COMPDYN 2011
ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis , V. Plevris (eds.)
Corfu, Greece, 25-28 May 2011

NONLINEAR SEISMIC ANALYSIS AND FATIGUE-ACCUMULATED DAMAGE OF STEEL FRAMES WITH END-PLATE BOLTED CONNECTIONS

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Keywords: End-Plate connections, seismic analysis, fatigue, damage seismic, nonlinear modes, hysteretic model.

Abstract. *Analyzing damage seismic in large and complex structures is one of the most challenging problems in steel moment-resisting frame with end-plate bolted connections. The existence of structural damage in an engineering structure leads to the modification of vibration modes and global eigenvalues are usually sensitive to the degree of local damage seismic in bolted connections.*

In this analytical study, a nonlinear time history analysis which takes into account nonlinear modes and frequencies was adopted. According to this approach, the nonlinear modes and frequencies can be determined by an iterative procedure which based on the method of equivalent linearization.

However, the damage analysis of seismic response requires the introduction of efficient structural models capable of describing actual behaviour and the application of an adequate algorithm. The paper presents a Fatigue Damage-Based Hysteretic FDBH model to evaluate the seismic performance of steel moment-resisting frames. The FDBH model is an evolutionary-degrading hysteretic model based on the low-cycle fatigue LCF damage index. Hence an algorithm was developed to characterize nonlinear behaviour in structures for the purpose of either damage identification of these structures.

1 INTRODUCTION

Steel moment-resisting frame structures often are used as part of the seismic force-resisting systems in buildings designed to resist earthquakes with substantial inelastic energy dissipation. End-plate bolted connections are one of the safest and most economical options for structural steel erection. It is therefore desirable to maintain the use and economy of bolted connections as an option for seismically loaded structures [3].

Beam-column connections in steel moment frames are proportioned and detailed to resist flexural, axial, and shearing actions that result as a building sways through multiple inelastic displacement cycles during strong earthquake ground shaking. However, such inelastic action causes fatigue damage and if long enough the cumulative effect may exhaust the ductile capacity leading to fracture [4].

However, analyzing damage seismic in large and complex structures is one of the most challenging problems in steel frames with end-plate connections. Many experimental approaches confirm the presence of changes in modal parameters such as natural frequencies, mode shapes because of the damaged elements of the structure [25]. The concept of nonlinear modes was introduced for the first time by Rosenberg in 1960 [24] and has a particular interest in the study of forced responses because there are close links between the properties of the modes (shape, number) and topology forced response curves of dynamical system. In general, to access the exact solutions of nonlinear system, complex nonlinear differential equations must be solved. The nonlinear systems do not generally have access to their exact solutions in the form. Then approached methods can be used such as Ritz-Galerkin method in which the nonlinear response of the system in the physical basis is approximated by a linear combination of the normal modes and the response of the system in modal coordinates [6].

Stupnicka [26] was assumed that the nonlinear normal coordinates are associated with nonlinear normal modes. Following, the principle of nonlinear modes is used by many researchers to analyze nonlinear systems with multiple degrees of freedom [25, 21]. Dynamic analysis by finite element FE is one means of estimating the changes in the modal parameters such as natural frequencies, mode shapes and modal damping. Global eigenvalues and nonlinear modes can be determined by an iterative procedure [25].

In this paper, a nonlinear time history analysis which considers the nonlinear modes and frequencies was adopted. According to this approach, the nonlinear modes and frequencies can be determined by an iterative procedure which based on the method of equivalent linearization [25].

Moreover, the damage analysis of seismic response requires the introduction of efficient structural models capable of describing actual behavior. The paper presents a Fatigue Damage-Based Hysteretic FDBH model to evaluate the seismic performance of steel moment-resisting frames. The FDBH model is a degrading model based on two damage indices. The plastic damage index (the first index) in this model is combined with a low-cycle fatigue LCF damage index (the second index). The model can be able to calculate the fatigue-accumulated damages of structures. Hence, an algorithm was developed to evaluate the nonlinear seismic behavior of steel moment frames. Finally, a two-story steel frame will be used as an example to illustrate the proposed technique.

2 STRUCTURAL DAMAGE OF STEEL MOMENT FRAMES

The problem of damage in steel moment frames, particularly due to seismic excitation is still a challenge for engineer and researchers to investigate [16]. To understand the damage in the bolted connection, the researchers distinguish between two phenomena of damage.

The first one is the damage due to the phenomena of elasto-plasticity behaviour of the bolted connection under repeated seismic efforts. The earthquake forces cause inelastic deformations in the components of bolted connection [22]. The accumulation of plastic strains is one phenomenon of cyclic plasticity. Inelastic deformation in the end plate, in the bolts and in flanges of beams and columns can change the mechanical properties of bolted connection like the strength and the stiffness [27].

The nonlinear behaviour is mainly resulted from gradual yielding of connection and can lead to hysteretic loops at a connection under repeated loading [17]. This important feature of nonlinear connections in ductile steel frames has the capability of dissipating excitation energy by an amount equal to the enclosed loop area to stabilize the response under cyclic loading.

