## NEW MIXED AND PENALTY FINITE ELEMENT FORMULATIONS FOR STRAIN-GRADIENT MODELS

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**Keywords**: Finite Elements, Strain-gradient Models, Mixed Formulations, Penalty Formulations

Various finite elements based on mixed formulations [1, 2] and on penalty formulations [3] have been proposed for the solution of boundary value problems involving strain-gradient models. These formulations have been developed to address (real or perceived) shortcomings of the  $C^1$  formulations necessary for a displacement-only finite-element formulation for strain gradient models. The relevant literature, however, does not discuss in detail a number of important theoretical and implementation aspects.

In this work we first present the existing elements within a single mathematical framework, identifying some theoretical issues common to all of them, which affect their robustness and numerical efficiency. We then proceed to develop a new family of mixed and penalty elements that addresses these issues, while being simpler and computationally cheaper. We discuss also a number of implementation aspects that are important in order to improve the computational performance of the newly proposed elements. The new elements are then critically compared to some of the  $C^1$  elements proposed in the literature.

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## ACKNOWLEDGEMENT

This research was funded from the People Programme (Marie Curie Actions) of the European Union's Seventh Framework Programme under REA grant agreement n° 618096 (Generalised Continuum Models and Plasticity—GECOMPL). F. Gulib also acknowledges the support received from the School of Engineering of the University of Edinburgh.