6th European Conference on Computational Mechanics (ECCM 6)
7th European Conference on Computational Fluid Dynamics (ECFD 7)
11-15 June 2018, Glasgow, UK

EFFICIENT DISCONTINUOUS GALERKIN METHODS FOR UNDER-RESOLVED TURBULENT INCOMRESSIBLE FLOWS

Niklas Fehn[1], Wolgang A. Wall[1] and Martin Kronbichler[1]

[1] Institute for Computational Mechanics, Technical University of Munich,
Boltzmannstr. 15, 85748 Garching, Germany,
{fehn,wall,kronbichler}@lnm.mw.tum.de, http://www.lnm.mw.tum.de

Keywords: *incompressible Navier-Stokes, discontinuous Galerkin methods, large-eddy simulation, matrix-free implementation, high-performance computing*

We present robust and efficient high-order discontinuous Galerkin methods for the incompressible Navier-Stokes equations based on a high-performance, matrix-free implementation. Our discontinuous Galerkin (L_2-conforming) discretization is based on the local Lax-Friedrichs flux for the convective term, the symmetric interior penalty method for the viscous term, and central fluxes for velocity-pressure coupling terms and uses mixed-order polynomials for velocity and pressure [1, 2]. This basic DG discretization, however, is not sufficiently robust for under-resolved, turbulent flows. To stabilize the DG discretization in the under-resolved regime our approach is based on consistent penalty terms enforcing the incompressibility constraint as well as interelement continuity of the velocity field in a weak sense [1, 3]. This discretization approach has been validated for several benchmark problems such as the Orr–Sommerfeld stability problem, the 3D Taylor–Green vortex problem, and turbulent channel flow [3]. For efficient time integration, our approach is based on projection methods with a mixed implicit/explicit formulation of viscous and convective terms, respectively. For high-order methods to be more efficient than low-order methods, an efficient matrix-free implementation exploiting sum-factorization techniques on quadrilateral/hexahedral elements is inevitable. Our marix-free implementation is highly optimized for modern, cache-based computer architectures and uses vectorization over several elements to fully exploit the SIMD capabilities of modern CPU hardware [4, 5]. Outstanding performance characteristics of our implementation with a throughput of up to $3.5 \cdot 10^7$ DoFs/sec/core for matrix-free operator evaluation are presented. The solution of linear systems of equations is based on state-of-the-art iterative methods using efficient matrix-free preconditioners and geometric multigrid smoothers that exhibit optimal computational complexity. By the example of the 3D Taylor-Green vortex problem, a detailed analysis of the efficiency of high-order methods for under-resolved turbulent flows is presented. Compared to performance results published within the last 5 years for high-order DG discretizations of the compressible Navier-Stokes equations [6], our approach achieves an improvement in computational costs by a factor of 10–100 for the same effective spatial resolution and Reynolds number.

REFERENCES

[1] B. Krank, N. Fehn, W.A. Wall, and M. Kronbichler, A high-order semi-explicit discontinuous Galerkin solver for 3D incompressible flow with application to DNS and LES of turbulent channel flow, Journal of Computational Physics, Vol. 348, pp. 634–659, 2017.

[2] N. Fehn, W.A. Wall, and M. Kronbichler, On the stability of projection methods for the incompressible Navier–Stokes equations based on high-order discontinuous Galerkin discretizations, Journal of Computational Physics, Vol. 351, pp. 392–421, 2017.

[3] N. Fehn, W.A. Wall, and M. Kronbichler, Robust and efficient discontinuous Galerkin methods for under-resolved turbulent incompressible flows, arXiv preprint arXiv:1801.08103, 2018.

[4] M. Kronbichler, K. Kormann, I. Pasichnyk, and M. Allalen, Fast Matrix-Free Discontinuous Galerkin Kernels on Modern Computer Architectures, High Performance Computing, Springer International Publishing, Cham, pp. 237–255, 2017.

[5] M. Kronbichler and K. Kormann, Fast matrix-free evaluation of discontinuous Galerkin finite element operators, arXiv preprint arXiv:1711.03590, 2017

[6] Z. J. Wang et al., High-order CFD Methods: Current Status and Perspectives, International Journal for Numerical Methods in Fluids, Vol. 72, pp. 811–845, 2013