To describe the nonlinear behaviour of semi-rigid connection like the end-plate connection, the relationship between the moment and the relative rotation angle is used. Richard and Abbott [14] propose to represent the moment-rotation relationship by four parameters. The Richard-Abbott model represents only the monotonically increasing loading portion of the $M-\theta$ curves. However, the unloading and reloading behaviour of the $M-\theta$ curves is also essential for nonlinear seismic analysis. The subject was extensively addressed in the literature ([9]) where the unloading and reloading parts of the $M-\theta$ curves are theoretically developed using the Masing rule (see Fig.1). The unloading and reloading behaviour of an end-plate connection based on the Richard-Abbott model is described as:

$$M = M_a - \frac{(R_0 - R_p).(\theta_a - \theta)}{\left(1 + \left|\frac{(R_0-R_p).(\theta_a-\theta)}{2M_0}\right|^{\gamma}\right)^{\frac{1}{\gamma}}} - R_p.(\theta_a - \theta) \tag{1}$$

$$R = \frac{(R_0 - R_p)}{\left(1 + \left|\frac{(R_0-R_p).(\theta_a-\theta)}{2M_0}\right|^{\gamma}\right)^{\frac{\gamma+1}{\gamma}}} + R_p \tag{2}$$

where M is the connection moment, θ is the relative rotation between the connecting elements, R_0 is the initial stiffness, R_p is the plastic stiffness, M_0 is the reference moment, and γ is the curve shape parameter. (M_a, θ_a) is the load reversal point as shown in Fig.1.

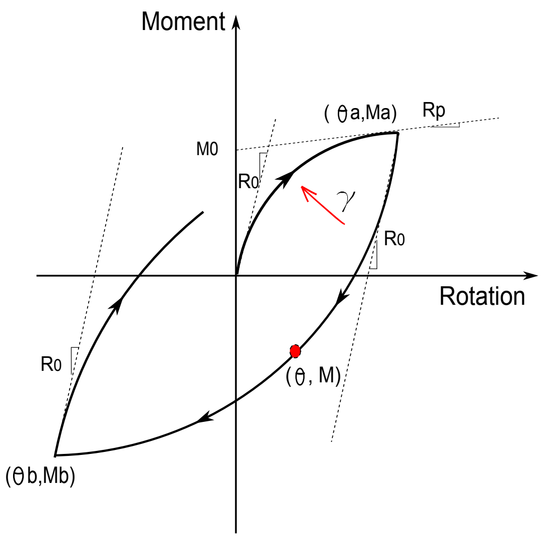

Figure 1: Nonlinear hysteretic model of end-plate connection.

The second phenomena of damage is the damage caused by low cycle fatigue. This type of fatigue causes progressive and cumulative damage in the stiffness of the bolted connection [5, 15]. Because of the stress concentration, cracks may appear in the welds which is characterised by the different stages of micro and macro crack propagation and final fracture [23]. Moreover, Bolts can loosen over time because micro-macro slip in the bolt-nut and the assembled plates [18]

Many mathematical relations have been proposed, particularly Manson-Coffin relation which describes linearly this function between the applied rotation and the number of cycles to rupture on a double-logarithmic scale [10, 11]. The information provided by the Rotation-Number of cycles curve is mainly used by engineers for the prediction of the lifetime and resistance of structures under repeated loading.

To evaluate damage more precisely, Miner's rule [11] is commonly used. According to this rule, applying a cycle n_x times with a stress amplitude which corresponds to a lifetime of N_x cycles is equivalent to consuming a portion n_x/Nf_x of the whole lifetime. This rule implies that rupture occurs when 100% of the lifetime is consumed. It describes also the phenomena of the cumulative linearity of fatigue damage if another application stress is employed. Though many models of damage have been proposed , Miner's rule is still widely used in the engineering field [15].

The fatigue problem occurs in the weaker parts of a structure and in the case of steel frames, as used in civil engineering, fatigue damage generally occurs at its connections. This indicates the importance of studying the resistance and fatigue damage of the beam-column connection in portal frame structures. Experimental tests for predicting the resistance of the beam-column connection have been published [19, 27] with the aim of identifying the key parameters of resistance of the beam-column connection. Some numerical studies have also been carried out using the finite element method to observe the behaviour of the fracture mechanism [20, 22].

The damage phenomena of elasto-plasticity behaviour and LCF due to cyclic loading may occur simultaneously. There is a strong interaction and a separation of the damage processes is impossible. Furthermore, each phenomenon alone is characterised by different aspects.

3 FBDH MODEL OF END-PLATE CONNECTION

In this study, The Richard-Abbott model has been modified to include degrading in the stiffness of the connection produced by the cumulative phenomenon of LCF. An evolutionary-degrading hysteretic model can be developed based on the LCF damage index D_n. In Richard-Abbott model, the cycle in $M-\theta$ curve begins with a secant stiffness R_0 and it changes in nonlinear way to R_p. But the connection loses part of its life in each cycle of excitation applied because of the cumulative phenomenon of LCF. For this reason the secant stiffness of the connection must be changed at the end of each cycle using an index of fatigue damage D_n to take into consideration the past lost life of the connection. The initial stiffness can be modified with the factor $(1-D_n)$ that takes into account the effect of cumulative fatigue (see Fig.2). The following equations present the developed model for the moment-rotation relationship and the secant stiffness:

$$M^* = M_a - \frac{(R_0.(1-D_n)-R_p).(\theta_a-\theta)}{\left(1+\left|\frac{(R_0.(1-D_n)-R_p).(\theta_a-\theta)}{2M_0}\right|^{\gamma}\right)^{\frac{1}{\gamma}}} - R_p.(\theta_a-\theta) \tag{3}$$

$$R^* = \frac{(R_0.(1-D_n)-R_p).(\theta_a-\theta)}{\left(1+\left|\frac{(R_0.(1-D_n)-R_p).(\theta_a-\theta)}{2M_0}\right|^{\gamma}\right)^{\frac{\gamma+1}{\gamma}}} + R_p \tag{4}$$

where M^* is the degraded connection moment and R^* is the degraded secant stiffness.

