wrapping and half of the concrete cover for partial wrapping is given before the initiation of jacket’s strains due to buckling of longitudinal bars since the concrete cover should be severely cracked in case of full wrapping and some spalling could appear in case of partial wrapping. Since, the displacement of the buckled longitudinal bar could be high and the phenomenon affects locally the jacket where the FRP material behavior could be considered linear-elastic, the results are compared to the deformation capacity of tensile coupons (dilation strains and buckling strains are studied independently). In the proposed algorithm detailed above, the failure criterion is used in two steps. Firstly, the circumferential strain due to dilation of concrete under compression is compared to a reduced FRP tensile coupon strain and secondly the induced circumferential strains due to buckling which locally accelerate the jacket’s rupture are compared to the deformation capacity of flat FRP tensile coupons. If one of these conditions are fulfilled the iteration procedure is terminated.

\[f_{i,\text{steel}}(\varepsilon_{\text{con}}) = k_{sl} \cdot \frac{E_{\text{con}} E_{A_{sh}} v(\varepsilon_{\text{con}})}{R_{\text{core}} E_{\text{con}} S + E_{s} A_{sh} [1 - v(\varepsilon_{\text{con}})] \cdot [v(\varepsilon_{\text{con}}) \cdot \varepsilon_{\text{con}} + 1]} \cdot \varepsilon_{\text{con}} \cdot \varepsilon_{sh} \leq \varepsilon_{yh}
\]

\[f_{i,\text{steel}}(\varepsilon_{\text{con}}) = k_{sl} \cdot 0.5 \rho_{sh} f_{sh}, \varepsilon_{sh} \geq \varepsilon_{sh} \geq \varepsilon_{yh}
\]

\[\rho_{sh} = \frac{4 A_{sh}}{D_{\text{core}} S}, \quad \varepsilon_{sh} = \frac{f_{sh}}{E_{s}}, \quad \varepsilon_{sh} = \varepsilon_{c,\text{core}} = \varepsilon_{r,\text{core}} = v(\varepsilon_{\text{con}}) \cdot \varepsilon_{\text{con}}
\]

\[v(\varepsilon_{\text{con}}) = \sqrt{\frac{1}{2} \left( \frac{\varepsilon_{con}}{\varepsilon_{c0}} \right)^{2} + 1.55 \left( \frac{\varepsilon_{con}}{\varepsilon_{c0}} \right)^{3}} \quad \varepsilon_{0} = 0.2, \quad \varepsilon_{co} = 0.002
\]

\[k_{sl} = \frac{45 \varepsilon_{3}^{3}}{45 \varepsilon_{3}^{3} + \beta \varepsilon_{st}}, \quad \varepsilon_{3} = \frac{D_{h}}{S}, \quad \beta = \frac{D_{h}}{D_{b}}, \quad \varepsilon_{st} = \frac{2 D_{h}}{\pi R_{\text{core}}}
\]

\[f_{c} = \frac{f_{cc} \cdot x \cdot r}{r - 1 + x^{r}}, \quad x = \frac{\varepsilon_{\text{con}}}{\varepsilon_{cc}}, \quad \varepsilon_{cc} = \varepsilon_{co} \left[ 1 + 5 \left( \frac{f_{cc}}{f_{co}} - 1 \right) \right], \quad r = \frac{E_{\text{con}}}{E_{\text{con}} - E_{\text{sec}}}
\]

\[E_{\text{sec}} = \frac{f_{cc}}{\varepsilon_{cc}}, \quad \frac{f_{cc}(f_{1})}{f_{co}} = 2.254 \sqrt{1 + 7.94 \frac{f_{1}}{f_{co}}} - 2 \frac{f_{1}}{f_{co}} - 1.254,
\]

\[\varepsilon_{r}(\varepsilon_{\text{con}}, f_{1}) = \frac{E_{\text{con}} \varepsilon_{\text{con}} - f_{c}(\varepsilon_{\text{con}}, f_{1})}{2 \beta f_{c}(\varepsilon_{\text{con}}, f_{1})}, \quad \beta = \frac{5700}{\sqrt{f_{co}}} - 500,
\]

