6th European Conference on Computational Mechanics (ECCM 6)
7th European Conference on Computational Fluid Dynamics (ECFD 7)
11 15 June 2018, Glasgow, UK

A NEW PROCEDURE FOR SOLVING PDES ON TRIMMED SURFACES AND ITS APPLICATION TO SHAPE OPTIMIZATION

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Key words: *Isogeometric Analysis, Trimming, Parametric shape optimization, Sparse-grid surrogate model*

This talk is divided in two parts. In the first one, we outline the details of a new procedure that allows us to solve effectively PDEs on trimmed geometries by isogeometric Galerkin solvers. Roughly speaking, we identify all Bézier elements cut by trimming curves/surfaces and tessellate them. This tessellation is performed either with a high-order Finite-Element-suitable triangulation of every cut element or by subdividing them into spline patches (tiles), that together are geometrically identical to the trimmed Bézier elements. We then use standard quadrature formulas on such reparametrization to evaluate the integrals of the (bi)linear forms, while the un-cut elements are instead treated as usual with a Gaussian tensorized quadrature rule. The resulting linear system is then solved in a standard way.

After having briefly discussed the validity of this approach, we turn our attention to some examples of parameteric shape-optimization that this new procedure allows to treat efficiently. In particular, we consider the problem of minimizing the maximum stress in a plate by choosing optimally the centers and the radii of a given number of circular holes. To this end, we consider a surrogate-model approach, which is composed of three steps. First, we solve the equation at hand (e.g., linear elasticity) for a number of carefully selected design-points (i.e., by fixing the radii and the locations of the holes at some prescribed values). Second, we compute a function that interpolates the multi-variate function that associates design-points (interpreted as coordinates in a parametric space) to stress values. Finally, we apply a minimization algorithm to the interpolating function. The underlying idea that the number of PDEs to be solved to build the surrogate model will be significantly smaller than the number of PDEs that one would have to solve if using the PDE solver at each step of the optimization phase (i.e., a PDE solve for each new candidate design-point). Among the possible strategies for design-point selection and interpolation, we elect to work with sparse grids, which are seen to be quite effective in delivering good approximations with a moderate number of design-points.