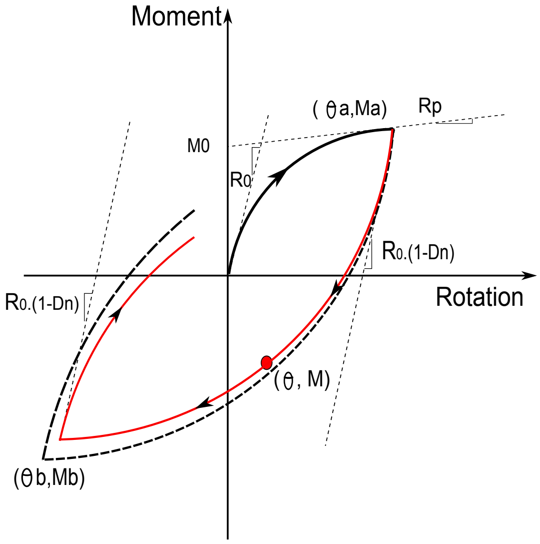

Figure 2: Fatigue Damage-Based Hysteretic model of end-plate connection.

It should be noted that the secant stiffness in Eq.(4) is modified and also with this idea, it is possible to combine the two indices by the equation:

$$D_p = 1 - \frac{R^*}{R_0} \tag{5}$$

where R_0 is the initial stiffness of the connection of connection and R^* is the tangent stiffness of the connection which depends on the index D_n.

To calculate the LCF damage index D_n, an analogous models based on the plastic connection rotation are used therein. A useful means of describing the LCF is expressed by Mander et al. [10] for a bolted connection. Thus using the well known Manson-Coffin relationship [7], the plastic rotation may be related to the number of cycles Nf by the following equation:

$$Nf = c.(\Delta\theta)^{-b} \tag{6}$$

where $\Delta\theta$ is the range of variation of rotation and c, b are parametres which depend on both the typology and the mechanical properties of the considered steel element. The parametres of Manson-Coffin relationship adopted in this study are ($c = 2.10^{-4}$) and ($b = 3$) [10].

Moreover, cumulative damage models should be used to assess deterioration and failure in structural components under arbitrary loading histories. Miner's rule is still the most used cumulative damage rule for its simplicity and efficiency in LCF region [11]. In the case of connection subjected to many cycles of rotation, Miner's rule is expressed by the following equation:

$$D_n = \sum_{x=1}^{X} \frac{n_x}{Nf_x} \tag{7}$$

where D_n is the LCF damage index, n_x is the numbre of applied cycles for a given rotation level x and Nf_x (see Eq.6) is the number of cycles to failure for roation level x according to the rotation-cycle curve given by Mander et al [10]. The rule is usually employed with the rainflow algorithm since it seems the best counting method [13].

The purpose of such model for the end-plate connection is to simulate inelastic response of structure under dynamic loading by combined damage indices.

4 NONLINEAR MODES AND FINITE ELEMENT

If a structure is properly modeled using finite element, structural damage manifests itself mathematically in the stiffness and mass matrices, and physically in its dynamic properties such as natural frequencies and mode shapes. Finite element methods (FEM) have become a standard technique for structural analyses for more than three decades. In this study, the damage in the structure is identified as a change in stiffness of the finite elements in the different beams of the FE model. The FE model of the structure is a two-dimensional model (see Fig.3) developed using the FE structural analysis program Structural Dynamics Toolbox (SDT) [28].

Consider a structure which is composed of a foundation r and a subset l of free-to-vibrate degree-of-freedom, shown in Fig.3. Ω is the interface between r and l. The equation of motion for a damped structure with N degrees-of-freedom (dof) is given as follows:

$$M\ddot{q} + \bar{C}\dot{q} + \bar{K}q = F(t) \tag{8}$$

where M, $\bar{C}$, $\bar{K}$ are respectively the mass, nonlinear damping and nonlinear stiffness matrix of the structure and $F(t)$ is the applied forces.

In this study, we adopt the hypothesis that the stiffness matrix of system is nonlinear and depends on the response of system and the nonlinear normal modes. In the case of seismic excitation applied to the foundation r, the only dynamic force of structure is the inertia force produced by basic movement. Consider the foundation of structure as immensely rigid. Therefore, the movement of the foundation r is defined as the rigid body displacements of structure.

The response of system in normal coordinate is defined as:

$$q(t) = \begin{Bmatrix} q_l \\ q_r \end{Bmatrix} \tag{9}$$

where q_l is the response of subset l and q_r is the response of foundation r.

The corresponding partitions of the mass, stifness matrix and the vector of external force are:

$$M = \begin{bmatrix} M_{ll} & M_{lr} \\ M_{rl} & M_{rr} \end{bmatrix}, \quad \bar{K}(q) = \begin{bmatrix} \bar{K}_{ll} & \bar{K}_{lr} \\ \bar{K}_{rl} & \bar{K}_{rr} \end{bmatrix}, \quad F(t) = \begin{Bmatrix} F_l = 0 \\ F_r \end{Bmatrix} \tag{10}$$

where F_r is the vector of reaction forces at the foundation r.

