A couple-stress formulation for the isogeometric boundary element method (IGABEM)

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Size effects have a great influence in heterogeneous materials such as composites, ceramics and bone tissues, to mention some examples. However, the classical approach in continuum mechanics does not take size effects into account, which can become problematic when the scale of the deformation field is comparable to the length scale of the microstructure [1].

To overcome this limitation, a generalised continuum approach based on the theory of couple-stress (or constrained Cosserat) elasticity [2, 3] is used in this work. This theory can be considered as a first step through the generalised classical elasticity theory where the constitutive equations and the strain-energy density depend on the strain and the gradient of rotation. In the couple-stress theory, the characteristic material length scale is introduced in the constitutive formulation, allowing for considering the size effects in the formulation.

In this work, we implement an isogeometric boundary element method (IGABEM) for plane-strain problems in isotropic Cosserat materials. We derived the Green's functions for a concentrated force and moment in closed form. A new boundary integral equation arises since the rotation and the displacements are independent on the boundary of the body. To validate our formulation, the benchmark problem of plate with an inclusion is analysed and the effects of the Cosserat lengths upon the macroscopic response of the material are investigated.

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