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A MULTILEVEL MONTE CARLO APPROACH WITH AN EMBEDDED ERROR ESTIMATOR FOR COMPUTATIONAL FLUID DYNAMICS APPLICATIONS

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The characterization and propagation of uncertainty in numerical simulations is of a paramount importance for Computational Fluid Dynamics (CFD) since, in many cases, the exact determination of the fluid properties and the operating conditions is impossible or impractical. Uncertainty Quantification (UQ) analysis provides tools to propagate the effect of uncertainty and generally relies on the realization of a certain number of deterministic simulations. On the other side, the accurate resolution of spatial and temporal scales within CFD can be extremely expensive, forbidding in most cases the possibility to perform more than a handful of realizations. Therefore, in the presence of high-dimensional input spaces, the overall UQ computational cost is prohibitive and the combination of less expensive solutions with reduced resolutions might help to alleviate the computational burden. In this direction, Multilevel Monte Carlo (MLMC) methods emerged as an effective way of accelerating the propagation of uncertainty of a standard Monte Carlo method [1]. The main idea is to redistribute the computational cost, i.e. the number of simulations to be performed, across several resolution levels according to the variance decay properties of the problem. The final result is an optimal sample profile which allocates the minimum number of simulations required on each level in order to match a target accuracy. Usually, in literature, this target accuracy is the Mean Square Error (MSE) which is the sum of the estimator variance and its bias. In this contribution, we aim to balance this two terms by means of an embedded Variational Multiscale estimators (VMS) approach which enables an on-line computation of the bias on each discretization level. The VMS method has been widely used as a stabilization method for CFD simulations using a Finite Element framework. Moreover, a posteriori error estimators based on the VMS approach have been defined for a variety of problems, including the Convection-Diffusion-Reaction (CDR) and Navier-Stokes (NS) equations [2]. A first study on the use of VMS error estimators to characterize the discretization error of simple QoIs in a MLMC framework will be performed for the CDR problem. The extension of this *a posteriori* error estimator for more complex (non-linear) QoIs and more complex problems, namely the NS problem, will be also considered.

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