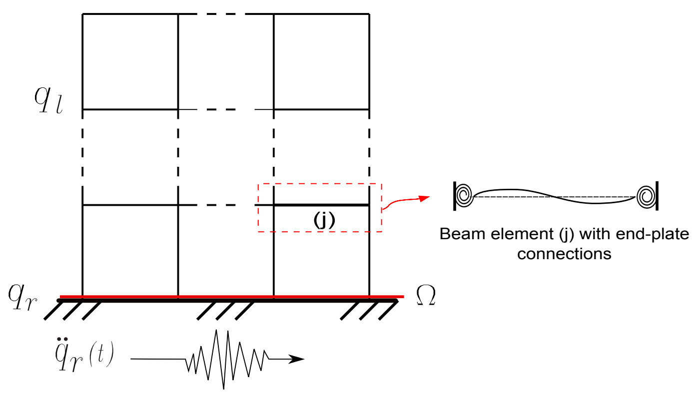

Figure 3: MDOF system

Rayleigh damping is used to represent damping in the structure. It can be written as:

$$\bar{C} = \alpha.M + \beta.\bar{K} \tag{11}$$

where α and β are the proportional constants.

According S. Setio [25], eigenvalues and eigenvectors for a nonlinear system cannot be obtained by solving the standard eigenvalue problem. As the solution of a nonlinear system depends heavily on the amplitude of motion, the frequencies and normal modes depend on the nonlinear modal amplitude. The introduction of the notion of nonlinear modes permits an extension of the method of linear modal synthesis to nonlinear cases in order to obtain the dynamical response of nonlinear multi-degree-of freedom systems.

The nonlinear normal modes and nonlinear frequencies must be calculated by an iterative procedure. In this paper, we adopted a procedure which is based on the method of equivalent linearization. For each time step of calculation, the stiffness and damping matrices of the system will be obtained which transform the nonlinear system into an equivalent linear system. The frequencies and nonlinear normal modes can then be calculated using a standard solution to the eigenvalues. A set of N nonlinear modes and frequencies are obtained according to their modal amplitudes.

Considering the FEM characteristic equation for the structure in Fig.3, the nonlinear modal problem can be written as following:

$$[\bar{K}(\eta_p, \bar{\varphi}_p(\eta_p)) - \bar{\omega}_p^2(\eta_p)M]\bar{\varphi}_p(\eta_p) = 0 \tag{12}$$

Where η_p, $\bar{\omega}_p(\eta_p)$, $\bar{\varphi}_p(\eta_p)$ are respectively the fixed structure response in modal coordinate, the nonlinear frequencies and the nonlinear normal modes.

S. Setio [25] assumes that the normal coordinates nonlinear are associated with nonlinear normal modes. In this case, the orthogonality property of eigenvectors used to treat the linear problem can be extended to nonlinear problems. This option allows to transform a system of N coupled equations of a system with N degrees of freedom in the physical basis of a set of N decoupled equations in the modal basis.

In the case of excitation at the base, the movement of the structure is the superposition of a movement training of rigid body and a movement relative to the base that can be expressed by the modes of structure fixed at the foundation r. Finally, the response of nonlinear system in normal coordinate can be obtained efficiently by superposition of modal response as follows:

$$q(t) = \Phi_r q_r + \bar{\varphi}_p(\eta_p)\eta_p \tag{13}$$

where Φ_r is rigid body modes matrix defined by the $r\ dof$ of the foundation.

The mode shapes matrix of a nonlinear structure fixed at the foundation is expressed as following:

$$\bar{\varphi}_p = \begin{bmatrix} \bar{\varphi}_{lp} \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{x}_l^{(1)} & \bar{x}_l^{(2)} & \dots & \bar{x}_l^{(p)} \\ 0 & 0 & \dots & 0 \end{bmatrix} \tag{14}$$

After substituting the transformation of Eq.(13) in the equation of motion (see Eq.(8)), we obtain the following system (given the properties of rigid modes $\bar{K}\Phi_r = 0$, $\bar{C}\Phi_r = 0$):

$$\begin{bmatrix} \Phi_r^t M \Phi_r & \Phi_r^t M \varphi_p \\ \bar{\varphi}_p^t M \Phi_r & \bar{\varphi}_p^t M \bar{\varphi}_p \end{bmatrix} \begin{bmatrix} \ddot{q}_r \\ \ddot{\eta}_p \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \bar{\varphi}_p^t \bar{C} \bar{\varphi}_p \end{bmatrix} \begin{bmatrix} \dot{q}_r \\ \dot{\eta}_p \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \bar{\varphi}_p^t \bar{K} \bar{\varphi}_p \end{bmatrix} \begin{bmatrix} q_r \\ \eta_p \end{bmatrix} = \begin{Bmatrix} F_r \\ 0 \end{Bmatrix} \tag{15}$$

And also we can write:

$$\begin{bmatrix} m_{rr} & L_{pr}^t \\ L_{pr} & [m_p] \end{bmatrix} \begin{bmatrix} \ddot{q}_r \\ \ddot{\eta}_p \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & [\bar{c}_p] \end{bmatrix} \begin{bmatrix} \dot{q}_r \\ \dot{\eta}_p \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & [\bar{k}_p] \end{bmatrix} \begin{bmatrix} q_r \\ \eta_p \end{bmatrix} = \begin{Bmatrix} F_r \\ 0 \end{Bmatrix} \tag{16}$$

where:
$m_{rr} = \Phi_r^t M \Phi_r$ is the rigid body mass matix.
$\bar{L}_{pr} = \bar{\varphi}_p^t M \Phi_r$ is modal participation factors matrix.
$[\bar{m}_p] = \bar{\varphi}_p^t M \bar{\varphi}_p$ is the generalized mass matrix.
$[\bar{c}_p] = \bar{\varphi}_p^t \bar{C} \bar{\varphi}_p$ is the generalized damping matrix.
$[\bar{k}_p] = \bar{\varphi}_p^t \bar{K}(\eta_p, \bar{\varphi}_p(\eta_p)) \bar{\varphi}_p$ is the generalized stiffness matrix.

