An entropy stable 3D curvilinear discontinuous Galerkin spectral element method for the resistive MHD equations

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The resistive magnetohydrodynamic (MHD) equations are a mixed type hyperbolic/parabolic system of partial differential equations (PDEs) useful to describe and predict the temporal evolution of compressible plasmas. Their applications are far reaching including modeling turbulence in the interstellar medium (ISM) or toroidal plasma fusion. As such, the plasmas exhibit a wide range of spatial and temporal scales. To overcome resolution issues, the nodal discontinuous Galerkin spectral element method (DGSEM) offers a high-order, highly parallelizable, three dimensional framework based on the collocation of interpolation and quadrature nodes to retain tensor product curvilinear hexahedral elements.

Additionally, numerical methods that discretely satisfy the second law of thermodynamics, so-called *entropy stable* schemes, are of interest as they ensure the numerics reflects the relevant physics. Entropy stable methods also offer increased robustness, particularly for under-resolved computations like turbulence. The nodal DGSEM can be an entropy stable numerical scheme provided:

- The derivative matrix satisfies the summations-by-parts (SBP) property.
- There exist two-point entropy stable finite volume flux functions that can be extended to high-order in a split form DG framework.

For MHD models (either ideal or resistive) discrete satisfaction of the divergence-free constraint of the magnetic field is critical to construct an entropy stable approximation. In fact, it is necessary to explicitly build the divergence-free constraint into the system of PDEs as a non-conservative term.

In this talk we present details of the continuous as well as discrete entropy analysis of the DGSEM applied to the resistive MHD equations where special attention is paid to the discretization of the non-conservative terms. To further remove errors in the divergence-free constraint we extend the MHD model to include a built-in generalized Lagrange multiplier (GLM) divergence cleaning mechanism that *does not* alter the provably entropy stable properties of the scheme. We then provide numerical verification of the theoretical properties as well as the increased robustness of the derived method compared to standard high-order DG methods. Finally, we introduce the open-source code FLUXO (github.com/project-fluxo/fluxo), jointly developed by F. Hindenlang at IPP Garching, the groups of G. Gassner at University of Cologne and of C-D. Munz at the University of Stuttgart.