FOURIER ANALYSIS OF HIGH-ORDER METHODS IN THE CONTEXT OF LES

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Large eddy simulation (LES) has been increasingly used to tackle vortex-dominated turbulent flows. In LES, the quality of the simulation results hinges upon the quality of the numerical discretizations in both space and time. It is in this context we perform a Fourier analysis of several popular methods in LES including the discontinuous Galerkin (DG), finite difference (FD), and compact difference (CD) methods. We begin by reviewing the semi-discrete versions of all methods under-consideration, followed by a fully-discrete analysis with explicit Runge-Kutta (RK) time integration schemes. In this regard, we are able to unravel the true dispersion/dissipation behavior of DG and Runge-Kutta DG (RKDG) schemes for the entire wavenumber range. The physical-mode is verified to be a good approximation for the asymptotic behavior of these DG schemes in the low wavenumber range. After that, we proceed to compare the DG, FD, and CD methods in dispersion and dissipation properties. Numerical tests are conducted using the linear advection equation to verify the analysis. In comparing different methods, it is found that the overall numerical dissipation strongly depends on the time step. Compact difference (CD) and central finite difference (FD) schemes, in some particular settings, can have more numerical dissipation than the DG scheme with an upwind flux. This claim is then verified through a numerical test using the Burgers' equation.

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