Eq.(16) can be expressed as the following developed form:

$$[\bar{m}_p]\ddot{\eta}_p + [\bar{c}_p]\dot{\eta}_p + [\bar{k}_p]\eta_p = -\bar{L}_{pr}\ddot{q}_r \tag{17}$$

$$F_r = m_{rr}\ddot{q}_r + \bar{L}_{pr}^t \ddot{\eta}_p \tag{18}$$

Modals movement equations of the excited structure are given by Eq.(17) where η_p is the fixed structure response in modal coordinates. The system is subjected to modal inertia forces $\bar{f}_p = -\bar{L}_{pr}.\ddot{q}_r$. The equation of movement for mode i has the following form:

$$\bar{m}_i\ddot{\eta}_i + \bar{c}_i\dot{\eta}_i + \bar{k}_i\eta_i = -\bar{L}_{ir}\ddot{q}_r \tag{19}$$

where $\bar{m}_i$, $\bar{c}_i$, $\bar{k}_i$, are respectively the modal mass, nonlinear modal damping and nonlinear modal stiffness and they can be calculated by the following equations:

$$\begin{aligned} \bar{m}_i &= \bar{x}^{(i)t} M \bar{x}^{(i)} \\ \bar{c}_i &= \bar{x}^{(i)t} \bar{C} \bar{x}^{(i)} \\ \bar{k}_i &= \bar{x}^{(i)t} \bar{K}(\eta_p, \bar{\varphi}_p(\eta_p)) \bar{x}^{(i)} \end{aligned} \tag{20}$$

In Eq.(19), $\bar{L}_{ir} = \bar{x}^{(i)t} M \Phi_r$ is the i^{th} row of the matrix $\bar{L}_{pr}$ and $-\bar{L}_{ir}.\ddot{q}_r = \bar{f}_i(t)$ is generalized force of the i^{th} mode. After solving the modal equations of motion (see Eq.(17)), the reactions at the base and the displacement of the system can be calculated.

To calculate the response of dynamical system, such as the system given by Eq.(19), which subjected to arbitrary seismic loads, temporal integration methods are considered the only methods applicable to nonlinear systems with many degrees of freedom. In this study, the Runge-Kutta method is adopted for the numerical solution of differential equations.

The incremental equilibrium equation of the dynamic system during the time Δt_n can be writhed as following:

$$\bar{m}_i.\Delta\ddot{\eta}_i + \bar{c}_{n,i}.\Delta\dot{\eta}_i + \bar{k}_{n,i}.\Delta\eta_i = \Delta\bar{f}_i(t) \tag{21}$$

where $\bar{c}_{(n,i)}$, $\bar{k}_{(n,i)}$ are the tangent properties defined at the time t_n.

In general, $\eta_{(n,i)}$, $\dot{\eta}_{(n,i)}$, $\ddot{\eta}_{(n,i)}$ at time t_n is known and Range-Kutta algorithm can be used to calculate $\eta_{(n+1,i)}$, $\dot{\eta}_{(n+1,i)}$, $\ddot{\eta}_{(n+1,i)}$ at time t_{n+1}. Once the modal displacements are obtained, the relative response of system in normal coordinates at the time t_{n+1} can be calculated by Eq.(13) as follows:

$$q_{n+1} = \sum_i^N \bar{x}^{(i)}.\eta_{n+1,i} \tag{22}$$

The stiffness matrix of system should be recalculated taking consideration nonlinear behaviour of connections. For this reason, the calculated displacements are used to calculate the internal forces of elements. From Eq.(22), the rotations of bolted connections are known and using the hysteretic model represented by Eqs.(1), (2) and (5), the connection moment $M_{j_{(n+1)}}$, the tangent stiffness $R_{j\,(n+1)}$, the LCF damage index $D_{n j\,(n+1)}$ and the plastic damage index $D_{p j\,(n+1)}$ of each bolted connection of the beam element (j) will be calculated at the time t_{n+1}.

A correction matrix Cr proposed by Monfortoon and Wu ([12, 8]) is adopted and it is used to modify each matrix beam of the system at the time t_{n+1}. It can take account of the flexibility of semi-rigid connections. The correction matrix Cr for the element (j) at the time t_{n+1} is presented as follows:

$$Cr_{j\,(n+1)} = \sum_q \sum_s c_{qs\,j\,(n+1)} \text{ with } c_{qs\,j\,(n+1)} = f(D_{p j,l\,(n+1)}, D_{p j,r\,(n+1)}, L_b) \tag{23}$$

where $D_{p j,l\,(n+1)}$, $D_{p j,r\,(n+1)}$ are the plastic damage indices of the connections (left l and right r, respectively) for the element (j) at the time t_{n+1}. L_b is the length of the beam element (j). The values of c_{qs} are represented by R. Hasan et al. [8] for a beam element with bolted connection at its ends.