\[E_{\text{con}} = 5700 \cdot \sqrt{f_{co}} \text{(MPa)}
\]
Table 1: Equations embodied in the iterative procedure.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
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<tbody>
<tr>
<td>( k_j = \left(1 - \frac{S_j}{2D}\right)^2 \approx \left(1 - \frac{S_j}{2D}\right)^2 )</td>
<td>(Coefficient for Partial Wrapping)</td>
</tr>
<tr>
<td>( f_{l,\text{cover}} = \frac{1}{2} k_j \rho_j E_j \varepsilon_c \rho_j = \frac{4t_j}{D} )</td>
<td></td>
</tr>
<tr>
<td>( f_{l,\text{core}} = f_{l,\text{cover}} + f_{l,\text{steel}}, f_{c,\text{av}} = \frac{A_{\text{core}}}{A_{\text{tot}}} f_{c,\text{core}} + \frac{A_{\text{cover}}}{A_{\text{tot}}} f_{c,\text{cover}} )</td>
<td>Iterative Procedure</td>
</tr>
<tr>
<td>( \varepsilon_{sl} = \max \left[ \left( \frac{0.035 \cos \theta + \theta}{\cos \theta - 0.035 \theta} \right) \frac{w}{D_b}, \left( \frac{0.07 \cos \theta + \theta}{\cos \theta - 0.07 \theta} \right) \frac{w}{D_b} - 0.035 \right] )</td>
<td>(Bae et al. 2005)</td>
</tr>
<tr>
<td>( \theta = \frac{6.9}{(L_{\text{buck}}/D_b)^2} - 0.05, \varepsilon_{sl} = \varepsilon_{\text{con}} )</td>
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### 4 CORRELATION WITH EXPERIMENTAL RESULTS

Three experimental studies have been included in this work for validation of the proposed iterative procedure. The first is one of the few extensive experimental studies on large scale FRP wrapped circular columns where different FRP configurations have been applied, for identical embedded steel reinforcement (Matthys et al. 2005). It includes 8 large-scale columns subjected to axial loading. The columns had a total length of 2 m, a longitudinal reinforcement ratio of 0.9% and 8 mm diameter stirrups spaced at 140 mm. All columns had circular cross section with 400 mm diameter. Different types of FRP reinforcement (CFRP, GFRP & HFRP) have been used to confine the columns. The comparison seems to be satisfactory (Figure 5, 6), although the solution has moderate success in resolving the problem of predicting the actual instance of jacket’s failure in terms of axial and circumferential ultimate strains. Some clarifications are in order for the last graph (Figure 7) which illustrates the model’s estimations of the circumferential strains in the FRP jacket owing to concrete dilation and to buckling of longitudinal bars at the ultimate axial strain reported in the tests for each specimen. These values are compared to the experimental rupture strain of the jacket (from strain gages) and to the deformation capacity of the flat tensile coupons which was reported, accordingly.

The second experimental study (Demers and Neale 1999) includes 16 reinforced concrete columns having a circular section 300 mm in diameter and 1200 mm high. These columns were confined by means of carbon-epoxy sheets and loaded concentrically in axial compression. The effects of various parameters on the structural behavior of the confined concrete columns are investigated. These parameters included the concrete strength, longitudinal steel reinforcement, steel stirrups, steel corrosion and concrete damage while the FRP configuration was kept constant. The comparison between model estimates and experimental results
depicted in Figure 8 and 9 also in this case could be characterized as satisfactory, moreover due to the fact that in this experimental study the lateral pressures from both confining materials (Steel and FRP) are provided based on circumferential strains obtained by strain gages applied on both FRP Jacket and Steel ties. (It should be underlined that the horizontal strain gages on the jacket were located midway between two successive stirrups). Among the 16 specimens in only one case (Specimen U40-4) the pressures coming from the ties were evidently higher than those of the FRP jacket and the model was able to detect that (Figure 10).

