A discontinuous skeletal method for Bingham fluids

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This work is motivated by the growing interest in the simulation of yield stress fluids for civil engineering materials, blood, foams, etc. To this aim, we propose a Discontinuous Skeletal (DiSk) method for the antiplane Bingham model, inspired by the Hybrid-High Order method introduced in [1] for linear elasticity. In particular, we focus on the lowest order case, where discrete velocity unknowns are constant polynomials: one per cell and one per face, and the cells unknowns are eliminated by static condensation. The main advantages are local conservativity and the possibility to use general meshes. We consider the Augmented Lagrangian method to solve the variational inequalities resulting from the discrete Bingham problem. We introduce constant Lagrange multipliers for the velocity gradient in each cell and for its jumps at each face. In comparison to Finite Element Methods, such as the use of Taylor-Hood elements [2], a crucial advantage of DiSk methods is that polytopal meshes are supported. We can exploit their use in performing local mesh adaptation, either locally refining around liquid-solid interfaces or coarsening in the solid regions. Numerical results are presented for circular and square domains and for different Bingham numbers. We show local adaptation can be exploited and the method is shown to capture regions of sharp transition between solid- and fluid-like regimes.

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