The nonlinear matrix for the beam element (j) at the time t_{n+1} can be calculated using the correction matrix with the following equation:

$$\bar{k}_{j(n+1)} = k_j.Cr_{j(n+1)} \tag{24}$$

where k_j the standard elastic stiffness matrix for the beam element (j) [8].

The matrix $\bar{K}$ can be assembled by the stiffness matrices of each beam and column element in the system with the following equation:

$$\bar{K}_{(n+1)} = \sum_{e=1}^{n_e} \bar{k}_{e(n+1)} \tag{25}$$

where n_e is the total number of elements in the MDOF system and members are generically identified by index e.

Because we do not have nonlinear behavior in the columns, the stiffness matrix of each column element $\bar{k}_e$ is equal to the standard elastic stiffness matrix for the column element k_e.

5 ALGORITHM OF ANALYSIS

An algorithm was developed to evaluate the seismic performance of steel moment resisting frames with end-plate connections (see Fig.4). Using this algorithm, the influence of the LCF damage on the behaviour of end-plate connection is studied. To evaluate the accumulation of LCF damage, two phases of analysis have been conducted in this algorithm.

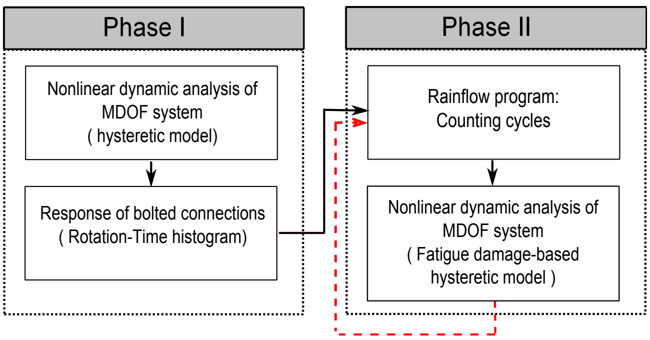

Figure 4: Flowcharte of the proposed algorithm

The objective of the first one is to prepare a nonlinear modal analysis of the structure using the hysteretic model. This phase is important to prepare the necessary data for the second phase. In this phase, the rotation-time histograms will be obtained. These histograms are necessary to prepare an analysis of LCF damage.

In the second phase, a cycle counting program (rainflow program) will be used through the Matlab toolbox offered by A. Nieslony [13]. This program analyzes the histograms to find the number of cycles counted, the corresponding rotational levels and the corresponding times. A FDBH model will be used, following in the second phase, to prepare a nonlinear modal analysis of the structure and to combine the damage caused by elasto-plastic behaviour and LCF.

In both phases, a modal analysis based on finite element code of SDT Toolbox is performed to find the dynamic response of the steel frame subjected to seismic excitation.

6 NUMERICAL EXAMPLE

In order to illustrate the application of the proposed method, a two-story steel frame is presented. The frame has end-plate bolted connections and fixed supports. The geometric, sectional properties and other pertinent information of the frames are given in Fig.5.

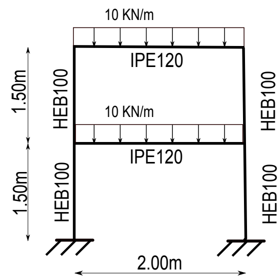

Figure 5: Two-story steel frame structure.

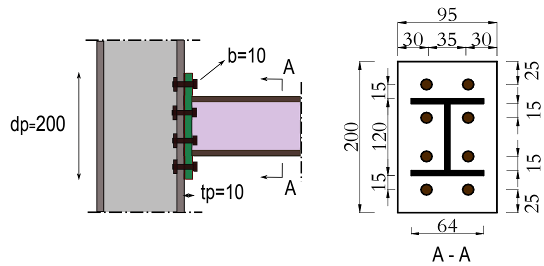

Figure 6: Parameters of the end-plate connection (in mm).

The cross sections are selected from the European section profiles. The important properties of steel used in this study are summarized in Table 1. The end-plate connection was designed for the required strength using yield-line theory.

Modulus of elasticity E (GPa)	Yield stress f_y (MPa)	Material density ρ (kg/m^3)
210	280	7850

Table 1: Properties of steel materials.

The connection parameters are represented in Fig.6 and they are set by the code EC3 to ensure a good ductile behaviour of the connection. The structure is subjected to gravity loads of the dead load plus a 10 (KN/m) live load, according to Fig.5, followed by an earthquake with a peak ground acceleration of $\ddot{q}$=0.1 g, 0.2 g and 0.36 g. An earthquake record (normalized El Centro) is used as ground motion input. A damping ratio of ξ= 0.05 is considered throughout.

The nonlinear behaviour and design limit states of the elements were modelled in accordance with the models presented above and the calculations were carried out by the code SDT Toolbox on Matlab 7.6 (R2008a).

The moment capacity of a connection depends on the strength of the individual connection elements. Various investigations have shown that the connection will begin to lose its ability to

Initial stiffness R_0 $(KN.m/rad)$	Maximum moment M_u (MPa)	Yielding moment M_y (MPa)
1.528×10^4	23.86	15.91

Table 2: Mechanical properties of the end-plate connection.

sustain further loading when one or more of the following failure modes occur: bolt failure (in tension), end plat plastic mechanism, beam and column flanges buckling. The lowest values of the moment corresponding to these failure modes will present the ultimate connection moment M_u. The equations used to evaluate M_u were adopted from EC3. The yielding moment of the connection can be evaluated as $M_y = 2/3M_u$.