The third and the last experimental study (Gallardo-Zafra R. & K. Kawashima, (2009)) contains a series of cyclic loading tests that was conducted on six reinforced concrete column specimens 400 mm in diameter and 1.350 mm in effective height. The specimens were grouped into A and B series where each series consisted of three specimens each; one was as-built while the second and the third were wrapped laterally by CFRP with a single layer and with two layers, respectively. The tie reinforcement ratio was 0.256% (150 mm spacing) for the A-series and 0.128% (300 mm spacing) for the B-series. Figures 12, 13, 14 and 15 depict the comparison with the two groups of cyclic tests on bridge piers having different levels of confinement in terms of lateral steel reinforcement and FRP jacketing. The modeling of the bridge piers was the same as in the original paper and has been performed using the “MatLab Finite Elements for Design Evaluation and Analysis of Structures” (FEDEAS Lab) developed by Professor F.C. Filippou of the Department of Civil and Environmental Engineering of the University of California Berkeley. While in the original proposal the fiber section had to be divided in the concrete core and concrete cover and two different stress-strain relations were applied for the concrete core (confined by both FRP & Steel) and concrete cover (confined by only the FRP), in this work since the material response is already averaged based on the different responses of those two regions, the same stress strain law is applied for each fiber (FEM). This fact gives a clear advantage to the proposed model. In addition to the force-displacement response of the cantilever columns also the response in the level of the section is provided for each specimen in terms of material stress-strain hysteresis. It can be seen that the agreement is very close to the experimental one with some deviation concentrated on the parts of reloading after reversal of the imposed displacement. This difference in response in terms of modeling can be explained based on the way the cracks on the concrete surface are described in the level of the material model. Since the crack is described as two event phenomenon, which means open or closed (while in reality it is not the case – imperfect crack closure) the contribution of concrete while the longitudinal steel reinforcement is in compression and the crack is closing gives this deviation in the response. The comparison with the originally proposed model of this experimental study is impressively the same. However, the proposed model describes rationally the procedure of the passive confinement based on the calculation of the lateral concrete expansion in terms of the different levels of lateral pressures coming from the two different materials (Steel and FRP). Moreover, the active (constant lateral pressure) confinement model proposed by Kawashima et al. (1999) is based on regression analysis of the experimental results of cylindrical specimens under compression and it is specifically calibrated for Carbon Fiber Composite material (CFRP). Finally, it doesn’t consider the confinement effect of the longitudinal reinforcement and the effect of partial confinement due to the vertical arching action of the adjacent stirrups along the member but also cases of partial FRP wrapping of the column.

An important comment that should be made before concluding, is related to some studies (Gallardo-Zafra and Kawashima 2009, Khaloo et al. 2008) that have reported for FRP and Steel confined concrete in circular RC sections a different behavior (softening) in respect to
the already recognized bilinear one (Carey and Harries 2005). The authors attribute that to the small scale of the reinforced concrete specimens used while the most important explanation which could lead to those results is the influence of concrete strength. According to Mandal et. al. (2005) the FRP wraps provide a substantial increase in strength and ductility, for low-to-medium-strength concrete, which shows a bilinear stress strain response with strain hardening. For high-strength concrete, however, enhancement in strength is very limited, with hardly any improvement in ductility. The response in this case shows a steep post peak strain softening.

![Figure 5: Correlation with experimental results (Matthys et al. 2005).](image1)

![Figure 6: Correlation with experimental results (Matthys et al. 2005).](image2)
Experimental and Model's Predicted FRP Strains at $\varepsilon_{\text{cu.exp}}$

Matthys et al 2005

- Full / CFRP-5 Layers
- Full / GFRP-6 Layers
- Partial / GFRP-4 Layers
- Full / HFRP-4 Layers

Figure 7: Correlation with experimental results (Matthys et al. 2005).

Large FRP Wrapped Circular Columns
(Height = 1.2 m, Diameter = 0.3 m)

- U25-1 3 layers of CFRP
  - $d_0 = 11.3$
  - $d_i = 6.4$
  - $s = 300$

- U25-3 3 layers of CFRP
  - $d_0 = 19.5$
  - $d_i = 6.4$
  - $s = 150$

Figure 8: Correlation with experimental results (Demers and Neale 1999).

Large FRP Wrapped Circular Columns
(Height = 1.2 m, Diameter = 0.3 m)

- U40-1 3 layers of CFRP
  - $d_0 = 11.3$
  - $d_i = 6.4$
  - $s = 150$

- U40-4 3 layers of CFRP
  - $d_0 = 25.2$
  - $d_i = 11.3$
  - $s = 150$

Figure 9: Correlation with experimental results (Demers and Neale 1999).
Figure 10: Correlation with experimental results (Demers and Neale 1999).

