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Applying Algebraic Multigrid Methods to Dynamic, Enriched, Conformal Decomposition Finite Element Methods for Multi-material Transport

David R. Noble[1] and Raymond S. Tuminaro[2]

[1] Sandia National Laboratories*, Fluid and Reactive Processes, drnoble@sandia.gov
[2] Sandia National Laboratories*, Computational Mathematics

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Enriched finite element methods, including the Conformal Decomposition Finite Element Method (CDFEM) provide powerful tools for multi-phase and multi-material transport problems. The methods provide discretizations that dynamically adapt to the moving phases or materials to accurate capture interfacial physics and discontinuities. The elements that are crossed by these interfaces are enriched to describe these discontinuities. Additional unknowns are assigned to one or more of the mesh entities (elements, nodes, sides, or edges) that are associated with these interfacial elements, and additional equations are formulated for these unknowns.

CDFEM is an enriched finite element method that can describe arbitrarily discontinuous physics across dynamic interfaces. A level set is used to describe the location of the moving interface. Nodes are added at the intersection of the level set surface with the edges of the input mesh, and a conforming mesh is generated automatically. Standard unstructured mesh data structures are generated for the resulting conformal mesh in terms of element blocks and side sets. This general framework allows the physics code to describe either weak or strong discontinuities across the interface using standard finite element methods.

Algebraic multigrid (AMG) has been shown to be an efficient and scalable approach for solving finite element equations. The application of AMG to CDFEM for multi-material transport is complicated by two issues. As an interface comes arbitrarily close to the background nodes, the equations for the added degrees-of-freedom become linearly dependent on the existing degrees-of-freedom. In addition, interfaces with strong discontinuities use multiple degrees-of-freedom at the interface to describe the discontinuities. In this way, the number of degrees-of-freedom at nodes on the interface is greater than the rest of the domain.

To tackle these problems with multigrid, we consider an approach that applies AMG to a scalar problem to generate initial grid transfer operators. This scalar problem corresponds to a distance Laplacian that is constructed automatically using coordinate information, the matrix, and a Boolean table indicating which degrees-of-freedom are defined at each nodal location. The grid transfers for this Laplace problem are then used to build interpolation/restriction operators for the original incompressible flow application.

Numerical results for thermal transport between materials in imperfect contact and multi-phase flow with surface tension will be presented. These results demonstrate both the accuracy of the proposed methods and the linear solver convergence rates.

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