Initial rotational stiffness of a connection is important and essential for the analysis semi-rigid frames. The rotational stiffness is directly related to the stiffness of each element in the connection. To evaluate the initial stiffness, the equations of EC3 are used. Based on the parameters of the connection (see Fig.6), the mechanical properties of the connection can be calculated (see Table 2). The plastic stiffness R_p is considered to be 0.

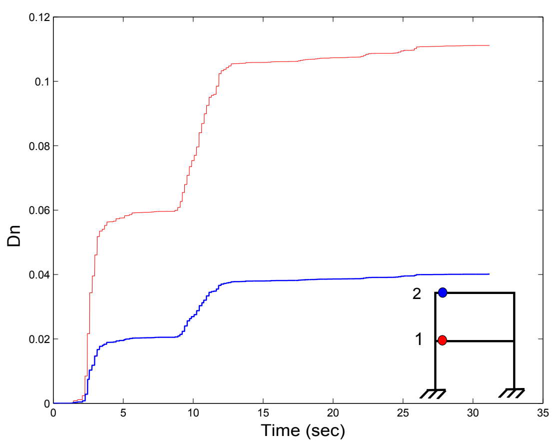

Figure 7: Low-cycle fatigue damage index of the connections (1), (2) for $\ddot{q}$=0.2 g considering the linear modes.

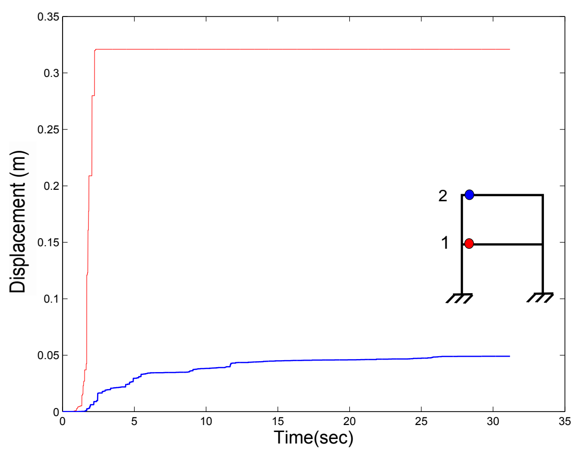

Figure 8: Low-cycle fatigue damage index of the connections (1), (2) for $\ddot{q}$=0.2 g considering the non-linear modes.

Two types of failure criteria are considered. The first one is the rotation of the connection and it must not exceed the maximum value given by the EC3 code. The maximum rotation of the connection in this study is $\theta_{max} = 0.025d_c/d_b$, according to the EC3. The second is the index D_n and it must not exceed the value of (1). If both criteria are not met, the connection will consider as a hinge. The index D_p can take value (1) but must not exceed it. In the other hand, the rigidity R_0 will be 0.

The LCF damage index of the connections (1), (2) for $\ddot{q}$=0.2 g is traced with considering and non-considering the nonlinear modes (see Figs.7 and 8). The cumulative index D_n reaches a value of 11% in the case of linear modes and it increases to a value of 32% in the case of nonlinear modes because of the modes and frequencies changes (see Figs.9).

To show the influence of the nonlinear mode in the behavior of the frame, the natural frequency of the structure as a function of time for the mode $i = 1$ is traced in Figs.9. The results are plotted for three levels of seismic excitation ($\ddot{q} = 0.1$ g, 0.2 g and 0.36 g). In the case $\ddot{q} = 0.1$ g, we note that the frequency remains constant over time (see Fig.9-a). This is because