Figure 11: Correlation with experimental results (Demers and Neale 1999).
Figure 12: Correlation with Experimental results A2 (Gallardo-Zafra R. & K. Kawashima, 2009).
Figure 13: Correlation with Experimental results A3 (Gallardo-Zafra R. & K. Kawashima, 2009).
Figure 14: Correlation with Experimental results B2 (Gallardo-Zafra R. & K. Kawashima, 2009).
Figure 15: Correlation with Experimental results B3 (Gallardo-Zafra R. & K. Kawashima, 2009).
5 CONCLUSIONS

- *fib*'s FRP-confined concrete model proposal has been enhanced to take into account the confining effect of the already existing steel reinforcement when retrofitting a reinforced concrete column with FRP jacketing. To this end, the transverse steel reinforcement has been considered not as imposing a constant value of confining pressure, rather, following the steel stress-strain law at each deformation step according with the BGL model (Braga et al. 2006), while also considering the confining contribution of longitudinal reinforcement.

- Similar to BGL model an important aspect of the model is that the cross-section tangential stresses (shear), which are generally neglected, have an essential role in ensuring plane strain conditions.

- In addition, compatibility in the lateral direction, inwards for confining pressures and outwards for lateral strains, between the two confining materials (FRP and Steel) has been established. Through this approach the difference in the lateral behavior of the concrete cover (confined with the jacket’s pressure) and the concrete core (confined by both the steel’s and FRP’s pressure) has been considered.

- This allows the application of the model also in cases of reinforcement repair and FRP retrofit where two different concrete strengths should be considered, one for the new layer of concrete applied externally and the other for the old concrete in the concrete core which may also be cracked due to former seismic loading.

- Moreover, in case of bridge column modeling with a fiber nonlinear beam-column element, apart from the immediate incorporation of shear deformations in the section level (in contrast to the standard fiber beam-column formulation), the averaged response of the two different regions -concrete core and concrete cover- gives an evolutionary advantage in terms of modeling since it allows the assignment of a unique stress-strain law for all the fibers/layers of the circular section (FEM).

- Finally, a two-condition failure criterion has been incorporated regarding dilation of concrete and buckling of longitudinal bars as independent events. Correlation with experimental results seems to be satisfactory, although the model has moderate success in predicting the actual instance of rupture of the FRP jacket.

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REFERENCES


**NOTATION**

\( \varepsilon_r \) radial strain

\( \varepsilon_c \) circumferential strain

\( \varepsilon_{con} \) concrete’s axial strain

\( \varepsilon_{co} \) concrete’s axial strain at unconfined concrete’s strength

\( \varepsilon_{cc} \) concrete’s axial strain at confined concrete’s strength

\( \varepsilon_{c,core} \) circumferential strain of the core

\( \varepsilon_{c,buck} \) circumferential strain due to buckling of longitudinal bars

\( \varepsilon_{r,core} \) radial strain of the core

\( \varepsilon_{r,cover} \) radial strain of the cover

\( R_{core} \) radius of the concrete core

\( c \) concrete cover

\( f_{co} \) concrete strength

\( f_{cc} \) confined concrete strength

\( \nu_o \) initial Poisson’s coefficient for concrete

\( \rho_{sh} \) steel hoop’s volumetric ratio
\( \rho_j \)  
FRP jacket’s volumetric ratio

\( f_{l,\text{core}} \)  
lateral confining pressure of the concrete core

\( f_{l,\text{cover}} \)  
lateral confining pressure of the concrete cover

\( f_{l,\text{steel}} \)  
lateral confining pressure of the steel reinforcement

\( f_{c,\text{av}} \)  
average axial concrete stress

\( f_{c,\text{core}} \)  
axial concrete core stress

\( f_{c,\text{cover}} \)  
axial concrete cover stress

\( L_{\text{buck}} \)  
buckling length

\( P_{cr} \)  
critical load

\( A_{\text{tot}} \)  
total area of the circular section

\( A_{\text{core}} \)  
total area of the concrete core of the circular section

\( A_{\text{cover}} \)  
total area of the concrete cover of the circular section

\( I_b \)  
longitudinal bar’s moment of inertia

\( f_{\text{yl}} \)  
yielding strength of longitudinal bar

\( w \)  
transversal displacement of the bar

\( \varepsilon_d \)  
axial strain in the bar

\( \varepsilon_{shu} \)  
ultimate steel hoop’s strain

\( D_h \)  
hoop’s diameter

\( D_{\text{core}} \)  
diameter of concrete core

\( D \)  
section’s diameter

\( S_j \)  
jacket’s clear spacing