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# IMPROVED CONVERGENCE OF FORWARD AND INVERSE SOFT TISSUE MODELS

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Soft tissues exhibit nonlinear and anisotropic mechanical behavior, which is modeled using exponential function in several constitutive models. In this talk, I will present methods that take into account this exponential nonlinearity to significantly improve the convergence of forward and inverse models.

For the forward model, I will start with a novel formulation that applies a transform on the discretized equations. That is, instead of solving the standard equations

$$R_i(\mathbf{x}) = f_i^{\text{int}}(\mathbf{x}) - f_i^{\text{ext}}(\mathbf{x}) = 0 \; \forall i = 1, \ldots, N, \tag{1}$$

we propose to solve

$$\mathcal{T}\left(f_i^{\text{int}}(\mathbf{x})\right) = \mathcal{T}\left(f_i^{\text{ext}}(\mathbf{x})\right) \; \forall i = 1, \ldots, N \tag{2}$$

for a pre-determined bijective transformation $\mathcal{T} : \mathbb{R} \to \mathbb{R}$. For an exponential nonlinearity, this transformation is its inverse, i.e. $\mathcal{T} \equiv \log$. This leads to a small modification in the residual vector, which can be implemented in any existing finite element solver and allows us to take $10 - 100$ times larger load steps. Thus, the forward problem is solved $10 - 100$ times faster [1].

For the inverse model, I will present two different scenarios – displacement controlled (DC) and force controlled (FC). In DC case, a displacement or strain is applied and force or stress is matched to calculate the elastic parameters. Inversely, in FC case, a force or stress is applied and displacement or strain is matched to calculated the elastic parameters. For the DC case, I will show that using a "log-norm" improves the convergence as well as sensitivity of the solution to data noise [2]. Whereas, for the FC case, I will show that a nonlinear parameter transformation with regular $L_2$ norm improves the convergence. The combination of these two techniques will allow us to calculate mechanical properties of soft tissues from clinical in-vivo imaging data in a fast, accurate, and reliable manner. I will demonstrate this with an application to heart valves.

## REFERENCES

[1] Yue Mei, Daniel E. Hurtado, Sanjay Pant, and Ankush Aggarwal. On improving the numerical convergence of highly nonlinear elasticity problems. *ArXiv e-prints*.

[2] Ankush Aggarwal. An improved parameter estimation and comparison for soft tissue constitutive models containing an exponential function. *Biomechanics and Modeling in Mechanobiology*, 16(4):1309–1327, Aug 2017.