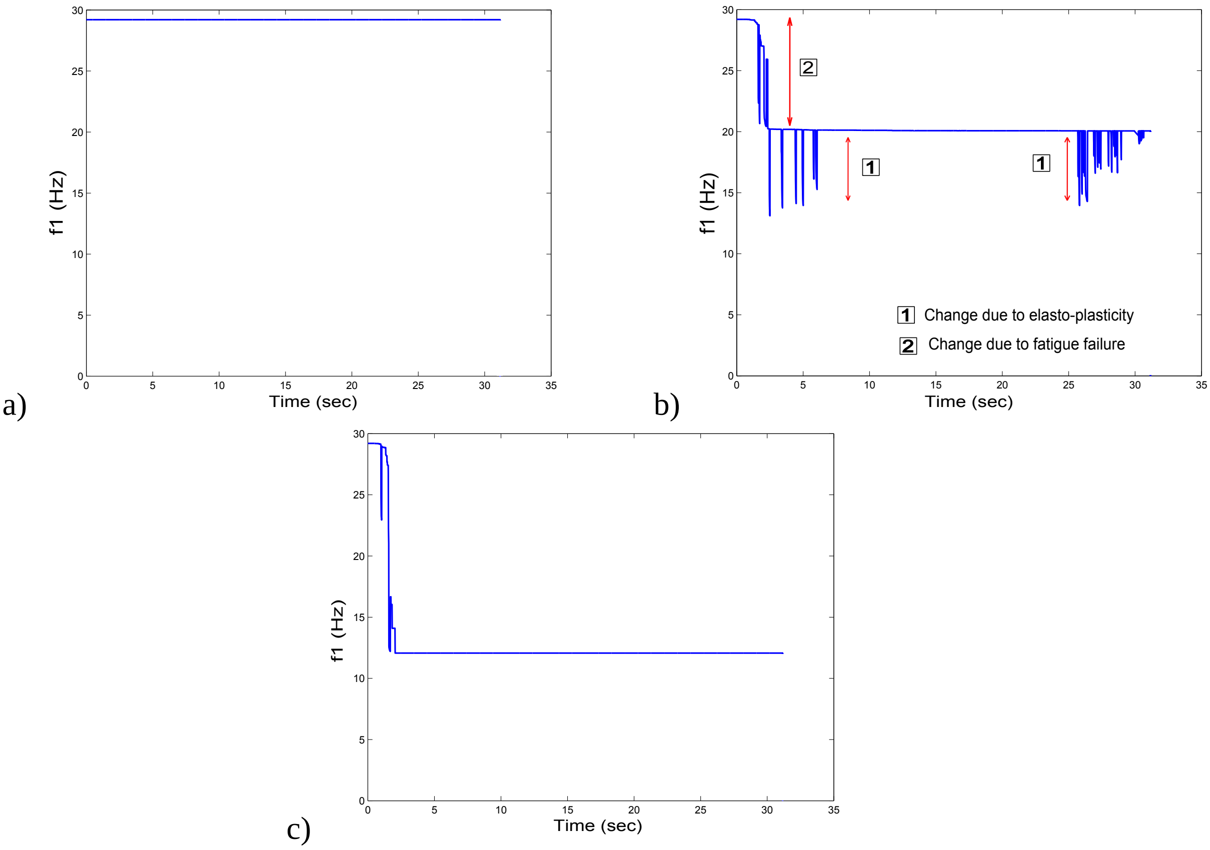

Figure 9: Natural frequency of the structure as a function of time for the mode $i = 1$ and for three cases: (a) $\ddot{q} = 0.1$ g, (b) $\ddot{q} = 0.2$ g and (c) $\ddot{q} = 0.36$ g.

the applied excitation is not very strong and it does not generate damage in modes. For the case $\ddot{q} = 0.2$ g, we can notice that two kinds of changes in the frequency of the frame (see Fig.9-b). The first is a drop in the value of frequency because the bolted connections in the frame are subjected to significant damage due to fatigue damage. The second is a cyclical change due to cyclic elasto-plastic behaviour of bolted connection into the frame.

The cyclical changes are less visible in the case of seismic excitation $\ddot{q} = 0.36$ g (see Fig.9-c). Because bolted connections (1), (2) are completely damaged and rotations reach the limit of ruin. It can also be seen that a greater drop in frequency is happened in this case.

It can be seen from Figs.9 that the analysis with nonlinear mode causes more damages in the system responses that were not taken into consideration before with the linear mode (see Figs.11 and 10).

7 CONCLUSIONS

This paper uses recent concepts in the structural damage evaluation to analyze structures under earthquake. An algorithm to study the influence of the LCF damage on the behavior of end-plate connection is used in this article and a FDBH model is adopted.

The results of this study confirm the presence of changes in modal parameters such as natural frequencies, mode shapes because of the damaged elements of the structure. Further, a drop in frequency due to the development of plastic hinges during the seismic excitation is observed in the proposed steel frame example.

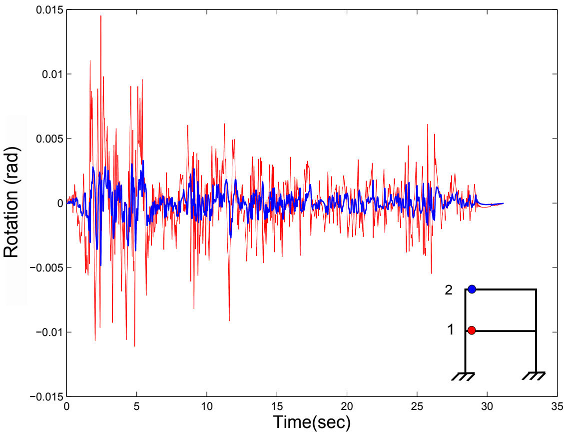

Figure 10: Rotation responses of the connections (1), (2) for $\ddot{q}$=0.2 g considering the nonlinear modes.

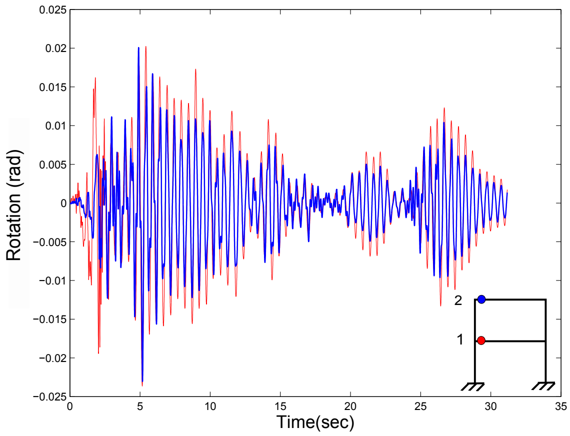

Figure 11: Rotation responses of the connections (1), (2) for $\ddot{q}$=0.36 g considering the nonlinear modes.

Moreover, the paper presents a contribution to the solution of nonlinear steel frame structures subjected to seismic excitation by the method of nonlinear modal synthesis. The introduction of the concept of nonlinear mode leads to the possibility of extending the most procedures of the method of linear modal synthesis to nonlinear cases, which involves considerable simplification in the context of the analysis of nonlinear steel frame structures.

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