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Maximum Principle and Realizability of the Intrusive Polynomial Moment scheme

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Model parameters as well as initial conditions of computational models are often uncertain. In order to understand the influence of these uncertainties, methods of uncertainty quantification need to be employed. Using standard intrusive techniques when solving hyperbolic conservation laws with uncertainties can lead to oscillatory solutions as well as nonhyperbolic moment systems.

The Intrusive Polynomial Moment (IPM) method is a minimum-entropy closure of the moments system, which ensures hyperbolicity while restricting oscillatory over- and undershoots of specified bounds. Minimal entropy methods often suffer from realizability problems, meaning that the solution leaves the admissible bounds.

In this talk, we derive a realizability-preserving, second-order discretization of the IPM moment system which fulfills the maximum principle. This task is carried out by investigating violations of the specified bounds due to the errors from the numerical optimization required by the scheme. This analysis gives weaker conditions on the entropy that is used, allowing the choice of an entropy which enables choosing the exact minimal and maximal value of the initial condition as bounds. Solutions calculated with the derived scheme are nonoscillatory while fulfilling the maximum principle. The second-order accuracy of our scheme leads to significantly reduced numerical costs.

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