

AN ENERGY-CONSERVING ISOGEOMETRIC SPACE-TIME METHOD FOR THE WAVE EQUATION

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We present a novel isogeometric discretization method for the linear wave equation based on a space-time formulation. The computational domain is represented by a standard isogeometric geometry map, and a discretization space for the entire space-time cylinder built upon this domain is constructed by means of tensor product spline spaces. We show how the resulting space-time stiffness matrix, based on the novel space-time variational formulation, can be efficiently computed by exploiting the tensor product structure of the involved spaces.

By a judicious choice of the test function in the variational formulation, we show that we can guarantee the exact conservation of the wave energy at the final time. Here we take advantage of the particular properties of Isogeometric Analysis which make it very easy to construct test and trial spaces which are C^1 in time, and to prescribe both the initial values and the initial derivatives of the solution in an essential way by eliminating certain degrees of freedom.

In several examples, we illustrate the convergence behavior of the new method. We demonstrate proper combinations of spline spaces for the space and time components of the discretization which lead to optimal convergence of the method with respect to the approximation order of the space discretization.

We also present a time decomposition scheme where the space-time cylinder is split into a number of time “slabs,” each of which is discretized by the proposed novel method. This leads to exact energy conservation at each time break point and enables a possible interpretation of the method as a time-stepping scheme.