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HIGH ORDER COMPACT SCHEMES FOR FLUID FLOWS

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We consider the Navier-Stokes equations in irregular domains. The Navier-Stokes equation may be written in stream-function formulation as follows (see [1]).

$$\begin{cases} \partial_t \Delta\psi + \nabla^{\perp}\psi \cdot \nabla\Delta\psi - \nu\Delta^2\psi = f(x,y,t), \\ \\ \psi(x,y,t) = \psi_0(x,y). \end{cases} \tag{1}$$

Recall that $\nabla^{\perp}\psi = (-\partial_y\psi, \partial_x\psi)$ is the velocity vector. The no-slip boundary condition associated with this formulation is $\psi = \frac{\partial\psi}{\partial n} = 0 \;\; , \;\; (x,y) \in \partial\Omega \;\; , \;\; t > 0$ and the initial condition is $\psi(x,y,0) = \psi_0(x,y), \;\; (x,y) \in \Omega$.

In [1] we have introduced a high-order compact scheme for the Navier-Stokes equation in regular domains. We have tested the scheme for analytic solutions of the problem and for the driven cavity problem. We found out that for high Reynolds number the solution the solution does not converge to a steady state but becomes rather periodic in time.

Here we extend the fourth-order scheme [2] to irregular domains. The strategy used here is to present the biharmonic operator $\partial_x^4 + 2\partial_x^2\partial_y^2 + \partial_y^4$ as a combination of pure fourth-order derivatives in the x, y and the diagonal directions $\eta = (x+y)/\sqrt{2}, \;\; \xi = (y-x)\sqrt{2}$. Then, the pure fourth-order derivatives may be approximated via a compact scheme using the values of the function and its directional derivatives.

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