

## Towards robust approaches for Large Eddy Simulations in high order discontinuous Galerkin methods

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**Abstract.** We analyse the numerical dissipation introduced by three strategies typically considered for implicit LES in high order discontinuous Galerkin methods: upwind Riemann solvers, viscous discretizations for second order terms (BR1[1] and Interior Penalty (as in [2]), and spectral vanishing viscosity (SVV). First, we analyze these methods using a linear von Neumann analysis (for linear advection equations [3]) to characterize their properties in wavenumber space [4]. Second, we validate these observations using the 3D Taylor-Green vortex (TGV) Navier-Stokes problem with transitional/turbulent flow.

We show that the dissipation introduced by upwind Riemann solvers affects high wavenumbers and that it is neither linear nor monotonic. This dissipation may be increased until a critical high value to then decrease as the discretization tends to that of a conforming (i.e. continuous Galerkin) method.

Regarding the dissipation introduced by the discretisation of elliptic terms (second order derivatives), we show that this dissipation acts at low and medium wavenumbers (lower to the region affected by the Riemman solvers, see Figure 1, left).

Finally, we study the dissipation introduced by SVV operators on discontinuous Galerkin methods. We found that with an appropriate kernel (that selects the modes where artificial viscosity is applied) it is possible to control the dissipation introduced in the low and medium wavenumber region.

Combining these ideas, we propose a new SVV approach that uses a Smagorinsky based viscosity to compute the appropriate numerical viscosity (which varies with the wavenumber). When this viscous kernel is combined with a low dissipation Riemann solver [5,6], the method is capable of maintaining low dissipation levels in laminar flows, whilst modelling small eddies and providing the correct dissipation at all wavenumber ranges, see Figure 1, right.

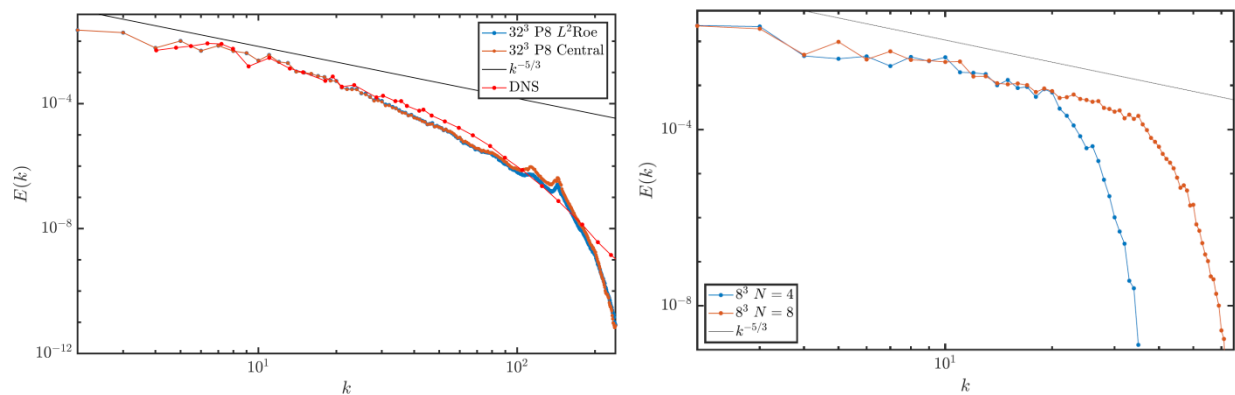


Figure 1: Kinetic energy spectra at  $t = 8$  for implicit LES using DG with skew-symmetric form: (left) non-linear fluxes (Central and Roe variants) for polynomial order  $N=8$  and (right) Smagorinsky-SVV kernel for  $N=4$  and 8. All cases are computed using an 83 Cartesian mesh.

## REFERENCES

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