## A fast multi-level method for aerodynamic uncertainty quantification due to surface imperfections

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Surface imperfections are inevitable even in the best manufacturing process. Maintaining strict manufacturing tolerance adds significantly to the production cost of components. Turbo-machinery performance is especially vulnerable to surface imperfections due to high sensitivity to the blade shape. The blades also undergo significant wear and tear over the lifetime of the machine. Therefore due consideration to surface imperfections has to be addressed early in the design.

There are two parts to the problem, (i) modelling the surface imperfections and (ii) estimating the change in aerodynamic performance due to the imperfections. In this work, we propose a fast approximate method to address the latter and use a synthetic disturbance model for the former. The proposed method approximates the uncertainty statistics (mean and standard deviation) using solutions from a hierarchy of coarsened meshes, the so-called Multi-Level Monte-Carlo method (MLMC). The method further achieves computational speedup using dimensionality reduction and approximate solution evaluation on coarse level meshes. The surface disturbance field is decomposed into the mean and perturbation modes using the principal component analysis (PCA). The PCA modes are truncated after ranking them based on their adjoint sensitivity to cost function (pressure loss, exit angle, etc). The random realisations of the surface imperfections are obtained using a linear combination of randomly perturbed truncated PCA modes.

We compute the flow and adjoint solution at each mesh level only for the mean surface. The cost-function is estimated for every sample in the random set using Taylor extrapolation augmented with the adjoint information. This requires deforming the volume mesh for every random surface realisation and one non-linear residual calculation on the perturbed volume mesh [1]. We deform the volume mesh using the fast inverse distance weighted interpolation method. The process is repeated for each mesh level until we reach convergence of mean and variance on the finest level. The number of function evaluations at each level is chosen using the optimality condition of MLMC for second-order finite volume schemes [2].

We demonstrate the method by quantifying the mean and variance of total-pressure loss due to surface imperfections on the VKI LS89 turbine cascade.

## References

[1] D. Ghate, *Inexpensive Uncertainty Analysis for CFD Applications*, PhD thesis, University of Oxford, 2014.

[2] H. Bijl, D. Lucor, S. Mishra, C. Schwab, Uncertainty Quantification in Computational Fluid Dynamics, Lecture Notes in Computational Science and Engineering, Vol. 92, 2013.